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Study on Combined shell Mechanics Analysis

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Abstract

The AUV combines mostly in ball shell, cylindrical shell, taper shells and other rotary shells by thread coupling, bolt coupling, wedge coupling and hoop coupling. This paper makes the finite element analysis and research on the mechanics mode of a certain AUV with the analytic method. Based on the basic equation of theory of thin shells, analysed every separated shells, and set up it's mechanics mathematical model, and analysed the combined shell with the finite element method. At last, the final result validated the mathematical model. The method presented is effective in analysing and dynamical designing of AUV structure.

Keywords: Combined shell, Mathematical model, FEA

During the work progress of AUV, such as torpedo and mine, the shell endures the hydraulic pressure. The research of vibration has important theory value and practical meaning on AUV. The AUV is combines mostly in ball shells, column shells, taper shells and other rotary shells by thread coupling, bolt coupling, wedge coupling and hoop coupling. It is shown in figure 1.

1. The basic theoretical equation of thin shell

A middle surface patch of thin shell and internal forces on the cross section are shown in figure 2. The parameters $N_1, N_2, N_{12}, M_1, M_2, M_{12}, Q_1, Q_2$ are the internal forces acted on α plane and β plane, k_1 and k_2 are the main curvatures on α direction and β direction, R_1 and R_2 are the radius of main curvature on the middle surface, and $k_1 = 1/R_1$, $k_2 = 1/R_2$, A and B are the Lame coefficients on α direction and β direction, p_1, p_2, p_3 are the component of loads on α direction , β direction and γ direction, u , v and w are the component of displacements on α direction, β direction and γ direction of any point on the middle surface of shell.

The balanceable equations of basic equation in the thin shell theory are:

$$\left.\begin{aligned}
&\frac{\partial}{\partial\alpha}(BN_1)-\frac{\partial B}{\partial\alpha}N_2+\frac{\partial A}{\partial\beta}N_{12}+\frac{\partial}{\partial\beta}(AN_{12})+ABk_1Q_1+ABp_1=0\\
&\frac{\partial}{\partial\beta}(AN_2)-\frac{\partial A}{\partial\beta}N_1+\frac{\partial B}{\partial\alpha}N_{12}+\frac{\partial}{\partial\alpha}(BN_{12})+ABk_2Q_2+ABp_2=0\\
&\frac{\partial}{\partial\alpha}(BM_{12})+\frac{\partial B}{\partial\alpha}M_{12}-\frac{\partial A}{\partial\beta}M_1+\frac{\partial}{\partial\beta}(AM_2)-ABQ_2=0\\
&\frac{\partial}{\partial\beta}(AM_{12})+\frac{\partial A}{\partial\beta}M_{12}-\frac{\partial B}{\partial\alpha}M_2+\frac{\partial}{\partial\alpha}(BM_1)-ABQ_1=0\\
&-AB(k_1N_1+k_2N_2)+\frac{\partial}{\partial\alpha}(BQ_1)+\frac{\partial}{\partial\beta}(AQ_2)+ABp_3=0
\end{aligned}\right\}\tag{1.1}$$

From the geometrical equations (1.2) and the physical equations (1.3) of basic equation in the thin shell theory, we can reason out the elastic equations (1.4).

$$\left.\begin{aligned}\varepsilon_1 &= \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{1}{AB}\frac{\partial A}{\partial \beta}v + k_1 w\\ \varepsilon_2 &= \frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{1}{AB}\frac{\partial B}{\partial \alpha}u + k_2 w\\ \varepsilon_{12} &= \frac{A}{B}\frac{\partial}{\partial \beta}(\frac{u}{A}) + \frac{B}{A}\frac{\partial}{\partial \alpha}(\frac{v}{B})\end{aligned}\right\} \tag{1.2}$$

$$\left.\begin{aligned}N_1 &= \frac{Eh}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)\\ N_2 &= \frac{Eh}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1)\\ N_{12} &= N_{21} = \frac{Eh}{2(1+\mu)}\varepsilon_{12}\end{aligned}\right\} \tag{1.3}$$

$$\left.\begin{aligned}&\frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{1}{AB}\frac{\partial A}{\partial \beta}v + k_1 w = \frac{(N_1 - \mu N_2)}{Eh}\\ &\frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{1}{AB}\frac{\partial B}{\partial \alpha}u + k_2 w = \frac{(N_2 - \mu N_1)}{Eh}\\ &\frac{A}{B}\frac{\partial}{\partial \beta}(\frac{u}{A}) + \frac{B}{A}\frac{\partial}{\partial \alpha}(\frac{v}{B}) = \frac{2(1+\mu)N_{12}}{Eh}\end{aligned}\right\} \tag{1.4}$$

The state of nonmomental theory supposed there are no both flexural moment and torsional moment on the any cross section of the thin shell, that is $M_1 = M_2 = M_{12} = M_{21} = 0$. Equations (1.1) are simplified (1.5).

$$\left.\begin{aligned}&\frac{\partial}{\partial \alpha}(BN_1) - \frac{\partial B}{\partial \alpha}N_2 + \frac{\partial A}{\partial \beta}N_{12} + \frac{\partial}{\partial \beta}(AN_{12}) + ABp_1 = 0\\ &\frac{\partial}{\partial \beta}(AN_2) - \frac{\partial A}{\partial \beta}N_1 + \frac{\partial B}{\partial \alpha}N_{12} + \frac{\partial}{\partial \alpha}(BN_{12}) + ABp_2 = 0\\ &-k_1 N_1 - k_2 N_2 + p_3 = 0\end{aligned}\right\} \tag{1.5}$$

1.1 The Cylindrical Shell

The α-axis point to the generatrix and the β-axis point to the circumference of cylindrical shell, then, $k_1 = 0$, $k_2 = 1/R$ and $A = B = 1$, the Gauss-Codazzi conditions are fulfiled. It is shown in figure 3. The balanceable equations and elastic equations of cylindrical shell nonmomental theory are:

$$\left.\begin{aligned}&\frac{\partial N_1}{\partial \alpha} + \frac{\partial N_{12}}{\partial \beta} + p_1 = 0\\ &\frac{\partial N_2}{\partial \beta} + \frac{\partial N_{12}}{\partial \alpha} + p_2 = 0\\ &-N_2 + Rp_3 = 0\end{aligned}\right\} \tag{1.6}$$

$$\left.\begin{aligned}&\frac{\partial u}{\partial \alpha} = \frac{N_1 - \mu N_2}{Eh}\\ &\frac{\partial v}{\partial \beta} + \frac{w}{R} = \frac{N_2 - \mu N_1}{Eh}\\ &\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} = \frac{2(1+\mu)N_{12}}{Eh}\end{aligned}\right\} \tag{1.7}$$

1.2 The Gyral Shell

The parameter C_1 is the curvature center of point M on the gyral shell generatrix. It is shown in figure 4. The curvatures are $k_1 = 1/R_1$ (α direction) and $k_2 = 1/R_2$ (β direction) on the middle surface. At the point M, $ds_1 = R_1 d\alpha$, $ds_2 = R_2 \sin\alpha d\beta$, $A = R_1$, $B = R_2 \sin\alpha$.

The Gauss-Codazzi conditions $\frac{\partial}{\partial\beta}(k_1 A) = k_2\frac{\partial A}{\partial\beta}$ and $\frac{\partial}{\partial\alpha}(k_2 B) = k_1\frac{\partial B}{\partial\alpha}$ are fulfiled: $\frac{dB}{d\alpha} = \frac{1}{k_1}\frac{dk_2 B}{d\alpha} = R_1\frac{d\sin\alpha}{d\alpha} = R_1\cos\alpha$, then $\frac{d(R_2\sin\alpha)}{d\alpha} = R_1\cos\alpha$. The balanceable equations and elastic equations of gyral shell nonmomental theory are:

$$\left.\begin{aligned} &\frac{1}{R_1}\frac{\partial N_1}{\partial\alpha} + \frac{ctg\alpha}{R_2}(N_1 - N_2) + \frac{1}{R_2\sin\alpha}\frac{\partial N_{12}}{\partial\beta} + p_1 = 0 \\ &\frac{1}{R_1}\frac{\partial N_{12}}{\partial\alpha} + \frac{2ctg\alpha}{R_2}N_{12} + \frac{1}{R_2\sin\alpha}\frac{\partial N_2}{\partial\beta} + p_2 = 0 \\ &-\frac{N_1}{R_1} - \frac{N_2}{R_2} + p_3 = 0 \end{aligned}\right\} \tag{1.8}$$

$$\left.\begin{aligned} &\frac{1}{R_1}\frac{\partial u}{\partial\alpha} + \frac{w}{R_1} = \frac{(N_1 - \mu N_2)}{Eh} \\ &\frac{1}{R_2\sin\alpha}\frac{\partial v}{\partial\beta} + \frac{\cos\alpha}{R_1\sin\alpha}u + \frac{1}{R_2}w = \frac{(N_2 - \mu N_1)}{Eh} \\ &\frac{1}{R_2\sin\alpha}\frac{\partial u}{\partial\beta} - \frac{\cos\alpha}{\sin^2\alpha}\frac{\partial v}{\partial\alpha} = \frac{2(1+\mu)N_{12}}{Eh} \end{aligned}\right\} \tag{1.9}$$

The ball shell is the special gyral shell, and $A = B = 1$, $k_1 = k_2 = 1/R$ in the ball shell.

2. The axial symmetrical bending equations of thin shell

2.1 The Axial Symmetrical Bending Equations Of Cylindrical Shell

The internal forces, displacements and strains are axial symmetrical in the cylindrical shell. The internal forces reduce to N_1 , N_2 , M_1 , M_2 , Q_1 , and the displacements reduce to u , w . The axial symmetrical bending equations of the cylindrical shell are:

$$\frac{d^4 w}{d\alpha^4} + \frac{Eh}{R^2 D}w = \frac{p_3}{D} \tag{2.1}$$

Dimensionless coordinate is brought in, $\xi = \lambda\alpha$, where $\lambda = (\frac{Eh}{4R^2 D})^{\frac{1}{4}}$, then

$$\frac{d^4 w}{d\xi^4} + 4w = \frac{4R^2}{Eh}p_3 \tag{2.2}$$

The approximate solution of equations (2.2) is made up of the nonmomental theory solution (w^*) and the edge effect solution (w^0), that is,

$$w = w^* + w^0 = w^* + e^{-\xi}(C_1\cos\xi + C_2\sin\xi) + e^{\xi}(C_3\cos\xi + C_4\sin\xi) \tag{2.3}$$

In the equation (2.3), the edge effect solution (w^0) is the solution on the effect of the flexural moment (M_0) and the lateral shearing force (Q_0) that are equally distributed along the boundary at the side of $\alpha = \xi = 0$,

$$\left.\begin{aligned} &w^0 = -\frac{M_0}{2\lambda^2 D}f_3(\xi) - \frac{Q_0}{2\lambda^3 D}f_4(\xi) \\ &\frac{dw^0}{d\alpha} = \lambda\frac{dw^0}{d\xi} = \frac{Q_0}{2\lambda^2 D}f_1(\xi) + \frac{M_0}{\lambda D}f_4(\xi) \\ &M_1 = M_0 f_1(\xi) + \frac{Q_0}{\lambda}f_2(\xi) \\ &Q_1 = Q_0 f_3(\xi) - 2\lambda M_0 f_2(\xi) \\ &M_2 = \mu M_1 \text{ , } N_2 = \frac{Eh}{R}w^0 \end{aligned}\right\} \tag{2.4}$$

Where, $f_1(\xi) = e^{-\xi}(\cos\xi + \sin\xi)$, $f_2(\xi) = e^{-\xi}\sin\xi$, $f_3(\xi) = e^{-\xi}(\cos\xi - \sin\xi)$, $f_4(\xi) = e^{-\xi}\cos\xi$.

2.2 The Axial Symmetrical Bending Equations Of Gyral Shell

The parameters of the gyral shell, $k_1 = 1/R_1$, $k_2 = 1/R_2$, $A = R_1$, $B = R_2\sin\alpha$, and on the condition of axial symmetrical bend, $N_{12} = M_{12} = Q_2 = 0$ and $p_2 = 0$, the axial symmetrical bending balanceable equations of the gyral shell are:

$$\left.\begin{aligned}&\frac{1}{R_1}\frac{dN_1}{d\alpha}-\frac{N_2}{R_1}+\frac{tg\alpha}{R_1}Q_1+tg\alpha p_1=0\\&\frac{1}{R_1}\frac{dM_1}{d\alpha}+\frac{M_2}{R_1}-tg\alpha Q_1=0\\&-\frac{N_1}{R_1}-\frac{N_2}{R_2\sin\alpha}-\frac{ctg\alpha}{R_1}\frac{dQ_1}{d\alpha}+p_3=0\end{aligned}\right\}\tag{2.5}$$

The approximate solution of equations (2.5) is made up of the general solution of the homogeneous equation and the special solution of the unhomogeneous equation. The special solution can be solved from the nonmomental theory equations, and the general solution , the edge effect solution, can be solved by hybrid method. Then the equations (2.5) simplified to the equations (2.6).

$$\left.\begin{aligned}&\frac{1}{R_1}\frac{dN_1^0}{d\alpha}-\frac{N_2^0}{R_1}+\frac{tg\alpha}{R_1}Q_1=0\\&\frac{1}{R_1}\frac{dM_1}{d\alpha}+\frac{M_2}{R_1}-tg\alpha Q_1=0\\&\frac{N_1^0}{R_1}+\frac{N_2^0}{R_2\sin\alpha}+\frac{ctg\alpha}{R_1}\frac{dQ_1}{d\alpha}=0\end{aligned}\right\}\tag{2.6}$$

The basic functions are supposed, $\omega=\frac{1}{R_1}(\frac{dw^0}{d\alpha}+u^0)$, $\phi=-R_2Q_1$.

The differential operator is supposed, $L[\omega]=[\frac{R_2}{R_1}\frac{d}{d\alpha}(\frac{1}{R_1}\frac{d}{d\alpha})+\frac{ctg\alpha}{R_1}\frac{d}{d\alpha}-\frac{ctg^2\alpha}{R_2}]\omega$.

The basic differential equations that the axial symmetrical bending edge effect of gyral shell are:

$$\left.\begin{aligned}&L(\omega)-\frac{\mu}{R_1}\omega=\frac{\phi}{D}\\&L(\phi)+\frac{\mu}{R_1}\phi=-Eh\omega\end{aligned}\right\}\tag{2.7}$$

To ball shell, the curvature radius $R_1=R_2=R$ are constants, and $\phi=-RQ_1$, then,

$$\left.\begin{aligned}&\frac{d^2\omega}{d\alpha^2}+\frac{d\omega}{d\alpha}ctg\alpha-\omega(ctg^2\alpha+\mu)=\frac{R^2}{D}Q_1\\&\frac{d^2Q_1}{d\alpha^2}+\frac{dQ_1}{d\alpha}ctg\alpha-Q_1(ctg^2\alpha-\mu)=-Eh\omega\end{aligned}\right\}\tag{2.8}$$

The effect of edge effect reduce rapidly with the distance increase to boundary, then the equations (2.8) simplified to the equations (2.9):

$$\left.\begin{aligned}&\frac{d^2\omega}{d\alpha^2}=\frac{R^2}{D}Q_1\\&\frac{d^2Q_1}{d\alpha^2}=-Eh\omega\end{aligned}\right\}\tag{2.9}$$

The basic differential equations that the axial symmetrical bending of ball shell are:

$$\frac{d^4Q_1}{d\alpha^4}+\frac{EhR^2}{D}Q_1=0\tag{2.10}$$

Dimensionless coordinate is brought in, $\eta=\vartheta\alpha$, where $\vartheta=(\frac{EhR^2}{4D})^{\frac{1}{4}}$, then,

$$\frac{d^4Q_1}{d\alpha^4}+4\vartheta^4Q_1=0\tag{2.11}$$

The internal forces expressions are:

$$\left.\begin{aligned} N_1^0 &= [Pf_3(\eta) + 2M\frac{\vartheta}{R}f_2(\eta)]ctg\alpha \\ N_2^0 &= 2P\lambda f_4(\eta) - 2M\frac{\vartheta^2}{R}f_3(\eta) \\ M_1 &= -P\frac{\vartheta}{R}f_2(\eta) + Mf_1(\eta) \\ M_2 &= \mu M_1 \\ Q_1 &= -Pf_3(\eta) - 2M\frac{\vartheta}{R}f_2(\eta) \end{aligned}\right\} \quad (2.12)$$

Where, $f_1(\eta) = e^{-\eta}(\cos\eta + \sin\eta)$, $f_2(\eta) = e^{-\eta}\sin\eta$, $f_3(\eta) = e^{-\eta}(\cos\eta - \sin\eta)$, $f_4(\eta) = e^{-\eta}\cos\eta$.

3. The analysisi of torpedo

The shell of torpedo is made up of ball shell, cylindrical shell, taper shells and other rotary shells by thread coupling, bolt coupling, wedge coupling and hoop coupling. All of them are rigid coupling. The radius of ball shell $R = 0.25$, the length of cylindrical shell $L = 5.50$, the thickness of shell $h = 0.005$, the elastic modulus $E = 7.47\times10^{10}\ pa$, the Poisson's ratio $\mu = 0.36$, inner pressure $p_3 = 10^6\ pa$.

It is shown the force analysis of the coupling of the ball shell and the cylindrical shell in figure 5.

From the balanceable equations of ball shell nonmomental theory, and $R_1 = R_2 = R$, $N_1 = N_2$, obtained the result: $(N_1^*)_B = (N_2^*)_B = \frac{Rp_3}{2}$;

From the balanceable equations of cylindrical shell nonmomental theory, obtained the result: $(N_2^*)_C = Rp_3$, $(N_1^*)_C = \frac{Rp_3}{2}$.

Obviously, the circumferential direction internal force is not continuous on the coupling circumference, that is $(N_2^*)_B \neq (N_2^*)_C$, so, there is a direct displacement, and the radial alterations are: $\delta a_B = \frac{R^2 p_3}{2Eh}(1-\mu)$, $\delta a_C = \frac{R^2 p_3}{Eh}(1-\frac{\mu}{2})$.

The direct displacement is not continuous, and the difference is $\delta a = \frac{R^2 p_3}{2Eh}$. Thus, there must be Q_0 and M_0 that are equally distributed along the circumference, so that the continuousness of the internal force and displacement are ensured. Based on the theory of Timashenko, the rotations of the ball shell and the cylindrical shell are same along the circumference, so $M_0 = 0$, and the discontinuousness is avoided enough by Q_0 .

The direct displacement of the ball shell brought by Q_0 is $\delta a_1' = -\frac{Q_0}{2\lambda^3 D}$, and the cylindrical shell is $\delta a_2' = \frac{Q_0}{2\lambda^3 D}$.

The difference is $\delta a' = -\frac{Q_0}{\lambda^3 D}$,where $\lambda = (\frac{Eh}{4R^2 D})^{\frac{1}{4}}$, $D = \frac{Eh^3}{12(1-\mu^2)}$.

According to the displacement continuous condition, $\delta a + \delta a' = 0$, then,

$$Q_0 = \frac{R^2 p_3}{2Eh}\cdot\lambda^3 D = \frac{p_3\lambda^3}{2}\cdot\frac{R^2 D}{Eh} = \frac{p_3\lambda^3}{2}\cdot\frac{1}{4\lambda^4} = \frac{p_3}{8\lambda}.$$

The parameters are counted, then,

$$D = \frac{Eh^3}{12(1-\mu^2)} = \frac{7.47\times10^{10}\times0.005^3}{12(1-0.36^2)} = 894,$$

$$\lambda = (\frac{Eh}{4R^2 D})^{\frac{1}{4}} = (\frac{7.47\times10^{10}\times0.005}{4\times0.25^2\times894})^{\frac{1}{4}} = 36, \quad \vartheta = (\frac{EhR^2}{4D})^{\frac{1}{4}} = 9$$

When $\xi = 0$,then $f_1(\xi) = f_3(\xi) = f_4(\xi) = 1$, $f_2(\xi) = 0$,

When $\eta = 0$,then $f_1(\eta) = f_3(\eta) = f_4(\eta) = 1$, $f_2(\eta) = 0$,

The results of the cylindrical shell are:

$$\left.\begin{array}{l} N_1 = \dfrac{p_3 R}{2} = 1.25\times 10^5 \ ,\ N_2 = \mu \dfrac{p_3 R}{2} + \dfrac{Eh}{R} w = 1.875\times 10^5 \\ M_1 = \dfrac{p_3}{8\lambda^2} f_2(\xi) = 0 \ ,\ M_2 = \mu M_1 = 0 \\ Q_1 = \dfrac{p_3}{8\lambda} f_3(\xi) = 3472 \end{array}\right\}$$

The results of the ball shell are:

$$\left.\begin{array}{l} N_1 = \dfrac{p_3 R}{2} = 1.25\times 10^5 \ ,\ N_2 = \dfrac{p_3 R}{2} + \dfrac{p_3 \vartheta}{4\lambda} f_3(\eta) = 1.875\times 10^5 \\ M_1 = -\dfrac{p_3 R}{8\lambda\vartheta} f_2(\eta) = 0 \ ,\ M_2 = \mu M_1 = 0 \\ Q_1 = -\dfrac{p_3}{8\lambda} f_3(\eta) = -3472 \end{array}\right\}$$

As a result, the circumferential direction internal force is not continuous on the coupling circumference.

References

P. Smithmaitrie & H.S. Tzou. (2004). Micro-control actions of actuator patches laminated on hemispherical shells. *Journal of Sound and Vibration.* 277, 157-164.

J.H. Ding & H.S. Tzou. (2004). Micro-electromechanics of sensor patches on free paraboloidal shell structronic systems *Mechanical Systems and Signal Processing.* 18. 367-380.

Xu, zhilun. (1982). *Elastomechanics*. The High Education Press.

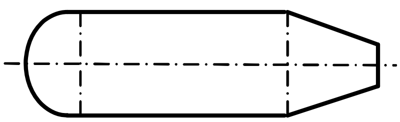

Figure 1. AUV shell

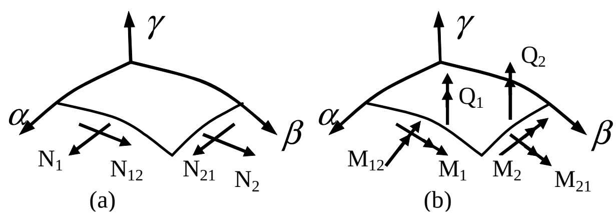

Figure 2. Space orthogonal coordinate system

Figure 4. Gyral

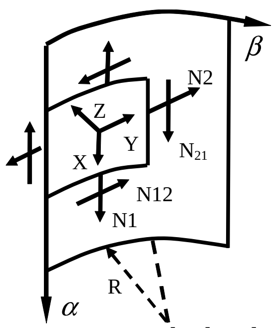

Figure 3. Cylindrical

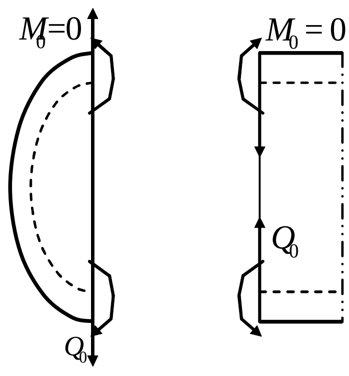

Figure 5. Force analysis of coupling