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Optimal Guaranteed Cost Control of an Uncertainty System and its Application

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Abstract

The non-fragile guaranteed controller design problem for an interval system and a given cost function is discussed. A sufficient condition is established such that the closed-loop system stability and cost function is guaranteed to be no more than a certain upper bound with all admissible uncertainties as well as a controller gain perturbation uncertainty. A modified interval system described by matrix factorization will lead to less conservative conclusions. An effective linear matrix inequality (LMI) approach is developed to solve the addressed problem. Furthermore, a convex optimization problem is formulated to design the optimal non-fragile guaranteed cost controller which minimizes the upper bound of the closed-loop system cost. The effectiveness of this approach has been verified on a missile launched underwater attitude control system design. Simulation results on a real example are presented to validate the proposed design approach.

Keywords: Interval system, Non-Fragile, Guaranteed cost control, LMI, Missile launched underwater

1. Introduction

A missile launched underwater is an essential naval device because it can be used in deep water and control large areas of sea. Being propelled by a rocket engine, the moving body's mass, weight center, moment of inertia, thrust and speed are all variable during the attacking process, so its motion dynamic model is time-variant. The motion of a moving body is a system with the property of model uncertainty. It has a control system that meets the requirements of large distance runs and little time high-precision tactics but is vulnerable to the impact of model uncertainty. The deviation between the moving body and the predicted ballistic trajectory is large, even if the direction is completely reversed and causes the instability of the control system. Kim and Yuh (2002, p.169–182) provided a neuro-fuzzy controller for autonomous underwater vehicles. Furthermore, Silvestre and Pascoal (2007, p. 883-895) adopted a LMI methodology for controller design in a frequency domain. More importantly, for the aforementioned controller design methods, not all have considered the model uncertainty problem. A non-fragile guaranteed cost control problem, resulting from uncertainty in an all system matrices, has not been studied.

In order to avoid the instability of the control system, the most commonly used method of predicting the motion test data is to extrapolate trends of identifying hydrodynamic parameters. This gives a basis for boundary prediction of the uncertain moving body and provides seaworthiness conditions to achieve the precision strikes purpose. The controller designs objective is to solve the problem that exists because of errors between the practical measurements and observation data of the hydrodynamic parameters identification model, and ensure that the moving body motion test has a good steady maneuverability. This paper considers the missile launched underwater model for an interval system, provides a non-fragile guaranteed cost controller design approach, and applies to a missile launched underwater control test, which is validated by a moving body's control trajectory numerical simulation.

The interval dynamic system is a typical parameter uncertain linear system. See, for example, the electrical machinery and the navigational control systems that may describe the interval dynamic system. In order for the design methods to be more useful, practical considerations need to be taken into account in the control design, for example, the inevitable model uncertainties, interval system, guaranteed cost control and even possible fragility. These issues will make the design more challenging. Recently, stability analysis and control synthesis of the interval systems have been discussed extensively (Wang, Anthony and Liu, 1994, p.1251-1255. Hu and Wang, 2000, p. 2106 – 2111. Mao and Liu, 2005, p.177-188). Hu and Wang (2000, p. 2106 – 2111) proposed a design method for a specific structure of a single-input interval system. Mao and Liu (2005, p.177-188) study the guaranteed cost control of the interval system but do not consider the fragility or the results presented by the proposed M_- matrix conditions. The existing approaches are limited and conservative and furthermore, in application, the Riccati equation algorithm cannot be guaranteed to be convergent. Very little open literature covering the research guaranteed cost control of interval system has been published and the fragility problem has not been considered.

Controller fragility problems have attracted considerable attention (Zhang, Zhou and Li, 2007. 5 p.118-5133). It is known that in most engineering systems, fragility is a common dynamic problem and is caused by many factors. Reduction in size and cost of digital control hardware result in limitations in available computer memory and word length capabilities of the digital processor. These further result in computational round off errors leading to controller implementation imprecision. The control system designers should consider that any controller that is part of a feedback system should be insensitive to some amount of error with respect to its gains.

In this paper, the LMI approach is used to study the design problem of non-fragile guaranteed cost controller of an interval system. The designed controller will be able to guarantee the robust stability of the closed-loop system in the event of plant and controller gain perturbation uncertainty. Its application is given to illustrate the effectiveness and necessity for the control design.

2. Model description and problem Statement

Within the coordinate system for the bodies measured from the center of buoyancy (Zhang, 1999, p.42-96), the missile launched underwater vertically dynamics and kinematics equation in vertical plane movement is established:

$$\dot{v}=k_{11}v^2+k_{14}\sin\Theta+k_{18}$$

$$\dot{\alpha}=\frac{1}{v}(k_{21}v^2\alpha+k_{22}v\omega_z+k_{23}\alpha+k_{24}\alpha\sin\Theta$$

$$++k_{25}\cos\Theta k_{26}\cos\theta+k_{27}\sin\theta+k_{28}+k_{29}v^2\delta_e)$$

$$\dot{\omega}_z=k_{31}v^2\alpha+k_{32}v\omega_z+k_{34}\alpha\sin\Theta+k_{35}\cos\Theta$$

$$++k_{36}\cos\theta+k_{37}\sin\theta+k_{38}+k_{39}v^2\delta_e$$

$$\dot{\theta}=\omega_z$$

$$\dot{y}_e=v\sin\Theta$$

$$\dot{x}_e=v\cos\Theta$$

$$\Theta=\theta-\alpha$$

Where $k_{ij}(1\le i\le 3, 1\le j\le 9)$ are the hydrodynamic parameters. α is the angle of attack, θ is the pitching angle, Θ is the angle of trajectory, ω_z is the pitching angular velocity, δ_e is the elevator deflection angle, x_e, y_e are the center of buoyancy coordinates of the ground coordinate system.

In a given speed and trim depth, the state-space equation of the vertical movement of small perturbation can be expressed as:

$$\dot{x}(t)=Ax(t)+Bu(t) \qquad x(0)=x_0 \tag{1}$$

$$A=\begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 0 \end{bmatrix} \quad B=\begin{bmatrix}0 & b_2 & b_3 & 0 & 0\end{bmatrix}^T$$

Where $x(t)=[v,\alpha,\omega_z,\theta,y]^T\in R^n$, $u\in R^p$ denote the state vector and the control vector respectively, y is the trim depth. A is the state matrix and coefficients $a_{ij}(1\le i\le 5, 1\le j\le 5)$ are hydrodynamic parameters. B is the input matrix and interval matrix $A\in R^{n\times n}$ and $B\in R^{n\times p}$ vary with the hydrodynamic parameters and can be shown as:

$$A\in\left[A^m, A^M\right]=:\left[a_{ij}\right],\ a_{ij}^m<a_{ij}<a_{ij}^M, i,j=1\cdots n.$$

$$B\in\left[B^m, B^M\right]=:\left[b_{ij}\right],\ b_{ij}^m<b_{ij}<b_{ij}^M, i=1\cdots n, j=1\cdots p.$$

Where

$$A^m=\left[a_{ij}^m\right]_{n\times n},\ A^M=\left[a_{ij}^M\right]_{n\times n} \text{ satisfying } a_{ij}^m<a_{ij}^M.$$

$$B^m=\left[b_{ij}^m\right]_{n\times p},\ B^M=\left[b_{ij}^M\right]_{n\times p} \text{ satisfying } b_{ij}^m<b_{ij}^M.$$

Let

$$A_0:=\left(A^M+A^m\right)/2,\ \bar{A}_{ij}:=\left(A^M-A^m\right)/2:\left[\bar{a}_{ij}\right]$$

$B_0 := \left(B^M + B^m\right)/2,\ \overline{B}_{ij} := \left(B^M - B^m\right)/2 : \left[\overline{b}_{ij}\right]$

Where A^m, A^M, B^m, B^M are known real matrices, $\overline{A}_{ij}$ denotes that the i, j th component is $\overline{a}_{ij}$ with all other entries being zeros, and can be factorized into $\overline{a}_{ij} e_i \times e_j^T$. A and B can be described as an equivalent form:

$$A = A_0 + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{ij}\overline{A}_{ij} = A_0 + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{ij}\overline{a}_{ij} e_i \times e_j^T = A_0 + D_0 F_A E_0$$

$$B = B_0 + \sum_{i=1}^{n}\sum_{j=1}^{p} \beta_{ij}\overline{B}_{ij} = B_0 + \sum_{i=1}^{n}\sum_{j=1}^{p} \beta_{ij}\overline{b}_{ij} e_i \times e_j^T = B_0 + D_1 F_B E_1$$

Where

$$F_A = diag\left[\lambda_{11}, \cdots \lambda_{1n}, \lambda_{21}, \cdots, \lambda_{2n}, \lambda_{n1}, \cdots \lambda_{nn}\right], -1 \le \lambda_{ij} \le 1, 1 \le i, j \le n$$
$$F_B = diag\left[\beta_{11}, \cdots \beta_{1p}, \beta_{21}, \cdots, \beta_{2p}, \beta_{n1}, \cdots \beta_{np}\right], -1 \le \beta_{ij} \le 1, 1 \le i, j \le p$$

are diagonal structured uncertainty matrices which satisfy

$$F_A^T F_A \le I_{n^2}, \quad F_B^T F_B \le I_{(n\times p)\times(n\times p)}. \tag{2}$$

Let e_i denote standard basis column vector in R^n with zero entry except for the i, j th component.

The controller gains perturbations with uncertainties are considered and the actual control input implemented is assumed to be $u(t) = \hat{K}x(t) = (K + \Delta K)x(t)$, K is the nominal control matrix, $\Delta K = D_k F_k E_k$ is the controller gain matrix form, D_k, E_k are known constant dimension matrices ands $F_k(t)$ represents the control gain variation and satisfies

$$F_k(t) F_k^{\mathrm{T}}(t) < \delta I \tag{3}$$

Where $\delta > 0$ is the controller gain perturbation bound.

The cost function associated with this system is:

$$J = \int_0^\infty \left[x^T(t)(Q + \hat{K}^T R \hat{K})x(t)\right]dt \tag{4}$$

Where Q and R are given weighting matrices.

Substitute (1) into (5) and the resulting closed-loop system is:

$$\dot{x}(t) = \left[(A_0 + D_0 F_A(t) E_0) + (B_0 + D_1 F_B(t) E_1)(K + D_k F_k(t) E_k)\right]x(t) \tag{5}$$

Lemma 1(Khargonekar, Petersen and Zhou, 1990, p. 356-361) Let Y_1, M, N and ψ be matrices of appropriate dimensions and assume ψ is symmetric, satisfying $\psi^{\mathrm{T}}\psi \le I$, then

$Y_1 + M\psi N + N^{\mathrm{T}}\psi^{\mathrm{T}}M^{\mathrm{T}} < \mathbf{0}$

if and only if there exists a scalar $\varepsilon > 0$ satisfying

$Y_1 + \varepsilon M M^{\mathrm{T}} + \varepsilon^{-1} N^{\mathrm{T}} N < \mathbf{0}$

3. Main results

The following theorem provides a sufficient condition for the existence of the non-fragile guaranteed cost controller and a design procedure for such controllers.

Theorem 1. A control law $u(t) = \hat{K}x(t)$ is said to be a non-fragile guaranteed cost control associated with cost matrix $P > 0$ for the system (1) and cost function (4) if the following matrix inequality

$$Q + \hat{K}^{\mathrm{T}} R \hat{K} + P(A + B\hat{K}) + (A + B\hat{K})^{\mathrm{T}} P < 0 \tag{6}$$

holds for all admissible uncertainties (2) and (3). The closed-loop cost function satisfies

$$J \le J^* = x_0^T P x_0 = tr(X^{-1}) \tag{7}$$

in which x_0 is the initial state.

Proof: Choose a Lyapunov function as

$$V(x) = x^{\mathrm{T}}(t) P x(t)$$

Then the time derivative of $V(\cdot)$ along any trajectory of the closed-loop system (5) is given by

$$\dot{V}(x) = x(t)^{\mathrm{T}}(P(A + B\hat{K}) + (A + B\hat{K})^{\mathrm{T}} P)x(t)$$

According to inequality (6), it is possible to obtain

$$\dot{V}(x) < x^{\mathrm{T}}(t)(-Q - K^{\mathrm{T}} R K)x(t) < 0$$

Therefore, the closed-loop system (5) is asymptotically stable. Furthermore, by integrating both sides of the above inequality from 0 to T,

$J \le V(x(0)) = x_0^{\mathrm{T}} P x_0 = J^*$

can be obtained. Thus, the theorem 1 is true.

Here, $J^* = x_0^T P x_0$ depends on the initial condition x_0. Assuming x_0 is a zero mean random variable satisfying $E\{x_0 x_0^T\} = I$, the guaranteed cost then also leads to $J^* = E\{J^*\} = E\{x_0^T P x_0\} = tr(P)$.

The objective of this paper is to develop a procedure for determining a state feedback gain matrix $\hat{K}$ which contains controller gain perturbation such that the control law $u = \hat{K}x$ is a non-fragile guaranteed cost control of the system (1) and cost function (4).

Theorem 2. There exist non-fragile guaranteed cost controllers for the system (1) and the cost function (4), if there exists scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ and $\varepsilon_3 > 0$, symmetric positive definite matrices $X > 0$ and a matrix Y such that

$$\begin{bmatrix} \Lambda_1 & Y^{\mathrm{T}} & B_0 D_2 & X E_0^{\mathrm{T}} & Y^{\mathrm{T}} E_1^{\mathrm{T}} & X E_2^{\mathrm{T}} & X \\ Y & -R^{-1} & D_k & 0 & 0 & 0 & 0 \\ D_2^T B_0^T & D_2^T & -\varepsilon_1^{-1} I & 0 & D_2^T E_1^T & 0 & 0 \\ E_0 X & 0 & 0 & -\varepsilon_2 I & 0 & 0 & 0 \\ E_1 Y & 0 & E_1 D_2 & 0 & -\varepsilon_3 I & 0 & 0 \\ E_2 X & 0 & 0 & 0 & 0 & -\rho^{-1}\varepsilon_1 I & 0 \\ X & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0 \tag{8}$$

Where

$$\Delta_1 = \left(A_0 X + B_0 Y\right)^{\mathrm{T}} + A_0 X + B_0 Y + \sum_{i=1}^{2} \varepsilon_i D_{i-1} D_{i-1}^{\mathrm{T}}$$

Furthermore, if $(\bar{\varepsilon}, \bar{X}, \bar{Y})$ is a feasible solution to the inequality (8), then $u = \hat{K}x$ is a non-fragile guaranteed cost controller of the system (1), where the feedback gain matrix $\hat{K}$ is given by $u(t) = YX^{-1}x(t)$ and the corresponding closed-loop cost function satisfies (7).

Proof: By manipulating the left-hand side in inequality (6), it follows that the inequality (6) is equivalent to

$$Y_1 + \Sigma_1 + \Sigma_1^{\mathrm{T}} < 0 \tag{9}$$

Where

$$Y_1 = \begin{bmatrix} Q + P(A+BK) + (A+BK)^{\mathrm{T}} P & K^{\mathrm{T}} \\ K & -R^{-1} \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} P B_0 D_2 \\ D_2 \end{bmatrix} F_k \begin{bmatrix} E_2 & \mathbf{0} \end{bmatrix}$$

By applying lemma 1, the above inequality holds for all F_k satisfying $F_k(t) F_k^{\mathrm{T}}(t) < \delta I$ if and only if there exists a constant $\varepsilon_1 > 0$ such that

$$Y_1 + \varepsilon_1 \begin{bmatrix} P B_0 D_2 \\ D_2 \end{bmatrix} \begin{bmatrix} P B_0 D_2 \\ D_2 \end{bmatrix}^{\mathrm{T}} + \rho \varepsilon_1^{-1} \begin{bmatrix} E_2^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} E_2 & 0 \end{bmatrix} < 0 \tag{10}$$

By manipulating the left-hand side in inequality (6) again and the above inequality (10) it follows from the Schur complement, that the above inequality is further equivalent to the following inequality

$$Y_2 + \Xi_1 + \Xi_1^{\mathrm{T}} + \Xi_2 + \Xi_2^{\mathrm{T}} < 0 \tag{11}$$

Where

$$Y_2 = \begin{bmatrix} \Psi_1 & K^{\mathrm{T}} & P B_0 D_2 \\ K & -R^{-1} & D_2 \\ (B_0 D_2)^{\mathrm{T}} P & D_2 & -\varepsilon_1^{-1} I \end{bmatrix} \quad \Xi_1 = \begin{bmatrix} P D_0 \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} E_0 & 0 & 0 \end{bmatrix} \quad \Xi_2 = \begin{bmatrix} P D_1 \\ 0 \\ 0 \end{bmatrix} F_B \begin{bmatrix} E_1 K & 0 & E_1 D_2 \end{bmatrix}$$

$$\Psi_1 = Q + P\left(A_0 + B_0 K\right) + \left(A_0 + B_0 K\right)^{\mathrm{T}} P + \rho \varepsilon_1^{-1} E_2^{\mathrm{T}} E_2$$

Applying lemma 1 again, the inequality (11) holds for all F_A, F_B satisfying $F_A^{\mathrm{T}}(t) F_A(t) \le I, F_B^{\mathrm{T}}(t) F_B(t) \le I$ if and only if there exists constants $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ such that

$$Y_2 + \varepsilon_2 \begin{bmatrix} P D_0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P D_0 \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_2^{-1} \begin{bmatrix} E_0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_0 & 0 & 0 \end{bmatrix} + \varepsilon_3 \begin{bmatrix} P D_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P D_1 \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_3^{-1} \begin{bmatrix} E_1 K & 0 & E_1 D_2 \end{bmatrix}^T \begin{bmatrix} E_1 K & 0 & E_1 D_2 \end{bmatrix} < 0$$

It follows from the Schur complement again, that the above inequality is further equivalent to the following inequality

$$\begin{bmatrix} \Lambda & K^T & PB_0D_k & (E_1K)^T & E_0^T & E_2^{\mathrm{T}} & I \\ K & -R^{-1} & D_k & 0 & 0 & 0 & 0 \\ (PB_0D_2)^{\mathrm{T}}P & D_2^T & -\varepsilon_1^{-1}I & (E_1D_2)^T & 0 & 0 & 0 \\ E_1K & 0 & E_1D_2 & -\varepsilon_3 I & 0 & 0 & 0 \\ E_0 & 0 & 0 & 0 & -\varepsilon_2 I & 0 & 0 \\ E_2 & 0 & 0 & 0 & 0 & -\rho^{-1}\varepsilon_1 I & 0 \\ I & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0$$

Where

$$\Lambda = P\left(A_0 + B_0K\right) + \left(A_0 + B_0K\right)^{\mathrm{T}} P + \varepsilon_2 PD_0D_0^{\ T}P + \varepsilon_3 PD_1D_1^{\ T}P$$

Defining the Matrixes $X = P^{-1}$, and considering the change of variable $Y = KX$ and by pre- and post-multiplying the left-hand side matrix in the above inequality by the matrix $diag\left\{P^{-1} \quad I \quad \cdots \quad I\right\}$, respectively, it can be concluded that the above matrix inequality is equivalent to (8). Thus the proof is complete.

Theorem 3. Consider system (1) with cost function (4). If the following optimization problem

$$\min_{\varepsilon_1,\varepsilon_2,\varepsilon_3,X,Y} \mathrm{tr(W)} \qquad (12)$$

subject to (i) (8)

(ii) $\begin{bmatrix} W & I \\ I & X \end{bmatrix} > \mathbf{0}$

has a solution $(\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{X}, \bar{Y})$, then the control law $u(t) = YX^{-1}x(t)$ is an optimal state feedback non-fragile guaranteed cost controller which ensures the minimization of the guaranteed cost $J \le J^* = x_0^{\mathrm{T}} P x_0 = tr(X^{-1})$ for the uncertain system (1).

Proof. If $(\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{X}, \bar{Y})$ is a feasible solution to the optimization problem (12), then $(\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{X}, \bar{Y})$ is also the feasible solution to the inequality (8), by theorem 2, $u(t) = YX^{-1}x(t)$ is a non-fragile guaranteed cost controller of the system (1).

It follows from the Schur complement that (ii) in (12) is equivalent to $W > X^{-1} > 0$. Thus, the minimization of the trace of W implies the minimization of the trace of the matrix X^{-1}, that is, the minimization of the guaranteed cost for the uncertain system (1). The optimality of the solution of the optimization problem (12) follows from the convexity of the objective function and of the constraints. Thus the proof is completed.

4. Application to motion control testing

Consider a certain missile launched underwater system model for interval system (1), given in a state space representation, where the system matrices are as follows:

$$A_0 = \begin{bmatrix} -0.6725 & 1.1448 & 0 & -1.1448 & 0 \\ -0.0124 & -3.1762 & 0.2163 & 8.3454e-004 & 0 \\ 0.0539 & 52.3465 & 5.7163 & 0.0258 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -20 & 0 & 20 & 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} 0 \\ -0.5481 \\ -17.5204 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{A}_{ij} = \begin{bmatrix} 0.21 & 0.33 & 0 & 0.33 & 0 \\ 0.003 & 0.96 & 0.006 & 2e-4 & 0 \\ 0.003 & 15 & 1.8 & 0.003 & 0 \\ 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \bar{B}_{ij} = \begin{bmatrix} 0 & 1.5 & 5 & 0 & 0 \end{bmatrix}^T$$

$D_k = \begin{bmatrix} 0.01 & 0 & 0.01 & 0 & 0.1 \end{bmatrix}$ $E_k = I_5$

Given condition $\delta = 0.4$, the weighted matrices of cost function (4) are $Q = I_1, R = I_5$.

By applying theorem 2 and solving the corresponding optimization problem theorem 3, respectively, the optimal guaranteed cost controller is obtained

$K = \begin{bmatrix} \text{-0.0082} & \text{-0.3522} & 1.9412 & 8.4362 & 1.0629 \end{bmatrix}$

and where $\varepsilon_1 = 1, \varepsilon_2 = 2, \varepsilon_3 = 3$

$$X = \begin{bmatrix} 1.2644 & -0.075922 & 0.13708 & 0.016271 & -0.39485 \\ -0.075922 & 1.0159 & -2.0284 & 0.40753 & 1.3216 \\ 0.13708 & -2.0284 & 16.993 & -1.6134 & -2.4166 \\ 0.016271 & 0.40753 & -1.6134 & 0.43226 & -0.34916 \\ -0.39485 & 1.3216 & -2.4166 & -0.34916 & 7.6195 \end{bmatrix}, Y = \begin{bmatrix} \text{-1.0047e-006} & 0.5481 & 17.52 & \text{-1.566e-006} & \text{-4.636e-006} \end{bmatrix}$$

The associated upper bound over the closed-loop cost function is $J^* = 13.8775$.

Assuming

$F_1 = F_d = \sin(5.00\pi t)$ $F_2 = -\cos(2.00\pi t)$ $F_k = 0.5916\sin(10.00\pi t)$ (Shown as a dashed line); $F_1 = F_d = F_2 = Fk = 1$ (continuous line); $F_1 = F_d = F_2 = Fk = -1$ (line of stars), carry out the dynamic response test with the three sets of data aforementioned. The results are shown in Figures 1 to 5.

Figures 1 to 5 show that the system is asymptotically stable for the all admissible uncertainties between the largest and smallest bound and it has a good dynamic performance.

The robust non-fragile optimal guaranteed cost controller obtained guarantees the asymptotic stability and the upper bound of cost function J^*. In this case, the matrices $\overline{A}_{ij}$ and $\overline{B}_{ij}$ perturbation range of parameters achieve 30%, showing that this method is effective, performs greater flexibility for the D_1, D_2, E_1, E_2 choice in practical applications and produces a less conservative conclusion in (Hu and Wang, 2000, p. 2106 – 2111) than was obtained from a square root in the interval matrix. It is also simpler than the algorithm in (Mao and Liu, 2005, p.177-188)

5. Conclusion

This paper provides a description about the dynamic interval system of missile launched underwater uncertainty model. The practice system model can be obtained through a series of tests and hydrodynamic parameter identifications. An effective LMI approach is developed to solve the problem of uncertainty and synthesis modeling error and simplify the controller design and calculation. The proposed design can guarantee the robust stability such that the cost function of the closed-loop system is guaranteed to be no more than a certain upper bound. It can be applied to a missile launched underwater motion control test to avoid other algorithms that are not always convergent and are more conservative. It also reduces the complexity of controller design and increases the control precision of attack target effectively.

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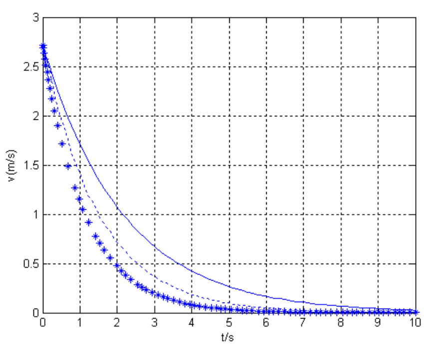

Figure 1. Three class of velocity

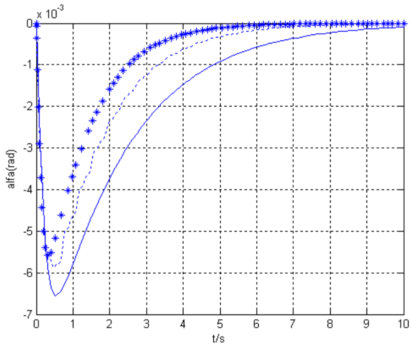

Figure 2. Three class of angle of attack

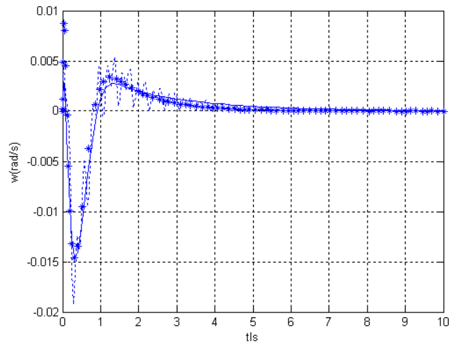

Figure 3. Three class of pitching angular velocity

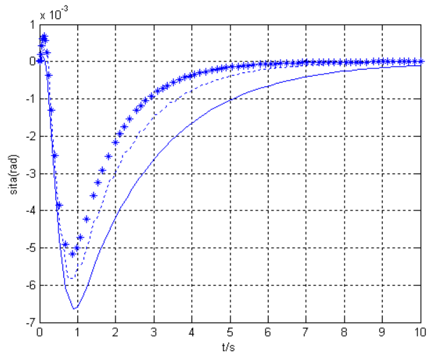

Figure 4. Three class of pitching angle

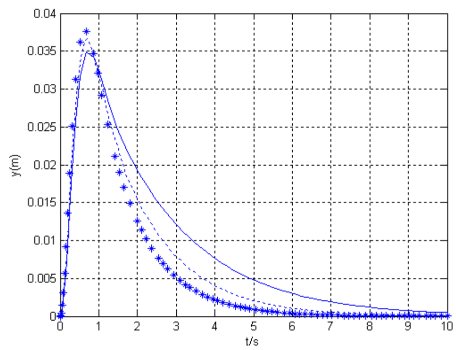

Figure 5. Three class of trim depth