Modern Applied Science

www.ccsenet.org/mas.html

Vol. 1, No. 3

September 2007

Global Exponential Stability of a Class of Neural Networks with Finite Distributed Delays

Jianzhi Sun

Department of Mathematics and Physics, Yanshan University, Qinhuangdao 066004, China

E-mail: jianzhisun@126.com

Huaiqin Wu

Department of Mathematics and Physics, Yanshan University, Qinhuangdao 066004, China

Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China

Abstract

In this paper, global exponential stability of a class of neural networks with finite distributed delays is investigated by matrix measure technique and Halanay inequality. Several sufficient conditions are given to guarantee globa exponential stability of the neural networks without assuming the differentiability of delay. At last, two examples are given to illustrate the applicability of our results.

Keywords: Neural networks, Global exponential stability, Matrix measure, Halanay inequality

1. Introduction

In recent years, various neural networks models such as Hopfield neural networks, cellular neural networks, and bi-directional associative memory networks have been extensively investigated and successfully applied to signal processing, pattern recognition, and associative memory and optimization problems. In such applications, due to finite switching speed of the amplifiers and communication time, time delays are actually unavoidable in the electronic implementation. It is known that the delays are a potential cause of the loss of stability to a system. On the other hand, it has also been shown that the process of moving images requires the introduction of delay in the signal transmitted through the networks. Therefore, it is of importance to investigate stability of neural networks with delays. In the literature, a lot of results have been established on global stability and global exponential stability of the equilibrium point for delayed neural networks (see, e.g., and references therein). To the best of our knowledge, few results on the global exponential stability of a class of neural networks with finite distributed delays have been reported in literatures. In this paper, the global exponential stability of this network were discussed, some sufficient conditions ensuring the global exponential stability of neural networks are derived, two examples are given to illustrate the effectiveness of our results.

The paper is organized as follows. In Section 2, the new network model is formulated; some preliminaries such as Halanay inequality, matrix measure are presented. In Section 3, some sufficient conditions ensuring the global exponential stability neural networks are given. In Section 4, two illustrative examples are provided to show the effectiveness of our results. Some conclusions are drawn in Section 5.

2. Preliminaries

In this paper, we consider the neural network model with distributed delays as

$$\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(x_j\left(t-\tau_j(t)\right)\right) + \sum_{j=1}^{n} \int_0^{\tau} c_{ij}(s)\, g_j\left(x_j(t-s)\right) ds + u_i, \quad i = 1,\ldots,n \qquad (2.1)$$

Here, n is the number of neurons in the indicated neural network, $x(t) = (x_1(t),\ldots,x_n(t))^T$ is the state vector of the network at time t, $g(x(t)) = (g_1(x_1(t)),\ldots,g_n(x_n(t)))^T$ is the output vector of the network at time t, $D = diag\,(d_1,\ldots d_n) > 0$, $A = \left(a_{ij}\right)_{n\times n}$ is the feedback matrix, $B = \left(b_{ij}\right)_{n\times n}$ is the delayed feedback matrix and $C(s) = \left(c_{ij}(s)\right)_{n\times n}$, $\int_0^{\tau} |c_{ij}(s)|\, ds$ is existent for $i, j = 1,\ldots,n$, $C = \left(\int_0^{\tau} c_{ij}(s) ds\right)_{n\times n}$, $U = (u_1,\ldots u_n)^T$ is the stimulus from outside of the network at time t, the

(2) Let $P \in R^{n\times n}$ is nonsingular, for any $x \in R^n$, $\|x\|_P^m = \|Px\|_m$, $\|\cdot\|_m$ denotes m-norm of R^n, then $\|\cdot\|_P^m$ is a vector norm of R^n, the matrix measure $\mu_P^m(A)$ and the norm $\|A\|_P^m$ induced by $\|\cdot\|_P^m$ satisfies respectively

$$\mu_P^m(A) = \mu_m\left(PAP^{-1}\right) \qquad \|A\|_P^m = \left\|PAP^{-1}\right\|_m$$

(3) For the 1-norm, 2-norm and ∞ - norm of R^n, the induced matrix measure are given by

$$\mu_1(A) = \max_j\left(a_{jj} + \sum_{i\neq j}^n |a_{ij}|\right)$$

$$\mu_2(A) = \max_j\left(\frac{\lambda_j\left(A + A^T\right)}{2}\right)$$

$$\mu_\infty(A) = \max_i\left(a_{ii} + \sum_{j\neq i}^n |a_{ij}|\right)$$

(4) $-\|A\| \le -\mu(-A) \le \operatorname{Re}\lambda(A) \le \mu(A) \le \|A\|$

3. Global exponential stability of equilibrium of the system (2.1)

In this section, we will derive sufficient conditions for the existence of equilibrium of the system (2.1). Furthermore, we will use lemma 1 and matrix measure to establish the exponential stability of system (2.1).

An equilibrium point of the system (2.1) is a constant vector $\left(x_1^*,...,x_n^*\right)^T \in R^n$ which satisfies the following equation

$$d_i x_i^* = \sum_{j=1}^n a_{ij} g_j\left(x_j^*\right) + \sum_{j=1}^n b_{ij} g_j\left(x_j^*\right) + \sum_{j=1}^n \int_0^\tau c_{ij}(s) g_j\left(x_j^*\right) ds + u_i, \quad i = 1,...,n \tag{3.1}$$

Theorem 3.1 Assume that

$$l\left(\|B\|_1 + \|C\|_1\right) < -\ \mu_1\left(-D + A^* L\right) \tag{3.2}$$

where $L = diag\left(l_1,...,l_n\right)$, $l = \max\{l_i, i = 1,...,n\}$. Then there exists a unique solution of the equation (3.1), i.e., the system (2.1) has a unique equilibrium point.

Proof. It follows from (3.2) that

$$l\left(\max_j \sum_{i=1}^n |b_{ij}| + \max_j \sum_{i=1}^n |c_{ij}|\right) < \min_j\left(d_j - a_{jj}^* l_j - \sum_{i\neq j}^n |a_{ij} l_j|\right)$$

i.e., $$\frac{1}{d_j}\left\{l\left(\max_j \sum_{i=1}^n |b_{ij}| + \max_j \sum_{i=1}^n |c_{ij}|\right) + a_{jj}^* l_j + \sum_{i\neq j}^n |a_{ij} l_j|\right\} < 1 \qquad j = 1,...,n$$

Define α as

$$\alpha = \max_j\left\{\frac{1}{d_j}\left[l\left(\max_j \sum_{i=1}^n |b_{ij}| + \max_j \sum_{i=1}^n |c_{ij}|\right) + a_{jj}^* l_j + \sum_{i\neq j}^n |a_{ij} l_j|\right]\right\}$$

It is obvious that $0 < \alpha < 1$.

Let $d_i x_i^* = v_i^*$, $i = 1,...,n$ in (3.1), then we have

$$v_i^* = \sum_{j=1}^n a_{ij} g_j\left(\frac{v_j^*}{d_j}\right) + \sum_{j=1}^n b_{ij} g_j\left(\frac{v_j^*}{d_j}\right) + \sum_{j=1}^n \int_0^\tau c_{ij}(s) g_j\left(\frac{v_j^*}{d_j}\right) ds + u_i, \quad i = 1,...,n \tag{3.3}$$

To finish the proof, it suffices to show that (3.3) has a unique solution. Consider a mapping $\Phi: R^n \to R^n$ defined by

$$\Phi(v_1,...,v_n)=\begin{pmatrix}\sum_{j=1}^{n}a_{1j}g_j\left(\frac{v_j}{d_j}\right)+\sum_{j=1}^{n}b_{1j}g_j\left(\frac{v_j}{d_j}\right)+\sum_{j=1}^{n}\int_0^{\tau}c_{1j}(s)g_j\left(\frac{v_j}{d_j}\right)ds+u_1\\ \vdots\\ \sum_{j=1}^{n}a_{nj}g_j\left(\frac{v_j}{d_j}\right)+\sum_{j=1}^{n}b_{nj}g_j\left(\frac{v_j}{d_j}\right)+\sum_{j=1}^{n}\int_0^{\tau}c_{nj}(s)g_j\left(\frac{v_j}{d_j}\right)ds+u_n\end{pmatrix}$$

For $v=(v_1,...,v_n)$, $\bar{v}=(\bar{v}_1,...,\bar{v}_n)\in R^n$, we have

$$\begin{aligned}\|\Phi(v_1,...,v_n)-\Phi(\bar{v}_1,...,\bar{v}_n)\|_1&=\sum_{i=1}^{n}\left|\sum_{j=1}^{n}\left(a_{ij}+b_{ij}+\int_0^{\tau}c_{ij}(s)ds\right)\left[g_j\left(\frac{v_j}{d_j}\right)-g_j\left(\frac{\bar{v}_j}{d_j}\right)\right]\right|\\&\le\sum_{j=1}^{n}\left(\sum_{i=1}^{n}|b_{ij}|+\sum_{i=1}^{n}|c_{ij}|+\sum_{i=1}^{n}a_{ij}\right)\frac{l_j}{d_j}|v_j-\bar{v}_j|\\&\le\sum_{j=1}^{n}\frac{l_j}{d_j}\left(\max_j\sum_{i=1}^{n}|b_{ij}|+\max_j\sum_{i=1}^{n}|c_{ij}|+\sum_{i=1}^{n}a_{ij}\right)|v_j-\bar{v}_j|\\&\le\sum_{j=1}^{n}\frac{1}{d_j}\left[l\left(\max_j\sum_{i=1}^{n}|b_{ij}|+\max_j\sum_{i=1}^{n}|c_{ij}|\right)+l_ja_{jj}^*+\sum_{i\neq j}^{n}|a_{ij}|\right]|v_j-\bar{v}_j|\\&\le\sum_{j=1}^{n}\alpha|v_j-\bar{v}_j|=\alpha\|v-\bar{v}\|_1\end{aligned}$$

this implies that $\Phi:R^n\to R^n$ is a global contraction on R^n endowed with the $\|\cdot\|_1$.Hence by contraction mapping principle, there exists a unique fixed point of the map $\Phi:R^n\to R^n$ which is a unique solution of the equation (3.3) from which the existence of a unique solution of (3.1) will follow. The proof is completed.

Consider two solutions $x(t)$ and $z(t)$ of system (2.1) corresponding to any initial values $x(t)=\phi(t)$ and $z(t)=\varphi(t)$ for $t\in[-\tau,0]$, Let $y(t)=x(t)-z(t)$, then we have

$$\dot{y}_i(t)=-d_iy_i(t)+\sum_{j=1}^{n}a_{ij}f_j(y_j(t))+\sum_{j=1}^{n}b_{ij}f_j(y_j(t-\tau_j(t)))+\sum_{j=1}^{n}\int_0^{\tau}c_{ij}(s)f_j(y_j(t-s))ds$$

$i=1,...,n$

or $$\dot{y}(t)=-Dy(t)+Af(y(t))+Bf(y(t-\tau(t)))+\int_0^{\tau}C(s)f(y(t-s))ds$$

where $f_j(y_j(t))=g_j(y_j(t)+z_j(t))-g_j(z_j(t))$, $\tau(t)=(\tau_1(t),...,\tau_n(t))$, the functions f_j satisfy the hypothesis H_i (i=1,2) and $f_j(0)=0$, $j=1,...,n$.

let $F_j(y_j(t))=f_j(y_j(t))/y_j(t)$, $F(y(t))=diag(F_1(y_1(t)),...,F_n(y_n(t)))$, then $0\le F_j(y_j(t))\le l_j$, $j=1,...,n$.

Theorem 3.2. Assume that $x(t)$ and $z(t)$ are two solutions of system (2.1) corresponding to any initial values $x(t)=\phi(t)$ and $z(t)=\varphi(t)$ for $t\in[-\tau,0]$, if the condition (3.2) is satisfied then $\|x(t)-z(t)\|_1\le\sup_{t_0-\tau\le s\le t_0}\|x(s)-z(s)\|_1e^{-\lambda(t-t_0)}\qquad t\ge t_0$

where λ is unique positive solution of the following equation

$$\lambda=-\mu_1(-D+A^*L)-l(\|B\|_1+\|C\|_1)e^{\lambda\tau}$$

Proof. Consider the rate of change of the 1-norm of $y(t)$

$$\begin{aligned}\frac{d^+\|y(t)\|_1}{dt}&=\lim_{s\to0^+}\frac{\|y(t+s)\|_1-\|y(t)\|_1}{s}=\lim_{s\to0^+}\frac{\|y(t)+sy'(t)+o(s)\|_1-\|y(t)\|_1}{s}\\&=\lim_{s\to0^+}\frac{\left\|\{I+s[-D+AF(y(t))]\}y(t)+sBf(y(t-\tau(t)))+s\int_0^{\tau}C(s)f(y(t-s))ds\right\|_1-\|y(t)\|_1}{s}\end{aligned}$$

$$\le \lim_{s\to 0^+}\frac{\|I+s[-D+AF(y(t))]\|_1-1}{s}\|y(t)\|_1+\|B\|_1\|f(y(t-\tau(t)))\|_1+\int_0^{\tau}\|C(s)f(y(t-s))\|_1 ds$$

$$\le \mu_1(-D+A^*L)\|y(t)\|_1+l(\|B\|_1+\|C\|_1)\sup_{t-\tau\le s\le t}\|y(s)\|_1$$

By lemma 1, if $l(\|B\|_1+\|C\|_1)<-\mu_1(-D+A^*L)$, then we have

$$\|y(t)\|_1 \le \sup_{t_0-\tau\le s\le t_0}\|y(s)\|_1 e^{-\lambda(t-t)} \qquad t\ge t_0$$

where λ is unique positive solution of the following equation

$$\lambda=-\mu_1(-D+A^*L)-l(\|B\|_1+\|C\|_1)e^{\lambda\tau}$$

This completes the proof.

By using Theorem 3.2, we can easily derive the following Corollaries.

Corollary 3.1. Assume that x^* is the equilibrium point of system (2.1), if the condition (3.2) is satisfied, then x^* is globally exponential stable.

Proof. Assume that $x(t)$ is a solution of the system (2.1) holding the initial condition (2.2), then

$$\|x(t)-x^*\|_1 \le \sup_{0-\tau\le s\le 0}\|x(s)-x^*\|_1 e^{-\lambda t}=e^{-\lambda t}\sup_{0-\tau\le s\le 0}\sum_{i=1}^{n}|\phi_i(s)-x_i^*| \qquad t\ge 0$$

$$\le\left(\sum_{i=1}^{n}\max_{-\tau\le s\le 0}|\phi_i(s)-x_i^*|\right)e^{-\lambda t}=\|\phi(s)-x^*\|e^{-\lambda t}$$

where λ is unique positive solution of the following equation

$$\lambda=-\mu_1(-D+A^*L)-l(\|B\|_1+\|C\|_1)e^{\lambda\tau}$$

This implies x^* is globally exponential stable. The proof is completed.

Corollary 3.2. The equilibrium point of system (2.1) x^* is globally exponential stable if there exist a positive diagonal matrixes $P=diag(p_1,p_2,...p_n)$ such that

$$l\left(\max_j\sum_{i=1}^{n}\frac{p_i}{p_j}|b_{ij}|+\max_j\sum_{i=1}^{n}\frac{p_i}{p_j}|c_{ij}|\right)<\min_j\left(d_j-l_j a_{jj}^*-\sum_{i\ne j}^{n}\frac{p_i l_j}{p_j}|a_{ij}|\right) \qquad (3.4)$$

Proof. Using lemma 2, we have

$$\mu_P^1(-D+A^*L)=\mu_1\{P(-D+A^*L)P^{-1}\}=\mu_1\{-D+PA^*LP^{-1}\}=\max_j\left(-d_j+l_j a_{jj}^*+\sum_{i\ne j}^{n}\frac{p_i l_j}{p_j}|a_{ij}|\right)$$

$$l(\|B\|_P^1+\|C\|_P^1)=l(\|PBP^{-1}\|_1+\|PCP^{-1}\|1)=l\left(\max_j\sum_{i=1}^{n}\frac{p_i}{p_j}|b_{ij}|+\max_j\sum_{i=1}^{n}\frac{p_i}{p_j}|c_{ij}|\right)$$

Hence, if the condition (3.4) holds, then we can conclude that

$$l(\|B\|_P^1+\|C\|_P^1)<-\mu_P^1(-D+A^*L)$$

Similar to the proof theorem 1, we have that if $x(t)$ and $z(t)$ denote two solutions of system (2.1) corresponding to any initial values $x(t)=\phi(t)$ and $z(t)=\varphi(t)$ for $t\in[-\tau,0]$, then

$$\|x(t)-z(t)\|_P^1 \le \sup_{t_0-\tau\le s\le t_0}\|x(s)-z(s)\|_P^1 e^{-\lambda(t-\tau)} \qquad t\ge t_0$$

where, λ is unique positive solution of the following equation

$$\lambda=-\mu_P^1(-D+A^*L)-l(\|B\|_P^1+\|C\|_P^1)e^{\lambda\tau}$$

Using the method of the proof Corollary 3.1, we have

$$\|x(t)-x^*\|_P^1 \le \sup_{0-\tau\le s\le 0}\|x(s)-x^*\|_P^1 e^{-\lambda t} = e^{-\lambda t}\sup_{0-\tau\le s\le 0}\sum_{i=1}^{n}\left|p_i\left(\phi_i(s)-x_i^*\right)\right| \qquad t\ge 0$$

$$\le \max_i(p_i)\left(\sum_{i=1}^{n}\max_{-\tau\le s\le 0}\left|\phi_i(s)-x_i^*\right|\right)e^{-\lambda t}$$

$$=\max_i(p_i)\|\phi(s)-x^*\|e^{-\lambda t}=M\|\phi(s)-x^*\|e^{-\lambda t}$$

where, $x(t)$ denotes a solution of the system (2.1) holding the initial condition (2.2)

This implies x^* is globally exponential stable. The proof is completed.

Corollary 3.3. The equilibrium point x^* of system (2.1) is globally exponential stable if.

$$\max_j\sum_{i=1}^{n}|b_{ij}|+\max_j\sum_{i=1}^{n}c_{ij}<\frac{1}{l}\min_i(d_i)-\max_j\left(a_{jj}^*+\sum_{i=1}^{n}|a_{ij}|\right) \qquad (3.5)$$

Proof. Using lemma 2, we have

$$\mu_1(-D+A^*L)\le\mu_1(-D)+\mu_1(A^*L)=\max_i(-d_i)+\|A^*\|_1\|L\|_1 \le -\min_i(d_i)+l\max_j\left(a_{jj}^*+\sum_{i=1}^{n}|a_{ij}|\right) \qquad (3.6)$$

From (3.5) - (3.6), we get that

$$l\left(\|B\|_1+\|C\|_1\right)<-\mu_1(-D+A^*L)$$

Applying Corollary 3.1, we can complete the proof.

4. Illustrative examples

Example 1. Consider the following system

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix}+\begin{pmatrix}0.1 & 0.2\\ 0.3 & 0.3\end{pmatrix}\begin{pmatrix}\tanh(x_1(t))\\ \tanh(x_2(t))\end{pmatrix}+\begin{pmatrix}0.05 & 0.03\\ 0.06 & 0.04\end{pmatrix}\begin{pmatrix}\tanh(x_1(t-\tau_1(t)))\\ \tanh(x_2(t-\tau_2(t)))\end{pmatrix}$$
$$+\int_0^1\begin{pmatrix}0.1(1-s) & 0.3(1-s)\\ 0.2(1-s) & 0.3(1-s)\end{pmatrix}\begin{pmatrix}\tanh(x_1(t-s))\\ \tanh(x_2(t-s))\end{pmatrix}ds+\begin{pmatrix}1\\ 1\end{pmatrix} \qquad (4.1)$$

where $g_1(x)=g_2(x)=\tanh(x)$, clearly satisfy hypothesis $H_1, l_1=l_2=1\cdot$ $\tau_1(t)=\tau_2(t)=\left|\sin\frac{\pi}{2}(t)\right|$, $\tau=1\cdot$

It is obvious that the delay is not differentiable,

$$D=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\quad A=\begin{pmatrix}0.1 & 0.2\\ 0.3 & 0.3\end{pmatrix}\quad B=\begin{pmatrix}0.05 & 0.03\\ 0.06 & 0.04\end{pmatrix}\quad C=\int_0^1\begin{pmatrix}0.1(1-s) & 0.3(1-s)\\ 0.2(1-s) & 0.3(1-s)\end{pmatrix}ds=\begin{pmatrix}0.05 & 0.15\\ 0.10 & 0.15\end{pmatrix}$$

We can easily check that

$$l\left(\|B\|_1+\|C\|_1\right)=0.45<0.50=-\mu_1(-D+A^*L)$$

It then follows from Corollary 3.1 that the equilibrium point of the system (4.1) is globally exponential stable.

Example 2. Consider the following system

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{pmatrix}=\begin{pmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{pmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{pmatrix}+\begin{pmatrix}0.1 & 0.1 & 0.6\\ 0.2 & 0.1 & 0.4\\ 0.2 & 0.9 & 0.4\end{pmatrix}\begin{pmatrix}\tanh(x_1(t))\\ \tanh(x_2(t))\\ \tanh(x_3(t))\end{pmatrix}$$

$$+\begin{pmatrix}0.02 & 0.02 & 0.03\\ 0.01 & 0.04 & 0.05\\ 0.01 & 0.07 & 0.08\end{pmatrix}\begin{pmatrix}\tanh(x_1(t-\tau_1(t)))\\ \tanh(x_1(t-\tau_2(t)))\\ \tanh(x_1(t-\tau_3(t)))\end{pmatrix}$$

$$+\int_0^1 \begin{pmatrix} 0.3(1-s) & 0.3\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.2\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \\ 0.1(1-s) & 0.2\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.3\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \\ 0.2(1-s) & 0.2\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.4\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \end{pmatrix} \begin{pmatrix} \tanh\left(x_1(t-s)\right) \\ \tanh\left(x_2(t-s)\right) \\ \tanh\left(x_3(t-s)\right) \end{pmatrix} ds + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(4.2)

where $g_1(x)=g_2(x)=g_3(x)\tanh(x)$, $l_1=l_2=l_3=1$, $\tau_1(t)=\tau_2(t)=\tau_3=\left|\cos\frac{\pi}{2}(t)\right|$, $\tau=1$.

$$D=\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad A=\begin{pmatrix} 0.1 & 0.1 & 0.6 \\ 0.2 & 0.1 & 0.4 \\ 0.2 & 0.9 & 0.4 \end{pmatrix} \qquad B=\begin{pmatrix} 0.02 & 0.02 & 0.03 \\ 0.01 & 0.04 & 0.05 \\ 0.01 & 0.07 & 0.08 \end{pmatrix}$$

$$C=\int_0^1 \begin{pmatrix} 0.3(1-s) & 0.3\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.2\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \\ 0.1(1-s) & 0.2\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.3\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \\ 0.2(1-s) & 0.2\left(1-\frac{1}{\pi}\sin\frac{\pi}{2}s\right) & 0.4\left(1-\frac{1}{\pi}\cos\frac{\pi}{2}s\right) \end{pmatrix} ds=\begin{pmatrix} 0.15 & 0.15 & 0.10 \\ 0.50 & 0.10 & 0.15 \\ 0.10 & 0.10 & 0.20 \end{pmatrix}$$

We can check that

$$l\left(\|B\|_1+\|C\|_1\right)=0.61>0.60=-\mu_1\left(-D+A^*L\right)$$

we cannot determine that the system (4.2) is globally exponential stable, if we use Corollary 1. However, let $P=diag(1,2,2)$, we have

$$l\left(\max_j\sum_{i=1}^3\frac{p_i}{p_j}|b_{ij}|+\max_j\sum_{i=1}^3\frac{p_i}{p_j}|c_{ij}|\right)=0.595<0.90=\min_j\left(d_j-l_ja_{jj}^*-\sum_{i\neq j}^3\frac{p_il_j}{p_j}|a_{ij}|\right)$$

So, it follows from Corollary 3.2 that the equilibrium point of the system (4.2) is globally exponential stable.

5. Conclusions

In this paper, several sufficient criterions have been derived to guarantee global exponential stability of the neural networks with distributed delays without assuming the differentiability of delay. Different from the normal method, i.e., constructing suitable Lyapunov function, these results are obtained based on matrix measure and Halanay inequality approach. Our results are easily checkable and valuable in the design of global exponential stability of neural networks.

Acknowledgment

The authors would like to thank the Associate Editor and the reviewers for their detailed comments,which considerably improved the presentation of this paper.

References

A., Cichocki & R., Unbehauen. (1993). *Neural Networks for Optimization and Signal Processing*. Chichester, U.K.: Wiley.

B., Yuhas & N., Ansari, Eds. (1993). *Neural Networks in Telecommunications.Norwell*. MA: Kluwer.

Cao, J, Huang, DS, & Qu, Y. (2005). Global robust stability of delayed recurrent neural networks. *Chaos, Solitons & Fractals*. 23(1):221-9.

Cao, J. & Wang, J. (2004). Absolute exponential stability of recurrent neural networks with time delays and Lipschitz-continuous activation functions. *Neural Networks*. 17(3):379-90.

Cao, J & Wang L. (2002). Exponential stability and periodic oscillatory solution in BAM networks with delays. *IEEE Trans Neural Networks*. 13(2):457-63.

Civalleri, PP, Gilli, LM & Pabdolfi, L. (1993). On stability of cellular neural networks with delay. *IEEE Trans Circ Syst I*. 40:157-65.

E. K. P. Chong, S. Hui, & S. H. Zak. (1999). An analysis of a class of neural networks for solving linear programming problems. *IEEE Trans. Automat.Control*. vol. 44, Nov. pp. 1095-2006.

Gopalsamy K and He XZ. (1994). Stability in asymmtric Hopfield nets with transmission delays. *Physica D*. 76:344-58.

Gopalsamy K, He X. (1994). Stability in asymmetric Hopfield nets with transmission delays. *Phys D*. 76:344-58.

Gopalsamy. (1992). *Stability and oscillations in delay differential equations of population dynamics*. Kluwer, Dordrecht.

Hasan SMR & Siong NK. (1995). A VLSI BAM neural network chip for pattern recognition application. *Proc IEEE Int Conf Neural Networks*. 1:164-8.

Kosko B. (1992). A dynamical system approach machine intelligence. In: Neural Networks and Fuzzy Systems. *Englewood Cliffs, NJ: Prentice-Hall*. p. 38-108.

M. Forti and A. Tesi. (1995). New conditions for global stability of neural networks with application to linear and quadratic programming problems. *IEEE Trans. Circuits Syst. I*. vol. 42, July, pp. 354-366.

M. P. Kennedy and L. O. Chua. (1988). Neural networks for nonlinear programming, *IEEE Trans. Circuits Syst. I*. vol. 35, May, pp. 554-562.

M.Forti, S. Manetti and A.Tesi. (2001). A New method to analyze complete stability of PWL cellular neural networks. *Int. J. Bifurc.Chaos*. vol. 11, Mar. J, pp.655-676.

R. Fantacci, M. Forti, M. Marini, and L. Pancani. (1999). Cellular neural network approach to a class of communication problems. *IEEE Trans. Circuits Syst. I*. vol. 45, Dec.,pp. 1457-1467.

Roska T and Chua LO. (1992). Cellular neural networks with delay type template elements and nonuniform grids. *Int J Circ Theory*. Appl; 20(4):469-81.

Wei, JJ, Ruan SG. (1999). Stability and bifurcation in a neural network model with two delays. *Phys D*. 130:255-72.