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Virtual Power Extraction Method of Designing Starting Control Law of Turbofan Engine

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Abstract

Virtual power extraction method (VPEM) of designing starting control law of turbofan engine was presented, and the computer program was developed. The VPEM of designing starting control law is based on the principle of VPEM of designing acceleration control law of turbofan engine, and combined with the method of extrapolation of component maps. The starting control law of some turbofan engine at single flight state and in whole starting envelope was designed by using the program, some computing results were analyzed. Computing results showed that VPEM is accurate and effective for designing starting control law of turbofan engine.

Keywords: Turbofan engine, Virtual power extraction method (VPEM), Starting, Starting control law

1. Introduction

The starting character affects directly the use of a turbofan engine, and one effective way to improve the starting character is to design the starting control law properly. There are lots of researches on starting character simulation technology (R. K. Agrawal, 1982, pp.194-201. Y. De-You, 1983, ISABE 83-7045. TU Qiu-ye, 1999, pp.21-24. WANG Zhan-xue, 2004, pp.444-448. WU Hu, 2007, pp.2068-2072), while there were few studies on design or optimization methods of the starting control law. According to starting character of common low speed component characters, R. K. Agrawal (1982, pp.194-201) built the common computing model to evaluate the starting character with the given fuel supply law in starting process. The common computing model of starting character was generalized (Y. De-You, 1983, ISABE 83-7045. TU Qiu-ye, 1999, pp.21-24. CHEN Yu-chun, 2002, pp.568-570. XIE Guang-hua, 2003, pp.232-235), and applied to the engines with different structure in different starting conditions. Wayne R Sexton (2001, pp.4-75) established the low speed component characters simulation method which is suitable for the starting character computing of turbo engines, and researched about the starting character of turbofan engines with variable geometry. The starting character computing did not relied on the common model any more, while adopted an improved model, which is based on the component character alike the engine performance computing. The precision of starting character computing was improved to large extent with this new technology. WANG Zhan-xue (2004, pp.444-448) and WU Hu (2007, pp.2068-2072) mainly focused on the starting character and the stability of the engine in the starting process on basis of the low speed component characters prediction method of Wayne R Sexton (2001, pp.4-75). In the studies mentioned, only XIE Guang-hua (2003, pp.232-235), with the test data and the given control law, has researched on the starting control law with the engineering experience and artificial iteration. So actually up to now, the research about the design and optimization method of starting control law is nearly blank, for the lacking of theory foundation and design methods.

A new method, Virtual Power Extraction Method(VPEM) of designing the acceleration and deceleration control law of turbo engine, was presented by Chen Yu-chun(2008, pp.327-332). CHEN (2009, pp.2242-2248) developed the improved VPEMs and researched about the acceleration of some turbofan engine. With the low speed component character, there is no essential difference between starting and acceleration or deceleration process, so the VPEM(CHEN Yu-chun, 2008, pp.327-332) and the improved VPEMs(CHEN Yu-chun, 2009, pp.2242-2248) can also be applied to design the starting control law. The VPEM model of designing starting control law of turbofan engine was presented in this paper combined with the improved VPEMs and low speed component character prediction model.

2. Model of the VPEM

CHEN(2008, pp.327-332) presented the VPEM of designing acceleration control law of turbojet engine and introduced the implementation of VPEM in detail: Based on the turbojet character computing program in existence (variable specific heat computing program based on component character), a virtual extra power extraction was introduced (the value of the power can be set optionally), and the control law of the engine was that the rotor speed was constant. When the power is larger than zero, since the power of the turbine is larger than that of the compressor, the fuel supply must be higher than the normal condition (without the power extraction), resulting in the operating point moving closer to the surge margin. On condition that the surge margin limit of the compressor, the total temperature limit of the turbine inlet and the rich oil flameout limit were not exceeded, the fuel supply, which met the power extraction as much as possible, was the fuel supply in acceleration process, in other words, the virtual power extracted was the acceleration power. To obtain the control law of accelerating, several speeds between the idle state and the maximum state were selected and computed.

To solve the inconvenience problem in using the pure VPEM presented above, three improved methods of VPEM were presented (CHEN Yu-chun, 2009, pp.2242-2248): Constant surge margin method, constant turbine inlet total temperature method and constant combustor excess air coefficient method. With the fixed value of the compressor surge margin, turbine inlet total temperature or the combustor excess air coefficient, the improved VPEMs could get directly the fuel supply of certain speed to design the control law of accelerating. The feature of the improved VPEMs are that, in the engine character computing, only the mass flow balance is needed, while the power balance between turbine and compressor need not be met (the power that the turbine power output minus the compressor power needed is the extra power extracted, namely the acceleration power). However, the improved methods are still the power extraction method in essence. The improved VPEMs can be applied to design the acceleration control law of turbojet, turbofan engines.

2.1 Concept of VPEM

VPEM, which does not depend on the transient calculating program, executes acceleration and deceleration control law design on basis of steady-state performance calculating program. It could turn the transient problem into steady-state problem to great extent. For single-spool turbojet engine, when the corrected shaft speed of the engine maintains at a certain constant, if a virtual power HPEXT (Addition to the normal power extraction) is extracted from the engine shaft, the steady-state operating point will move from the original point to a different position. Figure 1 shows that with the HPEXT increasing, steady-state operating points will move to higher points near the surge line, which is just similar to the trend in the acceleration process. When the HPEXT (which may be a negative value) decreases, steady-state operating points will move to lower points near the choke line, which is similar to the trend in the deceleration process. Based on the steady-state calculating program, the virtual power extraction HPEXT is introduced, and the acceleration and deceleration control law is designed by changing HPEXT. So this method is called virtual power extraction method (VPEM).

VPEM can ensure that the turbine steady-state operating point and the actual acceleration and deceleration process are approximately consistent, and the HPEXT is very close to the acceleration and deceleration power in actual process, so it is helpful to estimate the acceleration and deceleration time. If VPEM is applied to single-spool turbojet engine, the HPEXT is extracted directly from the engine shaft. To two-spool turbofan engine, HPEXT can also be extracted directly from the high-pressure rotor. And if it is necessary to ensure the operating points of fan, virtual power could also be extracted from low-pressure rotor at the same time.

As shown in Figure 1, HPEXT = 0 corresponds to the normal operating point A. Being HPEXT > 0, the turbine power is larger than the compressor power, and the fuel flow of the combustor exceeds that of normal state, which corresponds to the acceleration operating point B. In case of HPEXT < 0, the turbine power is lower than the compressor power, and the fuel flow of combustor is less than that of normal state, which corresponds to the deceleration operating point C. Being HPEXT ≠ 0, at certain number of corrected speeds between idling rating and maximum rating, certain number of points B and C which form the acceleration and deceleration line could be achieved by changing the HPEXT. The fuel supply law in the acceleration and deceleration line is just the acceleration and deceleration control law when HPEXT is cancelled, and the transient problem of the acceleration and deceleration control law is turned into steady-state problem.

2.2 Mathematical mode of VPEM

In traditional steady-state performance computing model, to describe the separate-exhaust turbofan engine, the error equations are:

$$E_1 = L_{TH} - L_{CH} = E_1(X_1, X_2, ..., X_6) \tag{1}$$

$$E_2 = L_{TL} - L_{CL} = E_2(X_1, X_2, ..., X_6) \tag{2}$$

$$E_3 = TFF_{HP} - TFF_{HPc} = E_3(X_1, X_2, ..., X_6) \tag{3}$$

$$E_4 = TFF_{LP} - TFF_{LPc} = E_4(X_1, X_2, ..., X_6) \tag{4}$$

$$E_5 = WG_{72} - WG_{72c} = E_5(X_1, X_2, ..., X_6) \tag{5}$$

$$E_6 = WG_{71} - WG_{71c} = E_6(X_1, X_2, ..., X_6) \tag{6}$$

Wherein, E_1,E_2 stand for high pressure turbine(HPT)/ high pressure compressor(HPC) power balance and low pressure turbine(LPT)/fan power balance equations, E_3,E_4 for combustor/HPT mass flow balance and HPT/LPT mass flow balance equations, E_5 for duct/ duct nozzle mass flow balance, E_6 for LPT /main nozzle mass flow balance, L_{TH} and L_{TL} for the power of HPT and LPT, L_{CH} and L_{CL} for the power of HPC and fan, TFF_{HP} for turbine flow function of HPT, TFF_{HPC} for turbine flow function of HPT calculated from the combustor exit parameters, TFF_{LP} for turbine flow function of LPT, TFF_{LPC} for turbine flow function of HPT calculated from the HPT exit parameters, WG_{72} for mass flow of bypass duct, WG_{72C} for mass flow of bypass duct calculated from duct nozzle parameters, WG_{71} for mass flow of LPT, WG_{71C} for mass flow of core calculated from main nozzle parameters. X_1-X_6 are independent variables which are related with operating points of fan, HPC, HPT and LPT. They vary with the different control law. For example, when the control law of fixed geometry turbofan engine is that the high-pressure-rotor speed is constant, X_1-X_6 are separately ratio of fan pressure ratio Z_F, ratio of HPC pressure ratio Z_C, low-pressure-rotor speed n_L, turbine entry temperature T_{t4}, turbine flow function of HPT TFF_{HP}, and turbine flow function of LPT $TFF_{LPP.}$

Virtual power extraction HP_{EXT1} (HPC spool) and HP_{EXT2} (FAN spool) are introduced into equations (1) and (2), and then the power balance equations become:

$$E_1 = L_{TH} - L_{CH} - HP_{EXT1} \tag{7}$$

$$E_2 = L_{TL} - L_{CL} - HP_{EXT2} \tag{8}$$

Equations (3) to (6) remain the same as the original ones, and the control law is that the high-pressure-rotor speed is constant. Using the traditional performance computing method to calculate the steady state operating points, VPEM can be implemented by changing the HP_{EXT1} and HP_{EXT2}. It is proved in follow that VPEM can achieve the optimal acceleration and deceleration process:

The power balance equations of two-spool turbofan in acceleration and deceleration model are:

$$E_1 = L_{TH} - L_{CH} - I_{PH}\omega_H d\omega_H / dt \tag{9}$$

$$E_2 = L_{TL} - L_{CL} - I_{PL}\omega_L d\omega_L / dt \tag{10}$$

Wherein, I_{PH} and I_{PL} stand for the moment of inertia of high-pressure-rotor and low-pressure-rotor separately,ω_H andω_L for angular velocity of high-pressure-rotor and low-pressure-rotor.

If the component volume inertia is ignored, the other error equations of transient model will be the same as those of the steady-state performance model.

By comparing equations (7), (9), and equations (8), (10), virtual power extraction HP_{EXT1} from high-pressure-rotor is equal to the acceleration power of high-pressure-rotor $I_{PH}\omega_H\ d\omega_H/dt$, and virtual power extraction HP_{EXT2} from low-pressure-rotor is equal to the acceleration power of low-pressure-rotor $I_{PL}\omega_L\ d\omega_L/dt$.

In the case of designing acceleration control law (HP_{EXT1}>0), with the increase of HP_{EXT1}, the steady-state operating points (corresponding to the transient points at the same corrected high-pressure-rotor speed) are forced to move to the surge margin (T_{t4} increasing and α decreasing at the same time). Therefore, if certain number of corrected low-pressure-rotor speeds are chosen and the surge margin limit ΔSMC of compressor, turbine inlet total temperature T_{t4} limit, and the excess air coefficient flameout limit α for combustor are satisfied, once HP_{EXT1} is increased as much as possible, the fuel flow supply of combustor at these steady states will be minimum-time control law. Because the acceleration power of high-pressure-rotor and the angular acceleration rate are maximized at any high-pressure-rotor speed, the total acceleration time is minimized. It is obviously shown in the next equation (11):

$$t_{acc} = \int_0^t dt = (\frac{\pi}{30})^2 I_{PH} \int_{n_{idle}}^{n_{max}} \frac{nh}{HP_{EXT1}} dnh \tag{11}$$

Wherein, n_{idle} stands for the idle rating speed, n_{max} for the maximum rating speed.

The design process is similar in deceleration. To the control law whose objective is the minimum of the acceleration and deceleration time, the law designed by VPEM is optimal. Complex mathematical methods based on traditional model are avoided, and this is the advantage of VPEM.

The further study shows that there are some inconveniences in the application of "pure VPEM" described above in the design of acceleration and deceleration control law. I.e. As ΔSMC, T_{t4} or α is presented in advance, the maximum HP_{EXT} should be obtained by artificial or computer's iteration, which means that the calculating process is very time-consuming. Therefore, it is necessary to improve the model. The improvement of the "pure VPEM" for turbojet and turbofan engine is listed as follows:

For equations (1) to (6), if combustor exit gas flow is corrected directly and be assumed as the corrected mass flow of high pressure turbine inlet (set $TFF_{\mathrm{HP}}=TFF_{\mathrm{HPc}}$, so $E_3=0$), and if high pressure turbine exit gas flow is corrected directly and be assumed as the corrected mass flow of low pressure turbine inlet (set $TFF_{\mathrm{LP}}=TFF_{\mathrm{LPc}}$, so $E_4=0$), equations (1) to (6) will be simplified to four-variables nonlinear equations (E_3, TFF_{HP}, E_4, and TFF_{LP} are eliminated). Furthermore, if n_L and Z_{C} are known (Z_{C} can be calculated by a given surge margin of HPC), equations (1) to (6) will be simplified to binary nonlinear equations:

$$E_5 = E_5(T_{t4}, Z_F) \tag{12}$$

$$E_6 = E_6(T_{t4}, Z_F) \tag{13}$$

If n_L and T_{t4} are known, equations (1) to (6) will also be simplified to binary nonlinear equations:

$$E_5 = E_5(Z_C, Z_F) \tag{14}$$

$$E_6 = E_6(Z_C, Z_F) \tag{15}$$

When equations (12) to (13) or (14) and (15) were solved, the mass flow balance can be satisfied, but the power balance is not satisfied. The difference of HPT power and HPC power is the virtual power extraction HP_{EXT1}, and the difference of LPT power and fan power is the virtual power extraction HP_{EXT2}. The modified performance computing model of turbofan is still VPEM essentially, but for given n_L Z_{C} (according to ΔSMC) or T_{t4} or α could be restricted directly, and the control law can be designed conveniently.

As for fixed geometry single-spool turbojet, the three-variables nonlinear equations can also be simplified to a single nonlinear equation, and the acceleration and deceleration control law can be designed conveniently for given Z_{C} (according to ΔSMC) or T_{t4} or α .

And if the low speed component characters are obtained, the starting process does not have essential difference with the process of acceleration, so the VPEM and the improved VPEMs could also be used to design the starting control law.

2.3 VPEM model of designing starting control law

With the successful applications of the VPEM and improved VPEMs to design the acceleration control law of the turbojet and turbofan engines, this paper researched about the application of VPEM to design the starting control law of turbofan engine. It is shown that even if relatively accurate low speed component character is obtained, with the traditional simulation methods of character computing (WANG Zhan-xue, 2004, pp.444-448 and WU Hu, 2007, pp.2068-2072), it is still difficult to simulate the starting process, and more difficult to design the starting control law. The reasons are as follows:

(1)To most turbo engines, the steady operating points with the lower speed than the idle state are hard to get. Since the decrease of the speed, the efficiency of each component decreased heavily, resulting in the temperature to maintain the speed would be quite high. During the iteration, the operating points would move to or even enter the surge margin, so it is very hard for the engine character computing to be balanced;

(2)Secondly, with the decrease of the speed, the condition of combustor inlet gets worse, and the combustion efficiency decreases. So it is probably that the turbine inlet temperature could not be achieved to its requiring value, no matter how the fuel increases;

(3)Below the self-operating speed (ignite speed is normally lower than self-operating speed), the traditional engine character computing method cannot achieve the balanced computing, for if the starter power is not in consideration, the power balance would not be met. However, there is not essential relationship between designing starting control law and starter character;

(4) To design the starting control law with traditional methods, there are the same problems, the functional optimization problem (CHEN Yu-chun, 2009, pp.2242-2249) based on the dynamic character computing, as designing acceleration or deceleration control law: the optimum function relationship between the fuel supply (or some parameter related to the fuel) and the speed which suits the starting process should be computed.

Based on the four reasons above, it is hard to achieve designing starting control law and optimization with the traditional methods.

If the VPEM is applied to the starting character simulation and designing control law, the problems met in the traditional methods would be solved. The reason is that the power balance is not required in the improved VPEMs, as described in section 2.2. In the engine steady character computing process which is based on VPEM, the imbalanced power is the acceleration power (if the turbine residual power is positive value) or the minimum required starter power (if the turbine residual power is minus).

In the starting process of the engine, when combustor is ignited and starter stops working, the engine will accelerate to idle state itself. The control law after starter stop working is very important. This paper mainly studied on the design of the starting control law at this phase. The model of VPEM of designing starting control law of the turbofan engine after ignition is followed:

(1)According to the character prediction method of low speed components (Wayne R Sexton, 2001, pp.4-75), the characters of the fan, compressor, the high pressure turbine and low pressure turbine in low speed needed of the turbofan engine are predicted, and suitable character of the combustion is adopted;

(2)Using steady performance computer program, "the steady performance" below the idle state of the engine (note: "the steady performance" is useless in fact) was calculated to get the fuel supply in the steady state. If the calculation of steady performance was divergent, check out the three parameter values, namely the compressor surge margin ΔSMC, inlet temperature of turbine T_{t4} and excess air coefficient of combustion α, whether exceeded their limitation, and reset the parameter value which exceeded firstly with its limitation value (CHEN Yu-chun, 2009, pp.2242-2249), then continue to compute "the steady performance" below the idle state. The steady fuel supply below the idle state is marked as the lower boundary of the solution space in designing starting control low. The engine with the fuel supply under the lower boundary can not start to accelerate automatically.

(3)According to the improved VPEMs (CHEN Yu-chun, 2009, pp.2242-2249), set the values of ΔSMC, or T_{t4}, or α in the starting process, the engine operating points where compressor and combustion can work stably were obtained, as well as the fuel supply, which was marked as the upper boundary of solution space in designing starting control low.

(4)According to the starting envelop of engine, as well as the selected starting control law pattern, the influence of the altitude, Mach number and atmosphere to the upper boundary of the solution space were analyzed obtain the starting control law suitable for the whole starting envelop.

3. Examples and Analysis

3.1 Characters prediction of low speed components

According to the studied characters of fan, compressor, high pressure turbine and low pressure turbine of turbofan engine in the paper, adopting the extrapolated method (Wayne R Sexton, 2001, pp.4-75), the component characters below the idle state are predicted respectively. To save the space, only the predicted results of compressor (Figure 2(a), (b)) and high pressure turbine (Figure 3(a), (b)) are shown as follows.

It should be mentioned that the character prediction method of low speed turbine in WANG Zhan-xue's paper (2007, pp.444-448) were according to the methods of Wayne R Sexton, (2001, pp.4-75), as well as other published papers in domestic (omitted). However, there is a mistake in these papers: the efficiency of turbine remains the same when corrected speed of turbine is lower than the existed lowest corrected speed. The reason for this mistake is that the prediction method of the turbine torque was not completely comprehended in these papers. It showed clearly by Wayne R Sexton (2001, pp.4-75), in the process of predicting the character of turbine below the lowest corrected speed, the exponent *m* of the turbine torque variation with the corrected speed should be adjusted artificially to make the torque of the actual lowest corrected speed "most close" to the torque of the lowest corrected speed, which was obtained by the torque of second lowest corrected speed varying with the exponent *m*. To examine the torque curve obtained with the actual toque curve whether "most overlap", the technique is applicable if any better idea unavailable.

The model of combustor character in the paper is from R. K. Agrawal (1982, pp.194-201).

3.2 Designing starting control law at single flight state

The starter worked from 0 to 35% of the high pressure rotor speed. This paper only designed the starting control law when the starter cranking was finished. According to the method shown in 2.2, the steady operating line of the fuel supply fuel below idle state of the turbofan engine was obtained in the standard atmosphere, H=0km, Ma=0.0; With the improved VPEMs, the surge margin of high compressor was set as ΔSMC=1%, then the corresponding fuel supply was obtained(the upper boundary). Adopting the pattern of starting control law $W_{fb}/P_{t3}=f(n_H)$ (the relationship between ratio of fuel supply of combustion W_{fb} to total pressure of compressor outlet P_{t3} and high pressure rotor speed n_H), the solution space of starting control law of turbofan engine was obtained , shown in Figure 4.

If one certain starting control law of engine is needed at H=0km, Ma=0.0 in standard condition, a straight line or a broken line or a curve line in the solution space in Figure 4 would be the one, in consideration of the actual condition of engine(control system and combustion character). If a minimal starting time control law is needed, the line would be the upper boundary.

It should be mentioned that, when VPEM is applied in designing starting control law of the two spool turbofan engine, the low pressure rotor speed n_L is linear to the high pressure rotor speed n_H: $n_L=n_{L,idle}\times n_H/n_{H,idle}$ (the subscript "idle" is idle state). It has been proved by CHEN Yu-chun (2009, pp.2242-2249) that the different speed of low pressure rotor would affect the fuel supply W_{fb}, while have little effect on W_{fb}/P_{t3}, namely, no effect on the designing starting control law.

3.3 Designing starting control law in the starting envelope

To design one certain starting control law which guarantees the reliable start in all conditions, the computation must be implemented in the whole starting envelope and all climate conditions. With the starting envelope and climate conditions of some turbofan engine, the influence of the flight altitude H, flight Mach number Ma and temperature TH on the upper boundary of the starting control law were computed respectively, the results showed in Figure 5-7. It is shown in the figures that the upper boundary of the low speeds increased obviously with the altitude increasing, the upper boundary of the high speeds increased with the Mach number increasing, and the influence of the TH on the upper boundary was similar to that of H. For the minimum value of the upper boundary should not be exceeded (to avoid surge and stall in the starting process), so the minimum value of the upper boundary would be obtained by accumulating the data in Figure 5-7, as shown in Figure 8.

As described in section 2.3, with the lower boundary in Figure 4 and the upper boundary in Figure 8, the starting control law solution space of some turbofan in the whole starting envelope and all climate conditions was obtained, as shown in Figure 9 (the actual starting control law was also presented).

As shown in Figure 9, the actual starting control law was just in the starting control law solution space. But the computing results showed that the control law could move upper (closer to the upper boundary) in the low speed to shorten the engine starting time.

Unlike the design of acceleration or deceleration control law (CHEN Yu-chun, 2009, pp.2242-2249), the starting control law with VPEMs was designed without consideration about the influence of Mach number and atmosphere on the "steady operating points" below the idle state. The reasons are that most "steady points" do not need fuel supply when the Mach number increases. The speeds of these points are named windmill speeds, and the research on windmill speeds has exceeded the research field of this paper. It is referred from the research that although the flight conditions and climate conditions affect the "steady points" whose speeds are below idle speed, but this does not affect the feasibility of designing starting control law. The reasons are: (1) with the increase of the flight altitude, the engine inlet total temperature decreases, which leads the fuel air ratio to maintain the steady condition decreases appreciably. But the decrease extent is little enough to be neglected; (2) with the increase of the flight Mach number, the engine inlet total temperature increases and the turbine pressure ratio increases, so the fuel air ratio to maintain the steady condition decreases to great extent. If the starting control law designed in the static condition on the ground is applied to the flight condition with some Mach number, the starting speed will increase greatly. For the compressor surge margin predicted in the starting process is not precise, it is hard to evaluate whether it will cause "hot hanging", so it is unnecessary to consider the influence of Mach number on the "steady operating points"; (3) the influence of the atmosphere temperature on the "steady points" is similar to that of the flight altitude.

4. Conclusions

Based on the VPEM of designing the acceleration control law of turbo engine, combined with the predict method of the low speed engine component character, the VPEM of designing starting control law of turbofan engine was presented. With the comparison between the solution space of the some two spool separate exhaust turbofan and the actual starting control law, it was shown that the VPEM was of better precision and could be applied to design of the starting control law of turbofan engines in both single condition and the whole starting envelope in all climate conditions. To the engines with larger starting envelope or with windmill start, the application of VPEM is need to be researched further.

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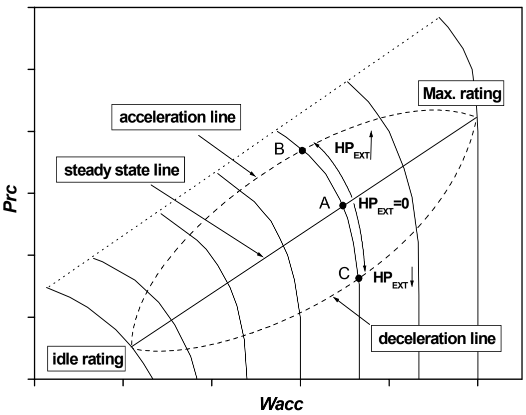
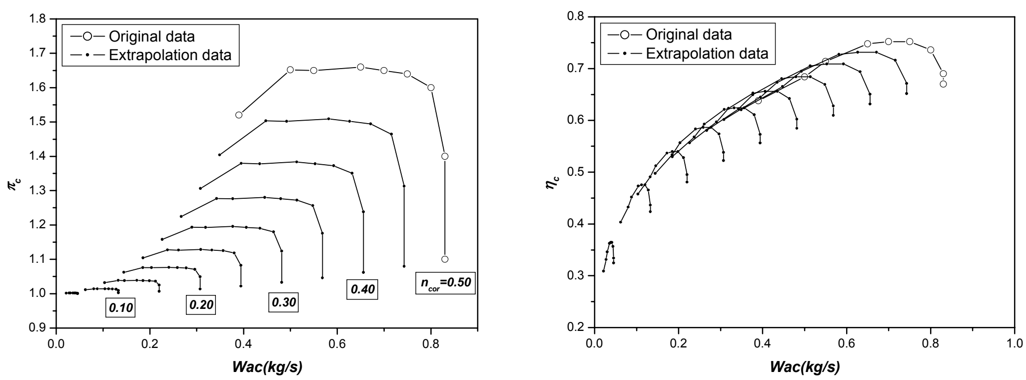
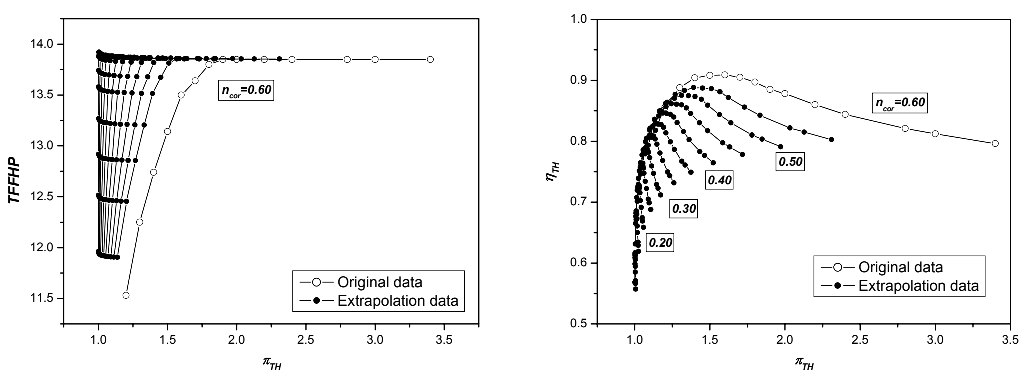


Figure 1. Influence of HPEXT on steady operating point



(a) (b)

Figure 2. Extrapolated data of compressor maps at low speed lines of some turbofan



(a) (b)

Figure 3. Extrapolated data of HPT maps at low speed lines of some turbofan

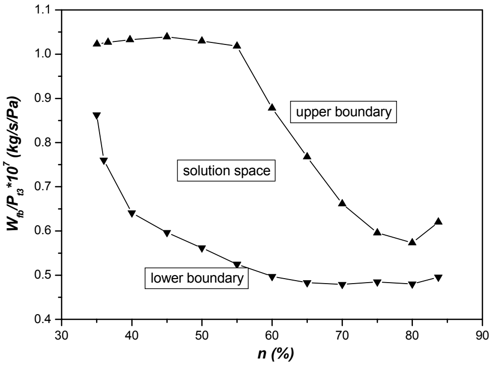


Figure 4. Solution space of designing starting control law of some turbofan

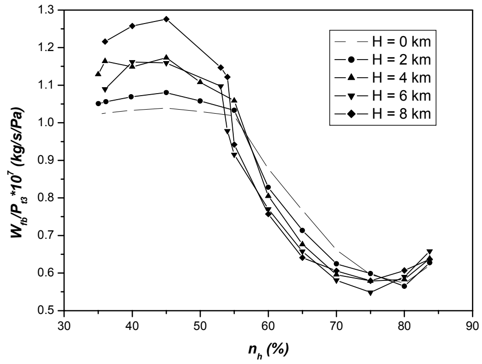


Figure 5. Influence of the flight altitude on the upper boundary

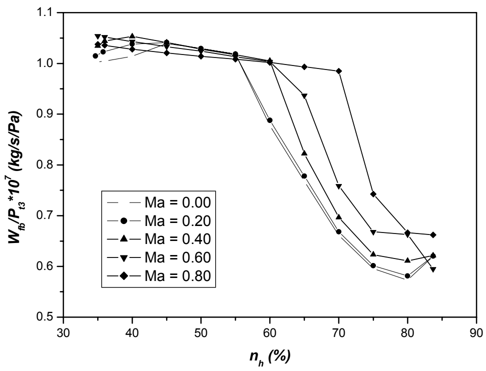


Figure 6. Influence of Mach number on the upper boundary

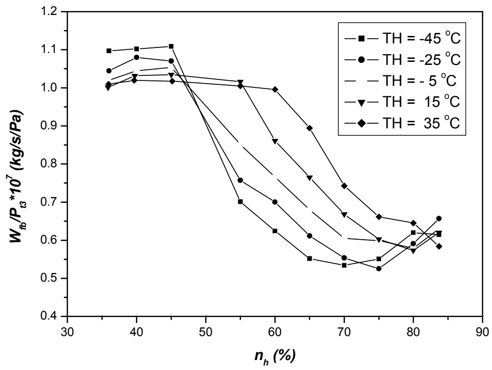


Figure 7. Influence of the temperature on the upper boundary

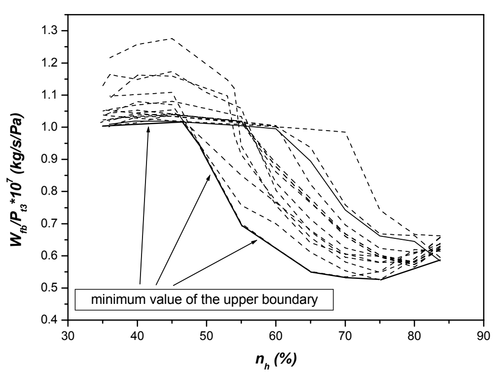


Figure 8. The starting control law upper boundary

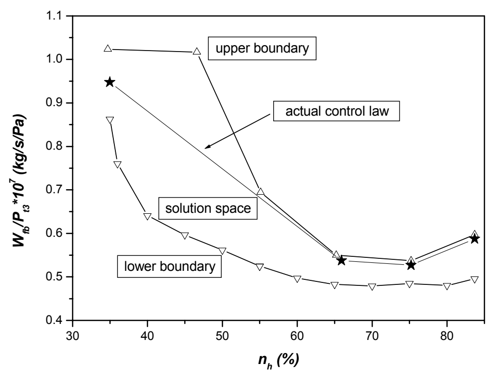


Figure 9. Solution space of designing starting control law and actual starting control law of some turbofan