R/S Analysis with Computer Algebra

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Abstract
In this paper, Hurst exponent is applied to study Shenzhen Stock Component (SSC), which shows that the market is a chaotic system and Hurst exponent is valuable in stock market. The program package presented can be used in calculating R/S.

Keywords: Chaos, Hurst exponent, R/S

1. Introduction
Generally, one of the most commonly used indicators to study the fractal and chaos theory is the Hurst exponent based on the analysis of rescaled range. Hurst discovered that instead of following Brown motion and Gauss distribution, the most of nature phenomena, including river levels, temperatures, rainfalls, and sunspots do follow biased random walk. Hurst extended Einstein’s work on Brownian motion that the distance that a random particle covers increases with the square root of time used to measure it:

\[ R = T^{0.5} \]  

(1)

Where \( R \) is the distance covered, \( T \) a time index. He found that the following was a more general form of equation (1):

\[ (R/S)_a = (bN)^H \]  

(2)

Where \( R/S \) is rescaled range, \( b \) is a constant, \( N \) is time index of a time series \( \{x_i\} \) \( (t=1,2,\ldots,N) \), \( H \) is called as Hurst exponent. Equivalently, in logarithm, equation (2) becomes

\[ \log(R/S) = H (\ln N + \ln b) \]  

(3)

R/S analysis is a nonparametric method raised by Hurst studying ample empirical analysis. The procedure for calculating \( R/S \) is as follow:

Step1: For a given time series \( \{x_i\} \) of length \( M \), divide this time period into \( A \) contiguous subperiods of length \( N \), such that \( AN = M \). Label each subperiod \( I_a \), \( a = 1,2,\ldots,A \). Each element in \( I_a \) is labeled \( X_{k,a} \) such that \( k = 1,2,\ldots,N \). For each \( I_a \) of length \( N \), the average value is defined as:

\[ e_a = \frac{1}{N} \sum_{k=1}^{N} X_{k,a} \]

Where \( e_a \) is average value of the \( M \) contained in subperiod \( I_a \) of length \( N \).

Step2: The time series of accumulated departures \( y_{k,a} \) from the mean value for each subperiod \( I_a \) is defined as:

\[ y_{k,a} = \sum_{i=1}^{k} (x_i - e_a) \; \quad (k = 1,2,\ldots,N) \]

Step3: The range is defined as the maximum minus the minimum value of \( y_{k,a} \) within each subperiod \( I_a \):

\[ R_a = \max_{k} \{ y_{k,a} \} - \min_{k} \{ y_{k,a} \} \]

Step4: The sample standard deviation calculated for each subperiod \( I_a \):

\[ S_a = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - e_a)^2 \right]^{1/2} \]
Step 5: Each range, $r_i$, is now normalized by dividing by the $S_u$ corresponding to it. Therefore, the rescaled range for each $I_u$ subperiod is equal to $R_u/S_u$. From step 1 above, we get $A$ contiguous subperiods of length $N$. Therefore, the average $R/S$ value for length $N$ is defined as:

$$(R/S)_N = \frac{1}{A} \sum_{i=1}^{A} (R_u/S_u)$$

We can now apply equation (3) by performing an ordinary least squares regression on $\ln(N)$ as the independent variable and $\ln(R/S)_N$ as the dependent variable. The slope of the equation is the estimate of the Hurst exponent $H$.

Simple discussions are given as follows: If $0 < H < 0.5$, the time series is anti-persistent. If $H = 0.5$, the time series is standard Brownian motion. If $0.5 < H \leq 1$, the time series is persistent. Let the measure of the length of the cycle $\nu$ be $\nu_i = \frac{(R/S)_N}{\sqrt{N}}$.

2. Design ideas

With applied computer algebra system Mathematica, we get some program packages as follows:

- Hrra [ndata_,num_]: the average $R/S$ value for each length $N$.
- Hurstzh [ndata_]: the estimate of the Hurst exponent.

Where the variable ndata is the original data, the variable num is the length $N$ for each $I_u$.

And the module hurstzh [ndata_] contains the module hrra[ndata_,num_].

3. Results

In this study, total 2392 Shenzhen stock components from 1992 to 2000 are used. From figure 2, we can easily observe the sequence data has a strong linear correlation. In order to eliminate the linearly dependence, applying $AR(1)$ regression to the component, we have

$$y_t = x_t - (a + bx_{t-1})$$

Where $a$ and $b$ is the coefficient of the $AR(1)$ regression model.

![Figure 1. Stock component original time series diagram](image)

![Figure 2. $x_{t+1} - x_t$ Phase diagram](image)
According to the above theory, we get \( y_t = x_t - (4.42191 + 0.998925x_{t-1}) \). And \( F = 9.41444 > F_{(a)}(1,2377) = 3.84 \) shows that there exists the dependency significant between \( x_t \) and \( x_{t-1} \) (figure 2). However, \( DW = 1.93 \) shows that the series \( \{y_t\} \) is not clear dependency, that is to say, the linearly dependency of stock component series has been removed. Thus, we get ideal results by R/S analysis method.

We obtain the values \( (R/S)_N \) and \( N \) (table 1) to be carried out fitting. Hence we get the fitted curve: \( f(x) = -0.520362 + 0.659262x \), thus, Hurst exponent \( H = 0.659262 > 0.5 \) indicates that Shenzhen stock market has distinct fractal characteristics.

Table 1. The value \( (R/S)_N \) and \( N \) of Shenzhen stock component series

<table>
<thead>
<tr>
<th>N</th>
<th>( (R/S)_N )</th>
<th>( V_N )</th>
<th>N</th>
<th>( (R/S)_N )</th>
<th>( V_N )</th>
<th>N</th>
<th>( (R/S)_N )</th>
<th>( V_N )</th>
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As shown in table 1, it is easily observed that the average orbital period of the system is 264 trade days (It can be judged by the Hurst exponent declining for the first time). To a certain extent, the objective of the state continued existence and the long-term memory cycle of Shenzhen stock market (SSMS) can be verified through this result.

4. Conclusion
Hurst exponent \( H = 0.659262 > 0.5 \) shows the fractal characteristics of SSMS. The calculations of Hurst exponent shows that the average orbital period of SSC is about 264 days(approximately 53 weeks), which shows SSMS has state persistence and long-term memory, and the memory cycle is 53 weeks, which means Hurst exponent is valuable in the investment in stock market. This is broadly consistent with the conclusion of some literature on SSMS. Furthermore, this conclusion confirmed the objective existence of the fractal characteristics of SSMS.

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References