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The Model-Matching Error and Optimal Solution in Locally Convex Space

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Abstract

The model-matching error and the optimal solution in the Hardy space are extended to the locally convex space, and the model-matching error and the optimal solution in the locally convex space are achieved. Thereby the ordinary H_∞-control theory is extended to with range in locally convex spaces through a form of a parameter vector. The algorithms of computing the infimal model-matching error and the infimal controller are presented.

Keywords: Locally convex space, Inner-outer function, Minimal realization, Infinal model- matching error

1. INTRODUCTION

Assume that R is the real field and R^n is the Cartesian product of n copies of R, here n is any positive integer, and that C is a complex plane.

To solve the problem for simplicity, we apply the $G(s)$ in the model matching problem to $G(s,\xi)$,where s in C, ξ in R^n,and $G(s,\xi)$ is in $C^\infty(R^n)$ (locally convex space) for each fixed s in C and in H_∞ for each fixed ξ in R^n. First, we extend several concepts.

Definition 1 The locally convex space VH_∞ consists of all complex-valued parameter functions $F(s,\xi)$ of a complex variable s and a parameter ξ which are analytic and bounded about s in $\mathrm{Re}\, s>0$ (for each fixed ξ in R^n). Similarly, we define the VH_∞-norm of $F(s,\xi)$ is

$$|||F|||_\infty = \sum_{k=1}^{\infty} \frac{g_k}{2^k(1+g_k)},$$

where $g_k = \sup_{-k<\xi<k} \|F(\bullet,\xi)\|_\infty$

Definition 2 The subset of VH_∞ consists of all real-rational functions of s and ξ,will be denoted by VRH_∞.

Definition 3 Let α denote the infimal model-matching error

$$\alpha = \inf\{|||T_1 - T_2QT_3|||_\infty : Q \in VRH_\infty\}. \tag{1}$$

A matrix Q in VRH_∞ satisfying $\alpha = |||T_1 - T_2QT_3|||_\infty$ will be called optimal,where α is a model-matching error.

When $T_i(s,\xi)$ are scalar-valued,then there is no need for both $T_2(s,\xi)$ and $T_3(s,\xi)$.So we may as well suppose $T_3(s,\xi)=1$.It is also assumed that $T_2^{-1}(s,\xi) \in VRH_\infty$ to avoid the trivial instance of the problem.

Returning to the model-matching problem, bring in an inner-outer factorization of $T_2(s,\xi): T_2(s,\xi) = T_{2i}(s,\xi)T_{2o}(s,\xi)$,

we have

$$|||T_1 - T_2Q|||_\infty = |||R - X|||_\infty. \tag{2}$$

We conclude that

$$\alpha = \inf\{\|| R - X \||_\infty : X \in VRH_\infty\} = dist(R, VRH_\infty). \tag{3}$$

Definition 4 The VL_p space, $1 \le p < \infty$,will be viewed as p th power integrable functions about s and ξ.When $p = \infty$,VL_∞ is the space of essentially bounded functions(for any fixed ξ in R^n).

Definition 5 The VRL_p space, VRL_p,will be viewed as a subset of VL_p,which consists of all real-rational functions of s and ξ.

Definition 6

(i) Let $F(s,\xi) \in VL_\infty$ and $g(s,\xi) \in VL_2$.Then the operator

$$\Lambda_{F(s,\xi)} : \Lambda_{F(s,\xi)} g(s,\xi) = F(s,\xi) g(s,\xi)$$

is called the Laurent operator.

(ii) A related operator is $\Lambda_{F(s,\xi)} \mid VH_2$,the restriction of

$\Lambda_{F(s,\xi)}$ to VH_2,which maps VH_2 to VL_2,where $F(s,\xi) \in VL_\infty$.

(iii) For t $F(s,\xi) \in VL_\infty$,the Hankel operator with symbol $F(s,\xi)$,denoted by $\Gamma_{F(s,\xi)}$, maps VH_2 to $VH_2^{\perp}$ and is defined as

$$\Gamma_{F(s,\xi)} = \Pi_1 \Lambda_{F(s,\xi)} \mid VH_2,$$

where $VL_2 = VH_2 \oplus VH_2^{\perp}$,and Π_1 is the projection from VL_2 onto $VH_2^{\perp}$.

Definition 7 We call $F(s,\xi)$ to be strong proper if $F(s,\xi) \in VRH_\infty$ and $\sup_{\xi \in R^n} |F(\bullet,\xi| < \infty$,strictly strong proper if $F(\infty,\xi) \equiv 0$.

Definition 8 We call $F(s,\xi)$ to be stable if $F(s,\xi) \in VRH_\infty$ and $F(s,\xi)$ has no poles in the closed right half-plane $\mathrm{Re}\, s \ge 0$ (for each fixed ξ in R^n).

If $F(s,\xi)$ is real-rational about s in $\mathrm{Re}\, s > 0$, then $F(s,\xi) \in VRH_\infty$ if and only if F is strong proper and stable (for each fixed ξ in R^n). Similarly, we define

$$G(s,\xi) = \begin{bmatrix} T_1(s,\xi) & T_2(s,\xi) \\ T_2(s,\xi & 0 \end{bmatrix}, K(s,\xi) = -Q(s,\xi),$$

then the model-matching problem is

$\|| T_1 - T_2 Q T_3 \||= \min imum$,

where $T_i (i = 1,2,3) \in VRH_\infty$.The constraint that K stabilizes G is equivalent to that $Q \in VRH_\infty$.

We shall give in the form of parameter valued case the algorithms of computing the model-matching error α and the optimal controller Q.

2. The minimal Realization

Definition 9 The linear time invarient system S_1 defined by

$$\dot{x}(t,\xi) = A(\xi) x(t,\xi) + B(\xi) u(t,\xi) \tag{4}$$

$$y(t,\xi) = C(\xi) x(t,\xi) \tag{5}$$

Where $A(\xi)$ is $n \times n$, $B(\xi)$ is $n \times m$,and $C(\xi)$ is $r \times n$ constant matrix depending on ξ,is said to be completely controllable if the $n \times mn$ controllability matrix

$$U(\xi) = [B(\xi), A(\xi)B(\xi), ..., A^{n-1}(\xi)B(\xi)] \tag{6}$$

has rank n ,denoted by $(A(\xi), B(\xi))$.

Definition 10 The system S_1 described by (1) and (2) is completely observable if the observability matrix

$$V^T(\xi)=[C(\xi),C(\xi)A(\xi),...,C(\xi)A^{n-1}(\xi)]^T \tag{7}$$

has rank n ,denoted by $(A(\xi),C(\xi))$.

Definition 11 Given an $r\times m$ matrix $G(s,\xi)$ whose elements are rational functions of s ,we wish to find matrices $A(\xi),B(\xi)$ and $C(\xi)$ depending on ξ ,having dimensions $n\times n, n\times m$ and $r\times n$ respectively, such that

$$G(s,\xi)=C(\xi)(sI_n-A(\xi))^{-1}B(\xi)\ (5) \tag{8}$$

where I_n is the unit matrix of order n.

$[A(\xi),B(\xi),C(\xi),0]$ is termed a realization of $G(s,\xi)$ of order n ,and is not, of course, unique. All such the above realizations will include matrices $G(s,\xi)$ having the least dimensions-be called the minimal realizations.

Definition 12 The Laplace transform of parameter-valued function $f(s,\xi)$ is defined by

$$F(s,\xi)=\int_0^\infty f(t,\xi)e^{-st}dt=Lf(t,\xi) \tag{9}$$

and the inverse Laplace transform of $F(s,\xi)$ is

$$f(t,\xi)=\int_{\sigma-j\infty}^{\sigma+j\infty}F(s,\xi)e^{st}ds=L^{-1}F(s,\xi) \tag{10}$$

we take the Laplace transform of (9) with zero initial conditions, we have

$$s\hat{x}(s,\xi)=A(\xi)\hat{x}(s,\xi)+B(\xi)\hat{u}(s,\xi)$$

and after rearrangement

$$\hat{x}(s,\xi)=(sI_n-A(\xi))^{-1}B(\xi)\hat{u}(s,\xi) \tag{11}$$

Since from (10) the Laplace transform of the output is

$$\hat{y}(s,\xi)=C(\xi)\hat{x}(s,\xi) \tag{12}$$

clearly

$$\hat{y}(s,\xi)=C(\xi)(sI_n-A(\xi))^{-1}B(\xi)\hat{u}(s,\xi)=G(s,\xi)\hat{u}(s,\xi) \tag{13}$$

where the $r\times m$ matrix

$$G(s,\xi)=C(\xi)(sI_n-A(\xi))^{-1}B(\xi) \tag{14}$$

Suppose $R(S,\xi)=[r_{ij}(s,\xi)]$ is an $p\times m$ strictly proper rationalfraction matrix of s (for any fixed ξ in R^n).

Theorem 1 A realization $[A(\xi),B(\xi),C(\xi),0]$ of a given transfer matrix $G(s,\xi)$ is minimal if $(A(\xi),B(\xi))$ is $c.c.$ and $(A(\xi),C(\xi))$ $c.o.$

Proof Let $U(\xi)$ and $V(\xi)$ be the controllability and observability matrices in (5) and (6) respectively. We wish to show that if these both have rank n then $R(s,\xi)$ has

least order n.

Suppose that there exists a realization $\{\overline{A}(\xi),\overline{B}(\xi),\overline{C}(\xi)\}$

of $R(s,\xi)$,with $\overline{A}(\xi)$ having order n_1. Since

$$C(\xi)(sI_m-A(\xi))^{-1}B(\xi)=\overline{C}(\xi)(sI_m-\overline{A}(\xi))^{-1}\overline{B}(\xi),$$

It follows that

$$C(\xi)e^{A(\xi)t}B(\xi)=\overline{C}(\xi)e^{\overline{A}(\xi)t}\overline{B}(\xi),$$

Which implies, using the series

$(e^{A(\xi)t}=I+tA(\xi)+\frac{t^2}{2}A^2(\xi)+...),$ that

$$C(\xi)A^i(\xi)B(\xi)=\overline{C}(\xi)\overline{A}^i(\xi)\overline{B}(\xi) \qquad i=0,1,2,...$$

Consider the product

$$V(\xi)U(\xi) = \begin{bmatrix} C(\xi) \\ C(\xi)A(\xi) \\ . \\ . \\ C(\xi)A^{n-1}(\xi) \end{bmatrix} \left[B(\xi), A(\xi)B(\xi), ..., A^{n-1}(\xi)B(\xi)\right]$$

$$= \begin{bmatrix} C(\xi)B(\xi) & C(\xi)A^{n-1}(\xi)B(\xi) \\ . & . \\ C(\xi)A^{n-1}(\xi) & C(\xi)A^{2n-2}(\xi)B(\xi) \end{bmatrix}$$

$$= \left[\overline{B}(\xi), \overline{A}(\xi)\overline{B}(\xi), ..., \overline{A}^{n-1}(\xi)\overline{B}(\xi)\right] = V_1(\xi)U_1(\xi).$$

By assumping, $V(\xi)$ and $U(\xi)$ both have rank n, so the matrix $V_1(\xi)U_1(\xi)$ also have rank n. However, the dimension of $V_1(\xi)$ and $U_1(\xi)$ are respectively $r_1 n \times n_1$ and $n_1 \times m_1 n$, where r_1 and m_1 are positive integers, so that the rank of ix $V_1(\xi)U_1(\xi)$ can not be greater than n_1. That is, $n < n_1$, so there can be no realization of $G(s,\xi)$ having order less than n.

3. INFIMAL MODEL-MATCHING ERROR

The Lyapunov equations are

$$A(\xi)L_c(\xi) + L_o(\xi)A^{\mathrm{T}}(\xi) = B(\xi)B^{\mathrm{T}}(\xi) \tag{15}$$

$$A^{\mathrm{T}}(\xi)L_o(\xi) + L_o(\xi)A(\xi) = C^{\mathrm{T}}(\xi)C(\xi) \tag{16}$$

Define the two controllability and observability gramians:

$$L_c(\xi) = \int_0^\infty e^{-A(\xi)t}B(\xi)B^{\mathrm{T}}(\xi)e^{-A^{\mathrm{T}}(\xi)t}dt,$$

$$L_o(\xi) = \int_0^\infty e^{-A^{\mathrm{T}}(\xi)t}C^{\mathrm{T}}(\xi)C(\xi)e^{-A(\xi)t}dt.$$

Theorem 2 $L_c(\xi)$ and $L_o(\xi)$ are the unique solutions of (12) and (13) respectively.

Proof Using the definition we have $A(\xi)L_c(\xi) + L_c(\xi)A^{\mathrm{T}}(\xi)$

$$= \int_0^\infty \begin{matrix} (A(\xi)e^{-A(\xi)t}B(\xi)B^{\mathrm{T}}(\xi)e^{-A^{\mathrm{T}}(\xi)t} + \\ e^{-A(\xi)t}B(\xi)B^{\mathrm{T}}(\xi)e^{-A^{\mathrm{T}}(\xi)}A^{\mathrm{T}}(\xi))dt \end{matrix} = B(\xi)B^{\mathrm{T}}(\xi) - \lim_{t\leftarrow\infty}(e^{-A(\xi)t}B(\xi)B^{\mathrm{T}}(\xi)e^{-A^{\mathrm{T}}(\xi)t}).$$

Since $A(\xi)$ is instable,

$$\lim_{t\to\infty}(e^{-A(\xi)t}B(\xi)B^{\mathrm{T}}(\xi)e^{-A^{\mathrm{T}}(\xi)t}) = 0.$$

So $L_c(\xi)$ are the unique solutions of (12).From the discussion above, the uniqueness is obvious.

$L_o(\xi)$ are the unique solutions of (13) follows similarly.

Q.E.D.

Definition 13 Suppose the linear operator

$$T : X \to Y,$$

it's the unique operator

$$T^* : Y^* \to X^*,$$

Satisfying

$$(T^* y^*, x) = (y^*, Tx), x \in X^*, y \in T^*,$$

T^* is called the adjoint of T.

Define

$$\begin{aligned} f(s,\xi) &= [A(\xi), \omega(\xi), C(\xi), 0], \\ g(s,\xi) &= [-A^{\mathrm{T}}(\xi), \lambda^{-1}(\xi)L_0(\xi)\omega(\xi), B^{\mathrm{T}}(\xi), 0], \end{aligned} \tag{17}$$

and

$$X(s,\xi) = R(s,\xi) - \lambda(\xi) f(s,\xi) / g(s,\xi). \tag{18}$$

So

$$f(s,\xi) = C(\xi)(sI - A(\xi))^{-1}\omega(\xi) \in VRH_2{}^{\perp},$$

and

$g(s,\xi) = B^{\mathrm{T}}(\xi)(sI + A^{\mathrm{T}}(\xi))^{-1}\lambda^{-1}(\xi)L_o(\xi)\omega(\xi) \in VRH_2$.

Theorem 3 [4] There exists a closest VRH_∞ -function $X(s,\xi)$ to a given VRL_∞ -function $R(s,\xi)$, and $\| R - X \| = \| \Gamma_R \|$.

Factor $R(s,\xi)$ as

$$R(s,\xi) = R_1(s,\xi) + R_2(s,\xi)$$

With $R_2(s,\xi)$ strictly proper and analytic in $\mathrm{Re}\, s < 0$ and $R_2(s,\xi)$ in VRH_∞. Then $R_1(s,\xi)$ has a minimal state-space realization

$$R_1(s,\xi) = [A(\xi), B(\xi), C(\xi), 0]$$

Define

$$L_c(\xi) = \lambda(\xi)\omega(\xi) \tag{19}$$

$$L_0(\xi) = \lambda(\xi)\nu(\xi) \tag{20}$$

Lemma 4 The function $f(s,\xi)$ and $g(s,\xi)$ satisfying equations

$$\Gamma_{R(s,\xi)} g(s,\xi) = \lambda(\xi) f(s,\xi) \tag{21}$$

$$\Gamma^*{}_{R(s,\xi)} f(s,\xi) = \lambda(\xi) g(s,\xi) \tag{22}$$

Proof to prove (21) start with (15). Add and subtract $sL_c(\xi)$ on the left-hand side to get

$$-(sI - A(\xi))L_c(\xi) + L_c(\xi)(sI + A^{\mathrm{T}}(\xi)) = B^{\mathrm{T}}(\xi)B(\xi)$$

Now pre-multiply by $C(\xi)(sI - A(\xi))^{-1}$ and pre--multiply by $(sI + A^{\mathrm{T}}(\xi))^{-1}\nu(\xi)$ to get

$$\begin{aligned} &-C(\xi)L_c(\xi)(sI + A^{\mathrm{T}}(\xi))\nu(\xi) + C(\xi)(sI - A(\xi))^{-1}L_c(\xi)\nu(\xi) \\ &= C(\xi)(sI - A(\xi))^{-1}B(\xi)B^{\mathrm{T}}(\xi)(sI + A^{\mathrm{T}}(\xi))^{-1}\nu(\xi) \end{aligned} \tag{23}$$

The first function on the left-hand side belong to VH_2 ; from (17) and (19) the second function equals $\lambda(\xi) f(s,\xi)$;and from (18) and (19) the function on the right-hand side equals $R_1(s,\xi)g(s,\xi)$. Project both side of (23) onto $VRH_2{}^{\mathrm{T}}$ to get

$$\lambda(\xi) f(s,\xi) = \Pi_1 R_1(s,\xi) g(s,\xi) = \Gamma_{R_1(s,\xi)} g(s,\xi) .$$

But $\Gamma_{R(s,\xi)} = \Gamma_{R_1(s,\xi)}$; hence (21) dolds.

Equation (22) is proved similarly starting with (16). Q.E.D.

From Lemma 4, we can conceive

Corollary 5 $\left\| \Gamma_{R(s,\xi)} \right\| = \lambda(\xi)$

Theorem 6 The infimum model-matching error α equals $\lambda(\xi)$, the unique optimal X equals

$$R(s,\xi) - \lambda(\xi)\frac{f(s,\xi)}{g(s,\xi)} .$$

Proof from Theorem 3 there exists a function $X(s,\xi)$ in VH_∞ such that

$$\|R - X\|_\infty = \|\Gamma_{R(s,\xi)}\| \tag{24}$$

It is claimed that every $X(s,\xi)$ in VH_∞ satisfying (24) also satisfies

$$R(s,\xi) - X(s,\xi)g(s,\xi) = \Gamma_{R(s,\xi)}g(s,\xi) \tag{25}$$

But (25) has a unique solution, namely,

$$X(s,\xi) = R(s,\xi) - \lambda(\xi)\frac{f(s,\xi)}{g(s,\xi)}.$$

Thus (21) and Theorem 3 imply

$$\alpha(\xi) = \lambda(\xi).$$

Therefore

$$X(s,\xi) = R(s,\xi) - \alpha(\xi)\frac{f(s,\xi)}{g(s,\xi)}.$$

Set

$$\alpha(\xi) = \lambda(\xi), \qquad Q(s,\xi) = T_2^{-1}(s,\xi)X(s,\xi). \tag{26}$$

Since $T_{20}(s,\xi), T_{20}^{-1}(s,\xi) \in VRH_\infty$,(26) sets up a one-to-one correspondence between functions $Q(s,\xi)$ in VRH_∞ and functions $X(s,\xi)$ in VRH_∞. An optimal $X(s,\xi)$ yields an optimal $Q(s,\xi)$ via (24)

For a single-input and single-output design in the form of parameter valued case, we have similar to ordinary computing method.

Example.

$$P(s,\xi) = \frac{(s-1)(s-2)}{(s+1)(s^2+s+1+\xi^2)} \in VRH_\infty, \omega_1 = 0.01,$$

$$\varepsilon = 0.1.$$

From the above method, we derive

$$K(s,\xi) = \frac{0.615(s+0.4)(s+1)(s^2+s+1+\xi^2)}{s^4+6.145s^3+12.54s^2+13.53s+0.0232}.$$

Note $K(s,\xi) \notin RH_\infty$, but $K(s,\xi) \in VRH_\infty$.

Step 1.

$$-P(s,\xi) = \frac{N(s,\xi)}{M(s,\xi)},$$

$$N(s,\xi) = -P(s,\xi), M(s,\xi) = 1 = X(s,\xi), Y(s,\xi) = 0.$$

Step 2.

$$W(s,\xi) = \frac{s+1}{10s+1}.$$

Step 3.

$$T_1(s,\xi) = \frac{(s+1)^k}{(10s+1)^k},$$

$$T_2(s,\xi) = -\frac{(s+1)^k(s-1)(s-2)}{(10s+1)^k(s+1)(s^2+s+1+\xi^2)},$$

$$V(s) = s+1.$$

Step 4. When $k=1$,

Step (1)

$$T_{21}(s,\xi) = \frac{(s-1)(s-2)}{(s+1)(s+2)},$$

$$T_{20} = -\frac{(s+1)(s+2)}{(10s+1)(s^2+s+1+\xi^2)}.$$

Step (2) $$R(s,\xi)=\frac{(s+1)^2(s+2)}{(10s+1)(s^2+s+1+\xi^2)}$$

the minimal realization is

$$A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix},\quad B=\begin{bmatrix}-\frac{12}{11}\\ \frac{12}{7}\end{bmatrix},\qquad C=\begin{bmatrix}1 & 1\end{bmatrix}.$$

Step (3) $$L_c=\begin{bmatrix}\frac{72}{121} & -\frac{48}{77}\\ -\frac{48}{77} & \frac{36}{49}\end{bmatrix},\qquad L_0=\begin{bmatrix}\frac{1}{2} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{4}\end{bmatrix}.$$

Step (4) $$L_cL_0=\begin{bmatrix}0.0898 & 0.0425\\ -0.0668 & -0.0853\end{bmatrix}.$$

Then

$$\alpha_1=0.2299>0.1.$$

When $k=2$,

Step (1) $$T_{21}(s,\xi)=\frac{(s-1)(s-2)}{(s+1)(s+2)},$$

$$T_{20}=-\frac{(s+1)(s+2)}{(10s+1)(s^2+s+1+\xi^2)}.$$

Step (2) $$R(s,\xi)=\frac{(s+1)^3(s+2)}{(10s+1)^2(s^2+s+1+\xi^2)}$$

the minimal realization is

$$A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix},\qquad B=\begin{bmatrix}-\frac{24}{121}\\ \frac{12}{49}\end{bmatrix},\qquad C=\begin{bmatrix}1 & 1\end{bmatrix}.$$

Step (3) $$L_c=\begin{bmatrix}\frac{24.12}{121.121} & -\frac{8.12}{121.49}\\ -\frac{8.12}{121.49} & \frac{12.3}{49.49}\end{bmatrix},\qquad L_0=\begin{bmatrix}\frac{1}{2} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{4}\end{bmatrix}.$$

Step (4) $$L_cL_0=\begin{bmatrix}0.0044 & -0.0025\\ 0.0031 & -0.0017\end{bmatrix}.$$

Then

$$\alpha_1=0.05113<0.1,\qquad \omega=\begin{bmatrix}1\\ -0.7209\end{bmatrix}.$$

Step (5) $$f(s,)=\frac{0.2791s-1.2791}{(s-1)(s-2)}$$

$$g(s)=\lambda^{-1}\frac{-0.0141s-0.0657}{(s+1)(s+2)}$$

$$X(s)=6.15\frac{(s+1)(s+2)(s+0.4)}{(10s+1)^2(s+4.66)}.$$

Step(6) Set

$$\alpha = \lambda = 0.05113$$

$$Q(s,\xi) = -6.15\frac{(s+0.4)(s^2+s+1+\xi^2)}{(s+1)(s+4.66)}$$

Step 5.

$$Q_\alpha(s,\xi) = -6.15\frac{(s+0.4)(s^2+s+1+\xi^2)}{(10s+1)(s+1)(s+4.66)},$$

$$K(s,\xi) = 0.615\frac{(s+0.4)(s+1)(s^2+s+1+\xi^2)}{s^4+6.145s^3+12.54s^2+13.53s+0.0232}.$$

References

Francis, B.A. (1987). A course in H_∞-Control Theory. Spring-Verlag, Berlin. Heidelberg. New York, 1987. 61-80.

Francis, B.A., & Doyle, J.C. (1987). Linear Control Theory with an H_∞-Optimality Criterion. *SIAM control and Optimization*, 1987. 25: 815-844.

Francis, B.A., & Zames, G. (1984). On H_∞-Optimal Sensitivity Theory for SISO Feedback System. *IEEE Trans. Auto Cont,* 1984. AC-29: 9-16.

Kerulen, B.V. (1993). H_∞-Control with Measurement-Feedback for Infimite-Dimensional Systems. *Journal Mathematical Systems*, Estimation and control, 1993.3: 373-411.