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The Stable Set and Weak Stable Set

For n-person Repeated Fuzzy Cooperative Games

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Abstract

In this paper,based on the fuzzy games,we define the imputation sequences of the n-person repeated games,and the domination,weak domination for the imputation sequences.Further,based on this theory,we define the core ,the weak core,the stable set,and the weak stable set of the n-person repeated fuzzy cooperative games.At last,some properties of the stable set and the weak stable set are given.

Keywords: Fuzzy game, Repeated cooperative game, Core, Weak core, Stable set, Weak stable set

Introduction

In the former literature about the cooperative game, how to allocate the total benefit among the the players have been studied widely.In 1974, Aubin, JP introduced Fuzzy games, and combine cooperative game with fuzzy coalition for the first time (Aubin, 1981, PP. 1-13). And in 1999, Jorge Oviedo developed the n-person repeated cooperative game, the core of this game was discussed in his paper.And based on the these theory, we develoepde n-person Repeated fuzzy cooperative games.Further,we define the weak core,the stable set and the weak stable set for n-person Repeated fuzzy cooperative games.Thus,we extend the study for the solution of Repedted fuzzy games.

1. Basic definitions

Definition1. Let $N := \{1,2,...,n\}$ be the set whose elements are called players. $[0,1]^n = [0,1]\times[0,1]\times...\times[0,1]$, v is a fuction on $[0,1]^n$,and $v:[0,1]^n \to R$,satisfy: $v\,(\Phi) = 0$,where $\Phi = (0,0,...0)$,then we call v is a characteristic function on $[0,1]^n$,and call $([0,1]^n, v)$ is a fuzzy cooperative game with player set N.We call v fuzzy game in short.

We dnote by FG^N the set of Fuzzy Games.And $d = (d_1, d_2,...d_n) \in [0,1]^n$ is a fuzzy coalition.The i th coordinate is called the participation degree of player i to fuzzy coalition d.And denote F^N all the possible fuzzy coalitions.

We denote $e^i \in R^n$ the i th coordinate is 1, and 0 otherwise.And we denote

$$e^N = \sum_{i\in N} e^i, \sup pd = \{i \in N : d_i > 0\}$$

We repedte the fuzzy game v m times(m may be ∞),and we denote (d, v_t, t) the fuzzy cooperative game of players in t th period,we call it the stage game of the n-person repeated fuzzy game.In order to make the stage-game and the repedted-game characteristic function to be measured in the same unities,we use a discount factor δ^t, $\delta \in (0,1)$,and we call $1-\delta$ the normalization factor.The imputation vector of the fuzzy coopertive game (d, v_t, t) shulde satisfy the follwing :

$$(1)\sum_{i=1}^{n} x_i{}^t = v_t(e^N) = (1-\delta)\delta^t v(e^N)$$

$$(2)\, d_i x_i{}^t \geq v_t(d \mid i) = (1-\delta)\delta^t v(d \mid i)$$

So we denote the core of the fuzzy cooperative game as

$c(v_t) = \{x \in R^n : \sum_{i \in N} x_i = v_t(e^N), \sum_{i \in N} d_i x_i \geq v_t(d)$, $\forall d \in [0,1]^n\}$

Let $d^t \in [0,1]^n$ ia a fuzzy coalition when v at time t.Then let $(d^0, d^1, \ldots d^t)$ be the coalition sequences when t from 0 to t.We denote $H^t = (L(u))^t$ the all possible coalitions when t th stage.where $L(u)$ is all the possible fuzzy coalitions.We denote H the set of all the coalition sequences.

Definition 2. Let $\theta = (d^0, d^1, \ldots d^m) = (d^t)_{t=0}^m \in H$ is a coalition sequence,then $w(\theta)$ is called the characteristic function of repeated fuzzy cooperative games,if it satisfies:

$w(\theta) = (1-\delta)\sum_{t=0}^{m} \delta^t v(d^t)$, and $w(\tilde{\theta})$=0,where $\tilde{\theta} = (0,0,\ldots 0)$

Definition 3. Let H be the set of coalition sequence,and w is the characteristic function of repeated fuzzy cooperative games.We denote (H, w) as repeated fuzzy cooperative game.

2. The core and the weak core for the n-person repeated fuzzy cooperative game

Definitin 4. An imputation sequence or payoff allocation sequence $x = (x^0, x^1, \ldots)$ for the repeated fuzzy cooperative game is a sequence of the stage game's imputation or payoff allocations.

Definition 5. Let (H, w) be a repeated fuzzy game, x, y are imputation sequences, $\theta = (d^0, d^1, \ldots d^m)$ is a coalition sequence if it satisfies:

$(1)\sum_{t=0}^{m} d_i{}^t x_i{}^t > \sum_{t=0}^{m} d_i{}^t y_i{}^t \qquad \forall i \in \Upsilon_k \sup p d^k$

$(2) \sum_{i \in \Upsilon_k \sup p d^k} d_i{}^t x_i{}^t \leq (1-\delta)\delta^t v(d^t)$

Then we call x dominates y through θ($x \phi_\theta y$)

Definition 6. Let $x = (x^0, x^1, \ldots x^m)$ is a imputation sequence of (H, w).where x^t is the imputation of t th stage game.then we define the core of the repeated fuzzy cooperative game as:

$$c(w) = \{x \in I(w) \mid \sum_{t=0}^{m} \sum_{i \in \Upsilon_k \sup p d^k} d_i{}^t x_i{}^t \geq (1-\delta)\sum_{t=0}^{m} \delta^t v(d^t), \sum_{i \in N} x_i{}^t = v_t(e^N) = (1-\delta)\delta^t v(e^N)\}$$

Definition 7. Let x, y be two imputations of the repeated fuzzy cooperative game. θ is a coalition sequence,if it satisfies

$(1) \sum_{i \in \Upsilon_k \sup p d^k} \sum_{t=0}^{m} d_i{}^t x_i{}^t > \sum_{i \in \Upsilon_k \sup p d^k} \sum_{t=0}^{m} d_i{}^t y_i{}^t > \qquad \forall i \in \Upsilon_k \sup p d^k$

$(2) \sum_{i \in \Upsilon_k \sup p d^k} d_i{}^t x_i{}^t \leq (1-\delta)\delta^t v(d^t), \qquad t \in [0,1,\ldots m]$

Then we call x dominate y weakly through θ, we denote by $x \overset{w}{\phi}_\theta y$.

Definition 8. All of the set of imputation sequences for repeated fuzzy cooperative game,if they are not dominated,then we call them the weak core of the repeated fuzzy cooperative game.And we denote by $\tilde{c}(w)$,i.e.

$\tilde{c}(w) = \{x \mid x \in I(w)$, there exist no θ and $y \in I(w)$,satisfy: $y \overset{w}{\phi}_\theta x\}$

3. The stable set and the weak stable set for n-person repeated fuzzy cooperative games

Definition 9. Let L be the set of some imputation sequences of repeated fuzzy game (H, w),i.e. $L \subset I(w)$,we call

L the stable set of (H,w),if it satisfies

(1) $\forall x=(x^0,x^1,\ldots x^m)$, $y=(y^0,y^1,\ldots y^m)\in I(w)$,there is no domination relationship between x and y (inner stability)

(2) $\forall\ x^{'}\in I(w)\setminus L$,there exist a coalition sequence θ and an imputation sequence $x\in L$,satisfy: $x\succ x^{'}$ (outer stability)

Definition 10. Let $\tilde{L}$ are the set of some imputation sequences of repeated fuzzy cooperative game,i.e. $\tilde{L}\subseteq I(w)$,we call $\tilde{L}$ the weak stable set of (H,w) if it satisfis:

$\forall x,y\in I(w)$, there is no weak dominationship between x and y .(inner stability)

$\tilde{x}\in I(w)\setminus\tilde{L}$, there is a coalition sequences θ and an imputation sequences $y\in\tilde{L}$,satisfy: $y\overset{w}{\underset{\theta}{\succ}}\tilde{x}$.(outer stability)

4. The properties of the stable set for n-person repeated fuzzy cooperative game

Theorem 1. To the same n-person repeated fuzzy cooperative game, $\tilde{c}(w)\subseteq c(w)$.

Proof. $\forall\tilde{x}\in\tilde{c}(w)$,we know from the definition of the weak core that there exist no coalition sequence θ and imputation sequence $\tilde{y}$,satisfy: $\tilde{y}\overset{w}{\underset{\theta}{\succ}}x$.If $\tilde{x}\notin c(w)$,then there exist a coalition θ and an imputation sequence y,and $y\underset{\theta}{\succ}\tilde{x}$,i.e.for all $i\in\bigcup_k s^k$,we have

$$\sum_{t=0}^{m} y_i^{\,t} > \sum_{t=0}^{m}\tilde{x}_i^{\,t}$$

To the inequlity above,for all $i\in\bigcup_k s^k$,we make sum to each side respectively.So we have

$$\sum_{i\in\bigcup_k \sup pd^k}\sum_{t=0}^{m} y_i^{\,t} > \sum_{i\in\bigcup_k \sup pd^k}\sum_{t=0}^{m}\tilde{x}_i^{\,t}$$,i.e. $y\overset{w}{\underset{\theta}{\succ}}\tilde{x}$,it is contradict.So $\tilde{x}\in c(w)$,then $\tilde{c}(w)\subseteq c(w)$.

Theorem 2. If the stable set of the n-person repeated fuzzy cooperative game $L\neq\Phi$, then $c(w)\subset L$

Proof.When $c(w)=\Phi$, clearly we have $c(w)\subset L$.

When $c(w)\neq\Phi$, $\forall x\in c(w)$,if $x\notin L$,then there exist coalition sequence θ and imputation sequence $y\in L$,satisfy: $y\underset{\theta}{\succ}x$,so $x\notin c(w)$.It is contradict. So, $c(w)\subset L$.

Theorem 3. For all $t(0\le t\le m)$,if $x\in I(w)$,and it satisfies $x\in c(w)$,then $x\in\tilde{c}(w)$.

Proof.If $x\notin\tilde{c}(w)$, then there exist a coalition θ and an imputation sequence $\tilde{x}\in I(w)$,and

It sastifies
$$\sum_{i\in\bigcup_k \sup pd^k}\sum_{t=0}^{m} d_i^{\,t}\,\tilde{x}_i^{\,t} > \sum_{i\in\bigcup_k \sup pd^k}\sum_{t=0}^{m} d_i^{\,t} x_i^{\,t} \tag{5}$$

And
$$\sum_{i\in\bigcup_k \sup pd^k}\sum_{t=0}^{m} d_i^{\,t}\,\tilde{x}_i^{\,t} \le (1-\delta)\delta^t v(d^t),\ 0\le t\le m, \tag{6}$$

To (6),we make sum in both sides respecr to t,and we have

$$\sum_{t=0}^{m}\sum_{i\in\bigcup_k \sup pd^k} d_i^{\,t}\,\tilde{x}_i^{\,t} \le \sum_{t=0}^{m}(1-\delta)\delta^t v(d^t) \tag{7}$$

Further becauce for all of the t $(0\le t\le m)$, $x\in c(w)$, so, for all of the t $(0\le t\le m)$,we have

$$\sum_{i\in\bigcup_k \sup pd^k} d_i^{\,t} x_i^{\,t} \ge (1-\delta)\delta^t v(d^t)$$

We make sum in both sides respect to t, and we have

$$\sum_{t=0}^{m} \sum_{i \in \bigcup_k \sup pd^k} d_i^{\,t} x_i^{\,t} \geq \sum_{t=0}^{m} (1-\delta)\delta^t v(d^t) \tag{8}$$

compare (7)and (8),we have

$$\sum_{i \in \bigcup_k \sup pd^k} \sum_{t=0}^{m} d_i^{\,t} \tilde{x}_i^{\,t} \leq \sum_{i \in \bigcup_k \sup pd^k} \sum_{t=0}^{m} d_i^{\,t} x_i^{\,t}$$

This is contradict with (5). then $x \in \tilde{c}(w)$.

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