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Calculation of Shrinkage Stress of Semi-rigid Base Courses

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Abstract

The calculation model of the shrinkage stress of semi-rigid base course was established. By analyzing the stressing status of the tiny element in the model, the correlative formulae were founded, and the expression of the shrinkage stress was deduced. The maximum value of the shrinkage stress of semi-rigid base course was calculated and the position of the maximum value was presented. Besides, the influencing factors of the maximum value of the shrinkage stress were analyzed.

Keywords: Semi-rigid base course, Shrinkage stress, Stress analysis

1. Introduction

With the advantages of higher strength, larger rigidity, more excellent integrity and water stability, the semi-rigid base course has become the major structure type of base courses in high-grade highways. However, the major maintenance problem of cracking is becoming more and more severer with more and more applications of this base course and this problem has affected the performance of roads badly. A large amount of study shows that the cracking of semi-rigid base courses is induced by thermal shrinkage and dry shrinkage of semi-rigid materials (Zhang, 1991, pp.16-21). Thermal shrinkage and dry shrinkage of semi-rigid base courses will cause shrinkage stress, and once the shrinkage stress exceeds the tensile strength of the semi-rigid materials, cracks will come into being (Zheng and Yang, 2003). To effectively prevent cracks of semi-rigid base courses, it is necessary to calculate the shrinkage stress of semi-rigid base course.

2. Basic assumptions in the calculation

(1) While subbase courses and base courses bring relative displacement in horizontal direction, the friction stress of a certain point on the interface is directly proportional to the horizontal displacement of the point (Wang, 1997), as is shown in Eq. (1):

$$\tau_x = -C_x \cdot u_x \tag{1}$$

where C_x = Friction coefficient (to subbase courses, $C_x = 0.6N/mm^3$), u_x =The horizontal displacement of point x in base courses (the minus of the formula shows that the direction of friction stress is always inverse to the displacement).

(2) It is assumed that the temperature and humidity of semi-rigid base courses is dropping equably from top to the bottom, in addition, the corresponding thermal shrinkage, thermal shrinkage stress, dry shrinkage and dry shrinkage stress are all epuable.

(3) By investigating the cracks observed in practical projects, it can be seen that shrinkage cracks of semi-rigid base courses are transverse and parallel each other mostly. So it is assumed that semi-rigid base courses are only restricted longitudinal but not transverse.

3. Formula deduction of the shrinkage stress

The cross section of the semi-rigid base course is rectangular. On a random point x of the semi-rigid base course, a length of tiny element is chosen as the study object, and its length, width and thickness are dx, B, and H respectively. A stressing model is established (see Figure 1), and the length of the base course for study is L. In the model, the average shrinkage stress in the section is expressed as $\sigma(x)$, and the shearing stress is expressed as τ_x, via. The friction stress between the base course and subbase course.

The equilibrium equation of the tiny element in horizontal direction is as follow:

$$\left[\sigma(x)+d\sigma(x)\right]BH-\sigma(x)BH+\tau_x Bdx=0 \quad (2)$$

The solution of Eq. (2) is

$$\frac{d\sigma(x)}{dx}+\frac{\tau_x}{H}=0 \quad (3)$$

When the temperature and humidity of the semi-rigid base course is decreasing, the displacement of section x in the base course, which is composed of restriction displacement and free displacement, is expressed as

$$u_x=u_\sigma+(\alpha_t t+\alpha_d\omega)x \quad (4)$$

where u_x = The displacement of section x in base course; u_σ = Restriction displacement, α_t = Thermal shrinkage coefficient, t = The temperature changes, α_d = Dry shrinkage coefficient, ω = The humidity changes. By differentiating Eq. (4), a differential equation is obtained

$$\frac{du_x}{dx}=\frac{du_\sigma}{dx}+(\alpha_t t+\alpha_d\omega) \quad (5)$$

By differentiating Eq. (5), the following second-order differential equation is obtained

$$\frac{d^2u_x}{dx^2}=\frac{d^2u_\sigma}{dx^2} \quad (6)$$

The shrinkage stress in the section can also be given as

$$\sigma(x)=E\frac{du_\sigma}{dx} \quad (7)$$

Where E = Elastic modulus of the base course. By differentiating Eq. (7) and combining with Eq. (6), a differential equation is obtained as

$$\frac{d\sigma(x)}{dx}=E\frac{d^2u_x}{dx^2} \quad (8)$$

By combining Eq. (8) with Eq. (3), the governing equation is expressed as

$$E\frac{d^2u_x}{dx^2}+\frac{\tau_x}{H}=0 \quad (9)$$

By combining with Eq. (1) and letting $\beta=\sqrt{C_x/EH}$, Eq. (9) can be written as

$$\frac{d^2u_x}{dx^2}-\beta^2u_x=0 \quad (10)$$

The general solution of differential equation (10) is evaluated as

$$u_x=C_1ch\beta x+C_2sh\beta x \quad (11)$$

The boundary conditions can be expressed as: ① If $x=0$, then $u_X=0$, ② If $x=L/2$, then $\sigma(x)=0$. From boundary condition ①, the value of C_1 can be easily calculated as $C_1=0$. By combining with Eq. (5), Eq. (7) can be written as

$$\sigma(x)=E(\frac{du_x}{dx}-\alpha_t t-\alpha_d\omega) \quad (12)$$

By differentiating Eq. (11), with $C_1=0$, the following equation is obtained

$$\frac{du_x}{dx}=\beta C_2ch(\beta x) \quad (13)$$

By combining Eq. (12) , Eq. (13) and boundary condition ②, C_2 can be calculated as $C_2 = \frac{\alpha_t t + \alpha_d \omega}{\beta ch(\beta L/2)}$, and then

u_x can be expressed as

$$u_x = \frac{\alpha_t t + \alpha_d \omega}{\beta ch(\beta L/2)} sh(\beta x) \tag{14}$$

By combining Eq. (12) and Eq. (14), the shrinkage stress of the section is calculated as

$$\sigma(x) = -E(\alpha_t t + \alpha_d \omega)[1 - \frac{ch\beta x}{ch(\beta L/2)}] \tag{15}$$

From Eq. (15), it can be seen that the shrinkage stress has maximum value when $x = 0$, and the maximum value is evaluated as

$$\sigma(x)_{\max} = -E(\alpha_t t + \alpha_d \omega)[1 - \frac{1}{ch(\beta L/2)}] \tag{16}$$

4. Influencing factors of the maximum value of the shrinkage stress

In the expression of $\sigma(x)_{\max}$, the value of t is negative because the temperature is dropping, and during the course of dry shrinkage, the humidity is decreasing gradually, so the value of ω is also negative. Thus, the value of the shrinkage stress is positive, viz. tensile stress. From Eq. (15), Eq. (16) and the expression of β, it can be seen that the shrinkage stress has maximum value in the middle of the semi-rigid base course, and the maximum value of shrinkage stress will increase with the increase of elastic modulus, temperature changes, humidity changes, thermal shrinkage coefficient, and dry shrinkage coefficient of the semi-rigid base course. Furthermore, the maximum value of shrinkage stress has relations with the degree of the restriction of the subbase course to the base course, which is depicted as C_x, the thickness H and length L. The maximum value will increase with the increase of C_x, H and L of the base course. However, all of theses relations are not linear.

5. Conclusions

The following are the main conclusions from the study:

The shrinkage stress in the middle of the semi-rigid base course has maximum value. The maximum value has relations with the elastic modulus, thermal shrinkage coefficient, dry shrinkage coefficient of semi-rigid base materials and temperature changes, water content changes of semi-rigid base courses. Besides, the maximum value also has relations with the degree of the restriction of the subbase course to the base course, the thickness and length of the base course. The expressions of shrinkage stresses of semi-rigid base courses presented in this study will have an important value for the design and construction of asphalt pavement with semi-rigid base courses and for the prevention of the cracking of semi-rigid base courses.

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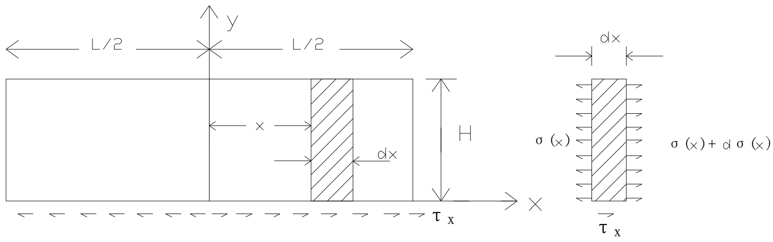

Figure 1. Stressing Model of Semi-rigid Base Course