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The Reliability Analysis of N-Unit Series Repairable System With One Replaceable Repair Facility and a Repairman Doing Other Work

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Abstract

A model of N-unit series repairable systems with a repairman doing other work is studied and the impact on the system reliability because of a replaceable facility is also considered. It is assumed that the life of each unit, the life of the facility are exponentially distributed, the repair time of the unit, the replace time of the facility and the work time of the repairman outside the system are all generally distributed. By using approach of supplementary variables and method of generalized Markov progress, some important reliability indicates of the system and the facility are obtained.

Keywords: Reliability, Vector Markov process, Laplace-transform

1. Introduction

In reliability analysis of repairable systems, it is usually assumed that the repairman has two states, either repairing the failed unit or idle. However, in actual practice, in order to increase system income, the administrator of system often will consider that the repairman should service for customer of outside system without affecting the system at the same time. The question is that, the system allows the repairman to service for customer of outside system, will how affect the reliability of indicators. If it is permitted, how to restrict the repairman's service for customers, will increase the system's total receipts without affecting the system. Theoretically, repairable system with replaceable repair facilities and repairman doing other work are in class of more general repairable systems. Liu and Tang studied an N-unit series repairable system with a repairman doing other work, more reliability indices of the system are obtained by using the Laplace-transform technique. In practical applications, it is necessary to consider the effect that caused by replacing the repair facility. In view of the reference (Liu and Tang), we consider the reliability analysis of N-unit series repairable system with one replaceable system with one replaceable repair facility and a repairman doing other work. It is assumed that the life of each unit, the life of the facility are exponentially distributed, the repair time of the unit, the replace time of the facility and the work time of the repairman outside the system are all generally distributed. By using approach of supplementary variables and method of generalized Markov progress, some important reliability indicates of the system and the facility are obtained.

2. The description of system

(1) The system consists of N-dissimilar-unit operate in a series configuration with a single repair facility. The lifetime X_i of the units has an exponential distribution $F_i(t)=1-e^{-\lambda_i t}, t\ge 0$. The repair time Y_i of failed unit has an arbitrary distribution $G_i(t)$ with mean $\mu_i(0\le\mu_i<\infty),(i=1,2,\cdots,n)$ At time t=0, all the units are new, and all are in working state while the repairman is idle. If a unit is waiting for the repairman or being repaired, the other unit won't fail any more and the system is down;

(2) The service time H for customer of the repairman has an arbitrary distribution $H(t)$ with mean $d(0\le d\le\infty)$;

(3) The time interval from the repairman idle to the customer arrival and the time interval from the first customer arrival to the second customer arrival have an exponential distribution $F(t)=1-e^{-ct}, t\ge 0$; The repairman is idle only when the system is in working state and there is no customer arrival. And the customer may arrival only when the system is in working state. If the repairman is busying with a customer and the other customer is waiting for the repairman or the system has failed units, another customer won't arrive any more. If a unit failed when a customer is waiting for the repairman, the customer will leave the system, and the failed unit is waiting for the repairman until the repairman completing the repair of the earlier customer.

(4) The lifetime U of the repair facility has an exponential distribution $U(t)=1-e^{-\alpha t}$ $(0\le\alpha<\infty, t\ge 0)$. The replacement time V of the repair facility has an arbitrary distribution $V(t)$ with mean $\beta(0\le\beta<\infty)$. It is assumed that the repair facility neither fails nor deteriorates in its idle periods. When the repair facility fails, it is replaced by a new one and the failed unit must wait for repair. When the replacement of the failed repair facility is completed, the new repair facility continues to repair the failed unit. The repair time for the failed unit cumulative repair time for failed unit exceeds Y_i;

(5) All the random variables are mutually independent. After being repaired, the failed unit is as good as new.

3. The equations of the system

Let $N(t)$ be the state of the system at time t, we have:

State 0 : the system works and the repairman idles;

State 1i : repairing the failure unit i;

State 2i : unit i has not been completed the repair and replacement of the repair facility;

State 3 : the system works and repairman services for customer;

State 4i : the failure unit i is waiting to be repaired and the repairman services for customer;

State 5: the system works ,the repairman services for customer and the other customers is waiting for service.

The introduction of supplementary variables:

Where $N(t)=3,4i,5,i=1,2,\cdots,n;$ let $X(t)$ be the hours that the customer has spent on the service for customer at time t, $0\le X(t)<\infty$.

Where $N(t)=1i,i=1,2,\cdots,n;$ let $Y_i(t)$ be the hours that the failure unit has been repaired at time t, $0\le Y_i(t)<\infty$.

Where $N(t)=2i,i=1,2,\cdots,n;$ let $Z(t)$ be the hours that the repair facility has been replaced at time t, $0\le Z(t)<\infty$.

Then $\{N(t),X(t),Y_i(t),Z(t),t\ge 0,i=1,2,\cdots,n\}$ constitute a vector Markov process.

The state probability of the system at time t is defined by

$P_0(t)=P\{N(t)=0\}$,

$P_{1i}(t,y)dy=P\{N(t)=1i, y\le Y_i(t)<y+dy\}$, $\quad i=1,2,\cdots,n;$

$P_{2i}(t,y,w)dydw=P\{N(t)=2i, Y_i(t)=y, w\le Z(t)<w+dw\}$, $\quad i=1,2,\cdots,n;$

$P_3(t,x)dx=P\{N(t)=3, x\le X(t)<x+dx\}$,

$P_{4i}(t,x)dx=P\{N(t)=4i, x\le X(t)<x+dx\}$, $\quad i=1,2,\cdots,n;$

$P_5(t,x)=P\{N(t)=5, x\le X(t)<x+dx\}$.

Where $\lambda=\lambda_1+\lambda_2+\cdots+\lambda_n$, we can obtain the following differential equations for the system from a probability analysis:

$$[\frac{d}{dt}+\lambda+c]P_0(t)=\int_0^\infty d(x)P_3(t,x)dx+\sum_{i=1}^{n}\int_0^\infty \mu_i(y)P_{1i}(t,y)dy, \qquad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_i(y)+\alpha\right]P_{1i}(t,y)=\int_0^\infty \beta(w)P_{2i}(t,y,w)dw, \qquad i=1,2,\cdots,n; \qquad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\beta(w)\right]P_{2i}(t,y,w)=0, \qquad i=1,2,\cdots,n; \qquad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+d(x)+c+\lambda\right]P_3(t,x)=0, \qquad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+d(x)\right]P_{4i}(t,x)=\lambda_i P_3(t,x)+\lambda_i P_5(t,x), \qquad i=1,2,\cdots,n; \qquad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+d(x)+\lambda\right]P_5(t,x)=cP_3(t,x) \qquad (6)$$

The boundary conditions are:

$$P_{1i}(t,0)=\lambda_i P_0(t)+\int_0^\infty d(x)P_{4i}(t,x)dx, \qquad i=1,2,\cdots,n;$$

$$P_{2i}(t,y,0)=\alpha P_{1i}(t,y), \qquad i=1,2,\cdots,n;$$

$$P_3(t,0)=cP_0(t)+\int_0^\infty d(x)P_5(t,x)dx$$

$$P_{4i}(t,0)=0 \qquad i=1,2,\cdots,n; \qquad P_5(t,0)=0$$

The initial conditions are: $P_0(0)=1,$ the others are 0.

4. The solutions of the equations

Remarks: This paper adopts the following notation:

$D^*(s)$ denotes the L transform of $D(t)$, and $\hat{D}(s)$ denotes the LS transform of $D(t)$, that is $D^*(s)=\int_0^\infty D(t)e^{-st}dt, \hat{D}(s)=\int_0^\infty e^{-st}dD(t)$, $\bar{D}(t)=1-D(t)$. Taking Laplace transforms on both sides of the above differential equations and using the boundary and initial conditions, we can obtain:

$$P_{1i}^*(s,y)=P_{1i}^*(s,0)e^{-\left[s+\alpha-\alpha\hat{H}(s)\right]y}\bar{G}_i(y), \qquad i=1,2,\cdots,n;$$

$$P_{2i}^*(s,y,w)=P_{2i}^*(s,y,0)e^{-sw}\bar{H}(w)=\alpha P_{1i}^*(s,y)e^{-sw}\bar{H}(w), \qquad i=1,2,\cdots,n;$$

$$P_3^*(s,x)=P_3^*(s,0)e^{-(s+c+\lambda)x}\bar{H}(x),$$

$$P_{4i}^*(s,x)=\frac{\lambda_i}{\lambda}P_3^*(s,0)e^{-sx}\left(1-e^{-\lambda x}\right)\bar{H}(x), \qquad i=1,2,\cdots,n;$$

$$P_5^*(s,x)=P_3^*(s,0)e^{-(s+\lambda)x}\left(1-e^{-cx}\right)\bar{H}(x),$$

$$P_{1i}^*(s,0)=\lambda_i P_0^*(s)\cdot A_1, \qquad i=1,2,\cdots,n;$$

$$P_3^*(s,0)=\frac{cP_0^*(s)}{1-\hat{H}(s+\lambda)+\hat{H}(s+\lambda+c)},$$

$$P_0^*(s)=\frac{1}{s+\lambda+c-A_3-\sum_{i=1}^{n}\lambda_i\hat{G}\left(s+\alpha-\alpha\hat{H}(s)\right)\cdot A_1}$$

where

$$A_1=1+\frac{c}{\lambda}\cdot\frac{\hat{H}(s)-\hat{H}(s+\lambda)}{1-\hat{H}(s+\lambda)+\hat{H}(s+\lambda+c)} \qquad A_2=\sum_{i=1}^{n}\frac{\lambda_i\left[1-\hat{G}_i\left(s+\alpha-\alpha\hat{H}(s)\right)\right]}{s+\alpha-\alpha\hat{H}(s)}$$

$$A_3=\frac{c\hat{H}(s+\lambda+c)}{1-\hat{H}(s+\lambda)+\hat{H}(s+\lambda+c)} \qquad A_4=1+\frac{c}{\lambda}\cdot\frac{1-\hat{H}(\lambda)}{1-\hat{H}(\lambda)+\hat{H}(\lambda+c)}$$

$$A_5 = \frac{c}{d} \cdot \frac{1}{1-\hat{H}(\lambda)+\hat{H}(\lambda+c)}$$

5. System reliability

Theorem 1: The system's point availability $A(t)$, The Laplace transform of $A(t)$ is given by

$$A^*(s) = P_0^*(s)\left[1+\frac{c}{s+\lambda}\cdot\frac{1-\hat{H}(s+\lambda)}{1-\hat{H}(s+\lambda)+\hat{H}(s+\lambda+c)}\right] \tag{7}$$

And the steady state availability of system is

$$A = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA^*(s) = \frac{A_4}{1+A_5+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\left(1+\frac{\alpha}{h}\right)A_4} \tag{8}$$

The rate of occurrence of failure (ROCOF) $W_f(t)$, and the Laplace transform of the ROCOF $W_f(t)$ is

$$W_f^*(t) = \lambda A^*(s) \tag{9}$$

And the steady-state ROCOF of the system is given by

$$W_f = \lambda A \tag{10}$$

Theorem 2: The time rate of the repairman services for customer at time t is $D(t)$, and the Laplace transform of $D(t)$ is given by

$$D^*(s) = \frac{c\hat{H}(s)P_0^*(s)}{1-\hat{H}(s+\lambda)+\hat{H}(s+\lambda+c)} \tag{11}$$

The mean number of the repairman services for customer of system in the steady-state in [0,t] is given by

$$N = \frac{\frac{c}{1-\hat{H}(\lambda)+\hat{H}(\lambda+c)}}{1+A_5+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\left(1+\frac{\alpha}{h}\right)\cdot A_4} \tag{12}$$

Theorem 3: The replacement time probability of the repair facility at time t is $\varphi(t)$, and the Laplace transform of $\varphi(t)$ is given by

$$\varphi^*(s) = \frac{\alpha\hat{H}(s)\cdot A_1\cdot A_2}{s+\lambda+c-A_3-\sum_{i=1}^{n}\lambda_i\hat{G}_i\left(s+\alpha-\alpha\hat{H}(s)\right)\cdot A_1} \tag{13}$$

In steady-state, the result is:

$$\lim_{t\to\infty}\varphi(t) = \lim_{s\to 0^+} s\varphi^*(s) = \frac{\frac{\alpha}{h}\cdot A_4\cdot\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}{1+A_5+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\left(1+\frac{\alpha}{h}\right)\cdot A_4} \tag{14}$$

Theorem 4: The repair facility's point replacement rate is $M_f(t)$, and the Laplace transform of $M_f(t)$ is given by

$$M_f^*(s) = \frac{\alpha\cdot A_1\cdot A_2}{s+\lambda+c-A_3-\sum_{i=1}^{n}\lambda_i\hat{G}_i\left(s+\alpha-\alpha\hat{H}(s)\right)\cdot A_1} \tag{15}$$

and in steady-state, the result is:

$$M_f = \lim_{t\to\infty} M_f(t) = \lim_{s\to 0^+} sM_f^*(s) = \frac{\alpha\cdot A_4\cdot\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}{1+A_5+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\left(1+\frac{\alpha}{h}\right)\cdot A_4} \tag{16}$$

6. The analysis of system benefit

Assuming that the system average revenue per unit time for the x_1, each unit fault caused a loss for the average x_2, the replacement of equipment once caused the average loss for x_3, repairman services for one customer bring a system of the average income of x_4, assuming that the system in a steady state, the average time units total receipts:

$$Y = Ax_1 - W_f x_2 - M_f x_3 + Nx_4$$

$$= \frac{\left(x_1 - \lambda x_2 - \alpha x_3 \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\right) \cdot A_4 + \frac{cx_4}{1 - \hat{H}(\lambda) + \hat{H}(\lambda + c)}}{1 + A_5 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\left(1 + \frac{\alpha}{h}\right) \cdot A_4} \tag{17}$$

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