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The Oscillation of Second-order Impulsive Delay Difference Equations with Continuous Arguments

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Abstract

In this paper, we considered the oscillation of second-order impulsive delay difference equation with continuous arguments, and the sufficient conditions are obtained for oscillation of all solutions and some results in the literatures are improved.

Keywords: Continuous arguments, Impulsive delay difference equation, Oscillation

1. Introduction

Recently, with the development of the medicine, biology mathematics and modern physics and so on, there have been many investigations into the study of delay difference equation. In particular, an extensive literature now exists on the oscillation theory for delay difference equation, and various applications have been found. But the study of the oscillation of impulsive delay difference equations with continuous arguments is very little, and this style of equations has very extensive application. Thus, to study the influence of impulsive to the system's stability has very important value applications. We refer to [1-6] and the references cited therein for more details.

In this paper, we consider the following second-order impulsive delay difference equations with continuous arguments

$$\begin{cases} \Delta_\tau^2 x(t) + \sum_{i=1}^{n} p_i(t) x(t-\sigma_i) = 0, & t \neq t_k, \\ x(t_k^+) - x(t_k) = b_k x(t_k), & t = t_k \end{cases} \tag{1}$$

Where

$\Delta_\tau x(t) = x(t) - x(t-\tau)$； $\Delta_\tau^2 x(t) = \Delta_\tau(\Delta_\tau x(t)) = x(t) - 2x(t-\tau) + x(t-2\tau)$

$\sigma_i > 0$， $i = 1,2,\Lambda$; τ is a nonnegative integer. The fixed moments of time t_k satisfy constants.

$0 \le t_0 \le t_1 \le t_2 \le \Lambda \le t_k \le \Lambda$ ， and $\lim_{k\to\infty} t_k = \infty$， $k = 1,2,\Lambda$ $b_k \neq -1$ $(k = 1,2,\Lambda)$ are

For $\delta \ge 0$， we define $r_\delta = \min\{\delta - 2\tau, \delta - \sigma_i, i = 1,2,\Lambda, n\}$.For all $\varphi \in C([r_\delta, \delta], R)$，

we define the function $y : [r_\delta, \infty) \to R$ as the equation（1）satisfy

$$y(t) = \varphi(t), \quad t \in [r_\delta, \delta] \tag{2}$$

If $y(t)$ is almost everywhere continuous in $[r_\delta, \infty)$,in $t_k (t > \delta)$ left continuous， $y(t_k^+)$ exist，for $t \in [\delta, \infty)$ satisfy (1)，for $t \in [r_\delta, \delta]$ satisfy initial condition (2)，then we easy to know that $\delta \ge 0$，

$\varphi \in C([r_\delta, \delta], R)$， (1) and (2) have the unique solution. The solution $x(t)$ of equation (1) is said to be oscillatory if the terms x_n are neither eventually positive nor eventually negative. Otherwise, the solution is called non-oscillatory.

The assistant equation

$$y(t) - 2\prod_{t-\tau \le t_k < t} (1+b_k)^{-1} y(t-\tau) + \prod_{t-2\tau \le t_k < t} (1+b_k)^{-1} y(t-2\tau) + \sum_{i=1}^{n} \prod_{t-\sigma_i \le t_k < t} (1+b_k)^{-1} p_i(t) y(t-\sigma_i) = 0 \tag{3}$$

the solution $y(t)$ of equation (3) satisfy initial condition (2) are almost everywhere continuous in $[r_\delta,\infty)$.

Lemma Suppose that $0 \le b < \frac{1}{4}$, and there exists $\{b_n\}$ satisfy $b_1 = \frac{b}{1-b}$ and $b_{n+1} = \frac{b}{1-b_n}$, for $n = 1,2,\Lambda$, then $\lim_{n\to\infty} b_n = \frac{1}{2}\left(1-\sqrt{1-4b}\right)$

2. Main results and proofs

Theorem 1 If $y(t)$ is the solution of equation(3), then $x(t) = \prod_{\delta\le t_k<t}(1+b_k)y(t)$ is the solution of equation (1) ; If $x(t)$ is the solution of equation(1), then $y(t) = \prod_{\delta\le t_k<t}(1+b_k)^{-1}x(t)$ is the solution of equation(3).

Proof: Since $y(t)$ is the solution of equation (3), then $x(t) = \prod_{\delta\le t_k<t}(1+b_k)y(t)$ is almost everywhere continuous in $[\delta,\infty)$. Let $t \ne t_k$, then

$$\Delta_\tau^2 x(t) + \sum_{i=1}^{n} p_i(t)x(t-\sigma_i) = x(t) - 2x(t-\tau) + x(t-2\tau) + \sum_{i=1}^{n} p_i(t)x(t-\sigma_i)$$

$$= \prod_{\delta\le t_k<t}(1+b_k)y(t) - 2\prod_{\delta\le t_k<t-\tau}(1+b_k)y(t-\tau) + \prod_{\delta\le t_k<t-2\tau}(1+b_k)y(t-2\tau) + \sum_{i=1}^{n}\prod_{\delta\le t_k<t-\sigma_i}(1+b_k)p_i(t)y(t-\sigma_i)$$

$$= \prod_{\delta\le t_k<t}(1+b_k)\left[y(t) - 2\prod_{t-\tau\le t_k<t}(1+b_k)^{-1}y(t-\tau) + \prod_{t-2\tau\le t_k<t}(1+b_k)^{-1}y(t-2\tau) + \sum_{i=1}^{n}\prod_{t-\sigma_i\le t_k<t}(1+b_k)^{-1}p_i(t)y(t-\sigma_i)\right] = 0$$

Let $t = t_k > \delta$, because $x(t)$ is right continuous, then

$$x(t_k^+) = \prod_{\delta\le t_i<t_k^+}(1+b_i)y(t_k^+) = (1+b_k)\prod_{\delta\le t_i<t_k}(1+b_i)y(t_k) = (1+b_k)x(t_k)$$

we obtain that $x(t)$ is the solution of equation(1).

Since $x(t)$ is the solution of equation (1), then $y(t) = \prod_{\delta\le t_k<t}(1+b_k)^{-1}x(t)$ is almost everywhere continuous in $[\delta,\infty)$. We easy to proof that it satisfy equation （3）. When it equal to point t_k, then

$$x(t_k^+) = \prod_{\delta\le t_i<t_k^+}(1+b_i)^{-1}y(t_k^+) = \prod_{\delta\le t_i<t_k}(1+b_i)y(t_k) = x(t_k)$$

i.e. $x(t)$ is right continuous in point t_k. Then we obtain that $y(t)$ is the solution of equation (3), and the proof is completed.

Corollary Assume that $b_k > -1$, then all solutions of equation (1) oscillate are equals to all solutions of equation (3) oscillate.

Proof: By the results of theorem 1, we can proof the corollary. Then the proof is completed.

Theorem 2 Assume that $0 < 2\tau \le \sigma_1 \le \sigma_2 \le \Lambda \le \sigma_n$ and $p_i(t) \in C(R^+, R^+)$, for all of $t > 0$ and $i = 1,2,\Lambda, n$, then $p_i(t) > 0$; for all $k = 1,2,\Lambda$, then $b_k \ge 0$, $1 - 2\prod_{t-\tau\le t_k<t}(1+b_k)^{-1} > 0$, such that

$$\liminf_{t\to\infty}\sum_{i=1}^{n+2}\gamma_i(t) = \alpha < \frac{1}{4} \tag{4}$$

$$\limsup_{t\to\infty}\sum_{i=1}^{n+2}\gamma_i(t) > 1 - \frac{1}{2}\left(1-\sqrt{1-4\alpha}\right) \tag{5}$$

Where

$$\gamma_i(t) = \inf\left\{1 - 2\prod_{t-\tau\le t_k<t}(1+b_k)^{-1};\ \prod_{t-2\tau\le t_k<t}(1+b_k)^{-1};\ \prod_{s-\sigma_i\le t_k<s}(1+b_k)^{-1}p_i(s), s\in[t-\tau,t]\right\}$$

Then every solution of equation (1) oscillates.

Proof: Firstly, the assistant equation（3）of equation (1) can derived to be

$$y(t) - y(t-\tau) + \left(1 - 2\prod_{t-\tau\le t_k<t}(1+b_k)^{-1}\right)y(t-\tau) + \prod_{t-2\tau\le t_k<t}(1+b_k)^{-1}y(t-2\tau)$$

$$+\sum_{i=1}^{n}\prod_{t-\sigma_i\le t_k<t}(1+b_k)^{-1}p_i(t)y(t-\sigma_i)=0 \tag{6}$$

for $1-2\prod_{t-\tau\le t_k<t}(1+b_k)^{-1}>0$ and $p_i(t)\in C(R^+,R^+)$, equation（6）can derived to be

$$y(t)-y(t-\tau)+\sum_{i=1}^{n+2}\gamma_i(t)y(t-\tau_i)=0 \tag{7}$$

Where

$$\tau_1=\tau,\tau_2=2\tau,\tau_3=\sigma_1,\tau_4=\sigma_2,\Lambda\ ,\tau_{n+2}=\sigma_n\cdot$$

Secondly, by the front conclude, if every solution of equation (7) oscillates, then every solution of equation (3) oscillates.

Next , we proof that every solution of equation (7) oscillates. Assume the contrary. Then equation（7）may have an eventually positive solution $y(t)$, then exist $t_1^*>0$，let $t>t_1^*$, such that $y(t)>0$，$y(t-\tau_i)>0$,for all $\varepsilon\in[0,\alpha)$ by（4）,exist $t^*>t_1^*+\sigma_n$， when $t>t^*$, then

$$\sum_{i=1}^{n+2}\gamma_i(t)>\alpha-\varepsilon \tag{8}$$

let $t_2^*=\max_{1\le i\le n+2}\{t^*+\tau_i,t_1^*\}$， integral from $t-\tau$ to t for equation (7), by the integral midst value theorem, then

$$\begin{aligned}0&=\int_{t-\tau}^{t}y(s)ds-\int_{t-\tau}^{t}y(s-\tau)ds+\sum_{i=1}^{n+2}\int_{t-\tau}^{t}\gamma_i(s)y(s-\tau_i)ds\\&=\int_{t-\tau}^{t}y(s)ds-\int_{t-\tau}^{t}y(s-\tau)ds+\sum_{i=1}^{n+2}\gamma_i(\xi_{it})\int_{t-\tau}^{t}y(s-\tau_i)ds\end{aligned} \tag{9}$$

where $t>t_2^*$， $t-\tau\le\xi_{it}<t$， $i=1,2,\Lambda\ ,n+2$.

Let $z(t)=\int_{t-\tau}^{t}y(s)ds$, then $z(t)$ is continuous and almost everywhere derivate in (t^*,∞) ,then

$$z'(t)=y(t)-y(t-\tau)=-\sum_{i=1}^{n+2}\gamma_i(t)y(t-\tau_i)<0，\quad t>t_2^*$$

for $z(t)>0$，then $z(t)$ is gradually reduce，and then

$$z(t-\tau)\le z(t-2\tau)\le z(t-\tau_i)，\quad t>t_2^*，\quad i=1,2,\Lambda\ ,n+2 \tag{10}$$

by (9), we have

$$z(t-\tau)=z(t)+\sum_{i=1}^{n+2}\gamma_i(\xi_{it})z(t-\tau_i) \tag{11}$$

by（8）,(10)and (11),we have

$$z(t-\tau)>\sum_{i=1}^{n+2}\gamma_i(\xi_{it})z(t-\tau_i)>\sum_{i=1}^{n+2}\gamma_i(\xi_{it})z(t-2\tau)>(\alpha-\varepsilon)z(t-2\tau)$$

$$t>t_2^*，\quad t-\tau\le\xi_{it}<t，\quad i=1,2,\Lambda\ n+2$$

i.e.

$$z(t)>(\alpha-\varepsilon)z(t-\tau)，\quad t>t_2^*+\tau \tag{12}$$

by (11) and (12), we have

$$z(t-\tau)>(\alpha-\varepsilon)z(t-\tau)+\sum_{i=1}^{n+2}\gamma_i(\xi_{it})z(t-\tau_i)>(\alpha-\varepsilon)z(t-\tau)+(\alpha-\varepsilon)z(t-2\tau)$$

then $z(t-\tau)>\dfrac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)}z(t-2\tau)$, i.e

$$z(t)>\frac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)}z(t-\tau)，\quad t>t_2^*+2\tau \tag{13}$$

by (11) and (13),we have

$$z(t-\tau) > \frac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)} z(t-\tau) + \sum_{i=1}^{n+2} \gamma_i(\xi_{it}) z(t-\tau_i) \quad > \frac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)} z(t-\tau) + (\alpha-\varepsilon) z(t-2\tau)$$

Thus, we have

$$z(t-\tau) > \frac{\alpha-\varepsilon}{1-\dfrac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)}} z(t-2\tau)$$

Let $b_1 = \dfrac{\alpha-\varepsilon}{1-(\alpha-\varepsilon)}$， $b_2 = \dfrac{\alpha-\varepsilon}{1-b_1}$， then

$$z(t-\tau) > \frac{\alpha-\varepsilon}{1-b_1} z(t-2\tau)\text{，} \quad t > t_2^* + 3\tau$$

by simple induction, in the end we have

$$z(t) > \frac{\alpha-\varepsilon}{1-b_{m-1}} z(t-\tau)\text{，} \quad b_{m-1} = \frac{\alpha-\varepsilon}{1-b_{m-2}}\text{，} \quad m = 2,3,\Lambda\text{；} \tag{14}$$

by (11) and (14) ,we have

$$z(t-\tau) > \frac{\alpha-\varepsilon}{1-b_{m-1}} z(t-\tau) + \sum_{i=1}^{n+2} \gamma_i(\xi_{it}) z(t-\tau_i)$$

$$\geq b_m z(t-\tau) + \sum_{i=1}^{n+2} \gamma_i(\xi_{it}) z(t-\tau_i) > b_m z(t-\tau) + \sum_{i=1}^{n+2} \gamma_i(\xi_{it}) z(t-\tau)$$

and

$$\sum_{i=1}^{n+2} \gamma_i(\xi_{it}) \leq 1 - b_m \tag{15}$$

As we all know，if $t \to \infty$, then $m \to \infty$， by the define of ε ,we know if $t \to \infty$ then $\varepsilon \to 0$. Thus, by the lemma and （15）,we have

$$\lim_{t\to\infty} \sup \sum_{i=1}^{n} \gamma_i(t) \leq \lim_{t\to\infty}(1-b_m) = \lim_{m\to\infty}(1-b_m) = 1 - \frac{1}{2}\left(1-\sqrt{1-4\alpha}\right)$$

This contradicts (5), then we prove that the positive solution of equation (7) doesn't exist. Similarly, we can proof that the negative solution of equation (7) doesn't exist. i.e. the every solution of equation (7) oscillates. Thus, the every solution of equation (3) oscillates. By the corollary, the every solution of equation (1) oscillates.

Then the proof is completed.

3. Conclusion

In this paper, by study the method of the oscillation first-order linear impulsive difference equation with continuous arguments, we study the second-order impulsive delay difference equations with continuous arguments. The sufficient conditions are obtained for oscillation of all solutions and some results in the literatures are improved.

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