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Forest Stands as Dynamical Systems: An Introduction

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Abstract

Forest management planning relies heavily on mathematical models that involve time. Concerns about climate change and ecosystem services have highlighted the limitations of traditional growth and yield prediction tools. Modern dynamical system theory provides a framework for a flexible representation of varying environments, as well as of responses to intensive silviculture and natural disturbances. Emphasis changes from trying to directly model functions of time to modelling rates of change. The fundamental concepts are introduced here in a non-technical manner. The theory is illustrated with a recent whole-stand growth model for even-aged stands, but it is noted that it applies to any system that evolves over time. It is shown also how a modular approach can improve balance and efficiency in the development of such models.

Keywords: modelling, stand dynamics, system theory, system dynamics, forest growth and yield, thinning, hybrid models

1. Introduction

Due to the long planning horizons involved, experience and experimentation are not as useful in forestry as in other disciplines. Therefore, mathematical modelling plays an important role, and has been used in one form or another for centuries (Lowood, 1990). In particular, growth forecasting through yield tables and their modern successors is an important prerequisite for rational forest management.

Simple yield tables or traditional growth and yield models are often all that is needed. However, by essentially specifying fixed functions of time, they fail to address satisfactorily issues that are becoming increasingly important, including a changing environment, growth and carbon cycling in immature stands, and response to disturbances (García, 2011). Dynamical systems theory provides an alternative point of view that can fulfil those needs. The ideas are not new, and are routinely applied in other areas. But perhaps due to unfamiliarity and entrenched traditions, their adoption in forest modelling has been slow (Weiskittel et al., 2011, Burkhart & Tomé, 2012). A non-technical explanation of the basic concepts is presented here. Although illustrated through growth modelling examples, the ideas are general and apply to any system that evolves in time.

2. From Trajectories to Rates of Change, and Back

Yield tables and many growth and yield models define functions of time, typically including stand height, number of trees, basal area, and volumes, for various ages and site qualities. Consider the height and basal area columns from a yield table, for a given site quality. The trajectory followed by a stand can be represented as in Figure 1, with points on the curve corresponding to ages.

But what about thinning, as in the lower curves of Figure 1? Frequent light thinnings, as commonly practised in Europe, are often approximated by smooth curves, or a small number of standard regimes may be represented in managed yield tables. Evaluation of a full range of management alternatives is impractical, in particular the timing and intensity of a few heavy thinnings typical of plantation forestry in many countries. Other disturbances, and the updating of projected yields with inventory data, cause similar discontinuities.

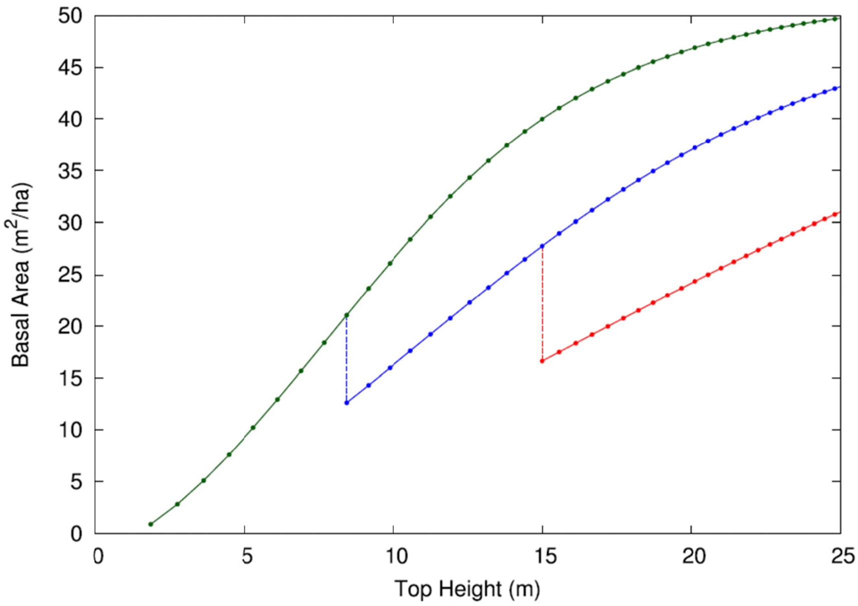

Figure 1. Unthinned trajectory (top), and trajectories with one and two thinnings. Annual steps shown as points on the curves

A deceptively simple trick solves the problem. Instead of trying to model the trajectories directly, we predict the change for a small time interval at every point. The arrows in Figure 2 indicate the change of state of the stand (height and basal area), depending on the current state. Trajectories are constructed by following the arrows. After a disturbance, one follows the arrow corresponding to the new point. Any combination of thinnings, or of other disturbances or updates, can be simulated in this way.

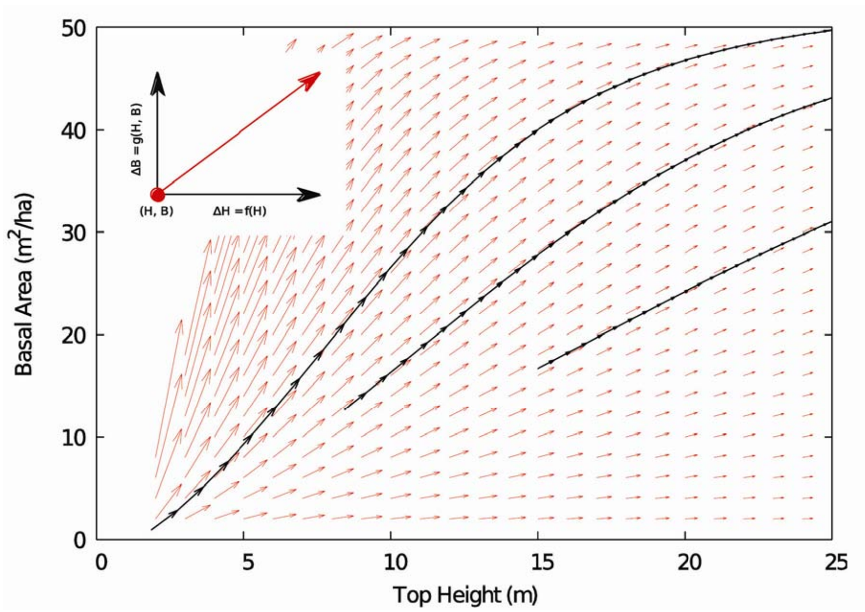

Figure 2. Rates of change: annual height and basal area increments. The rates generate a unique curve passing through any given point in the two-dimensional state space. The red arrows have been plotted originating at a rectangular grid

A dynamic model predicts the periodic (or annual, or infinitesimal) change in each of the state variables, e.g., height and basal area, as a function of the current state. The approximation can be improved by including additional state variables, more on that later. In the absence of disturbances, the trajectory is computed by iterating these relationships. A disturbance causes an instantaneous change of state, after which the same procedure is applied.

3. Inputs, Outputs, Dimensionality

The rate of change equations may depend on other variables, "inputs", such as a site quality or productivity index q. Note that q does not need to be constant, it can be time-dependent possibly due to climate or nutrient level changes. All that happens then is that the length and/or direction of the arrows vary over time. We simply use whatever arrow happens to be at the current point when we get there.

In addition to the state variables H and B, one may be interested in things like volume, or size distribution parameters ("outputs"). These may be estimated from the current values of the state variables. For instance, total volume may be obtained through stand volume functions such as $V = b_0 + b_1 B H$, or $\log V = b_0 + b_1 \log B + b_2 \log H$.

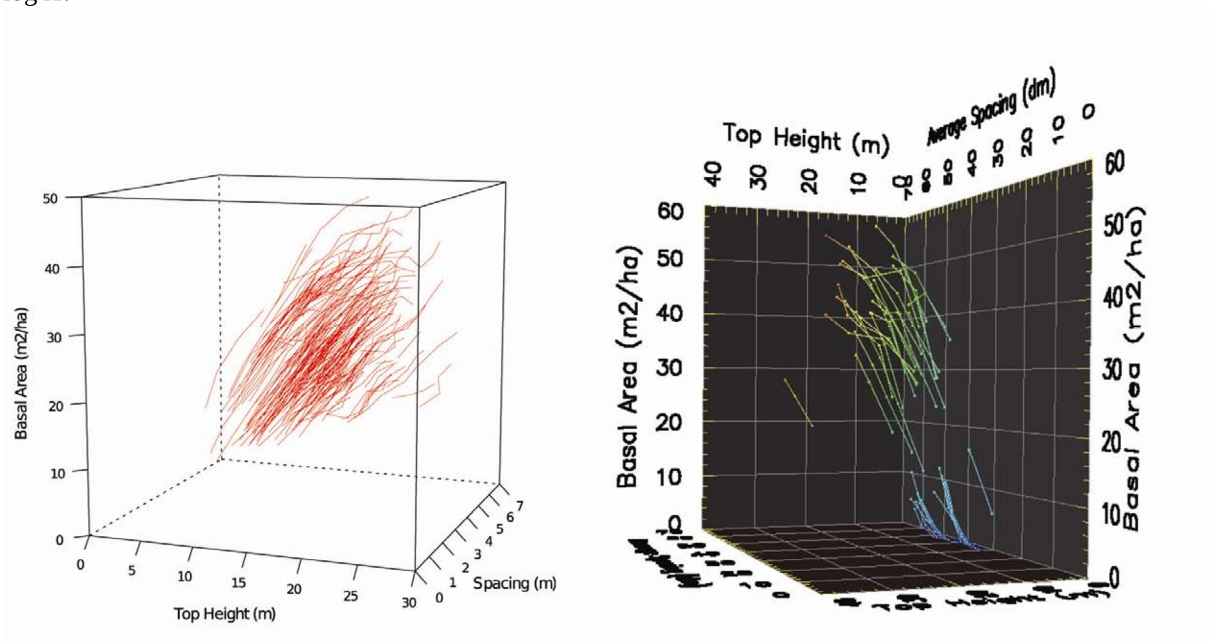

Figure 3. Three-dimensional trajectories observed in permanent sample plots. Left: thinned and unthinned loblolly pine data from García et al. (2011). Right: interior spruce plot measurements (García, 2011). A dynamic growth model predicts rates of change for the three variables at any point

So far, we have assumed that the behaviour of the two state variables is determined by their current values. For some purposes, a two-dimensional state may be a good enough approximation. Stand density management diagrams are a good example (Farnden, 1996). However, two stands with the same H and B, but different number of trees per hectare, may differ in their basal area growth. Also, merchantable volumes are affected by average tree size in addition to H and B. The model can be improved by adding the number of trees per hectare N (or the average spacing, or mean diameter) as a third state variable. The principles are the same, but now there is a 3-dimensional state space and 3 rate equations (Figure 3).

More state variables can be used. A stand immediately after thinning may not fully occupy the site, growing less than another stand with the same H, B and N but not recently thinned. Therefore, a fourth variable might improve accuracy, especially with heavy thinning and pruning (e.g., García et al., 2011). Iindividual-tree based models can be described in the same way, but they may contain hundreds of state variables, at least one diameter for each tree in the stand being modelled.

In the model, predicted rates and outputs are determined by the current state description. In other words, the state summarizes the relevant information about the system past. This should be seen more as a definition than as an assumption. In principle, it is always possible to add variables until they constitute a proper state, up to any desired degree of approximation. The appropriate dimensionality is a practical compromise between accuracy, parsimony, available data, and other considerations. For management purposes, it is also important to choose a scale at which reliable estimates of the initial state are possible, and at which the dynamics can be predicted (García, 2010).

4. Dynamical Systems and System Dynamics

The basic idea of modelling through rates of change probably originated with Isaac Newton in the 17th Century, and is standard in physics and engineering. In the 1960's, System Theory abstracted the general principles from the physical details, paving the way for wider applications (Zadeh & Polak, 1969; Kalman et al., 1969; Patten, 1971). There were contributions also from Cybernetics and Optimal Control Theory. Today the subject is part of Dynamical Systems Theory, although interest has shifted to chaotic behaviour and other aspects not directly relevant here (Wikipedia, 2012a).

The general idea is to describe a system through a suitable number of state variables. Then, a set of difference equations (Note 1) specify the change of these variables over a given time interval, depending on the current state and possibly also on one or more input variables (Figure 4, left). Continuous time can also be used, with infinitesimal time intervals, and in that case the rate of change is given by differential equations. Outputs are represented as functions of the current state.

The System Dynamics graphical notation of Forrester (1961) can facilitate communication, especially with less mathematically inclined researchers, practitioners, and students (Wikipedia, 2012b). There is also software that performs simulations based on the diagrams, requiring little or no mathematical or programming knowledge. The diagram represents each state variable as the stock of some material contained inside a box or compartment (Figure 4, right). Material moves in or out through pipes, with valves controlling the flow and therefore the rate of change in stock. Arrows indicate the dependence of a flow on stocks and on auxiliary variables (inputs or parameters). Arrows also define outputs as functions of stocks.

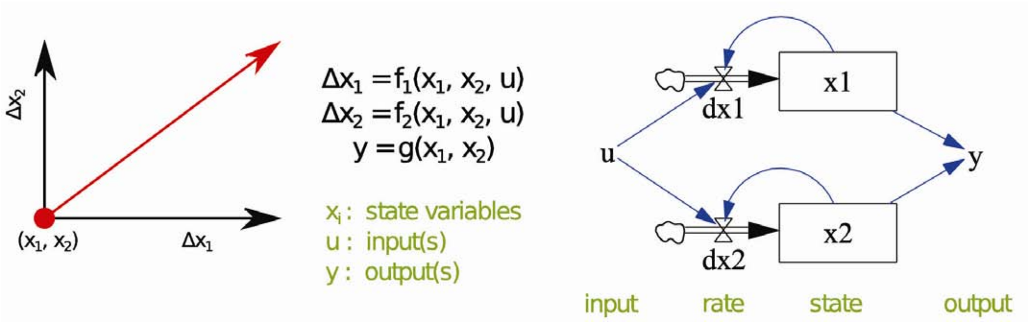

Figure 4. Left: system equations (rate equations, output function). Right: representation in a Forrester System Dynamics diagram

The stock/flow analogy might be stretched too far when dealing with variables such as height, and then a more general level/rate terminology may be more appropriate. Essentially, the stocks or levels correspond to state variables, and the flows or rates to difference or differential equations.

A convenient mathematical shorthand substitutes a single symbol for a list of numbers, a *vector*, usually distinguished by boldface or underlining. Notation can thus be simplified, especially in theoretical developments that are valid for any number of variables (Figure 5).

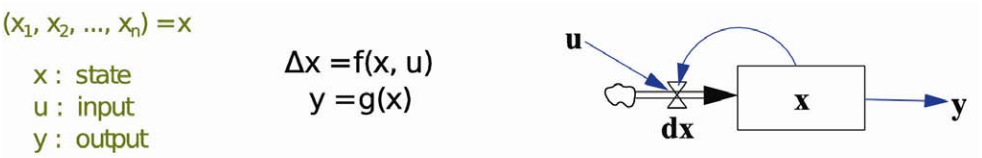

Figure 5. Writing lists of numbers as vectors (boldface) simplifies and generalizes the notation

5. An Example

How does this work in forest stand modelling? We illustrate with an even-aged whole-stand model from García (2011), García et al. (2011). With extensive data, as on the left-hand side of Figure 3, observed behaviour can be summarized by flexible purely empirical equations on three or four state variables, free from the influence of preconceived ideas. With sparse data (Figure 3, right), it is desirable to constrain the options under the guidance of eco-physiological theory and previous experience, through models that are parsimonious and consistent with biological knowledge. This is the case shown here. Although explained for this specific example, it should be clear that the principles are more generally applicable.

The stand is described by four state variables: top height (H), trees per hectare (N), the product of basal area and height $W = BH$ that is approximately linearly related to stem volume or biomass, and a variable R that represents relative canopy closure. We discuss the rates of change and interactions for each of these variables, shown clockwise from the top in Figure 6. In this description it is assumed that the stand consists of a single species.

Top-height growth rate depends on current height, and on the site quality parameter q (the arrows indicating the influence of q on all the rates will be omitted) (Note 2). It is known that, within limits, top height is not significantly affected by stand density, and therefore does not depend on the other state variables. Height development constitutes a self-contained sub-system, corresponding to a conventional site-index model.

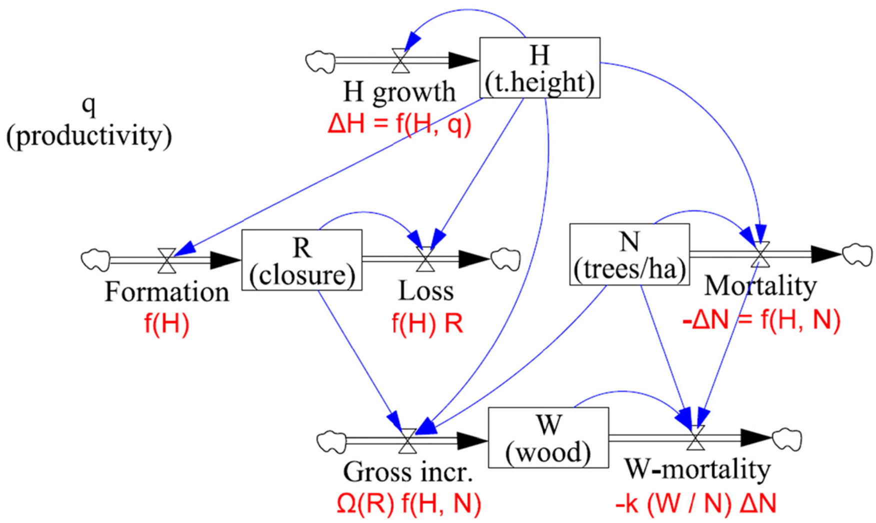

Figure 6. System Dynamics diagram for a stand growth model with 4 state variables. f( ) indicates a function of the variables inside the parenthesis, a different function in each equation

N decreases at a rate depending on H and N (and q, which is omitted to simplify). As before, age is excluded as a causal variable. Also excluded are stem diameter, basal area, or stem volume, variables commonly used in self-thinning relationships. These variables represent the amount of wood accumulated on the stems, much of it dead as heartwood; there are no good biological reasons why they should directly affect growth or mortality.

The change in W equals gross increment minus mortality. The mortality here is the mortality in number of trees times the mean tree size, reduced by a factor representing the size of dying trees relative to the survivors. In a fully closed stand ($R = 1$), gross increment may depend on H and N; again, we exclude age and W for biological reasons. If a stand is not yet fully closed, because it is young or has been recently thinned, growth is reduced by an "occupancy" factor Ω that depends on closure. Roughly, one may think of R as (relative) amount of foliage and fine roots, and of Ω as resource capture (light interception, water, nutrients).

Finally, a model for the changes in closure is needed. Thinking of foliage, it initially increases as height increases. As full closure approaches, foliage is lost from the crown base and older leaves/needles, tending to reach a balance.

The actual rate equations from García et al. (2011) are:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = q\left(48.76 / H^{0.07860} - 0.9271H\right)$$

$$\frac{\mathrm{d}N}{\mathrm{d}H} = -1.754 \cdot 10^{-11} H^{3.642} N^{2.518} / p^{3.037}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}H} = 0.1344\,pH\left(1-\Omega\right)$$

$$\frac{\mathrm{d}W}{\mathrm{d}H} = 0.2474\,p\Omega H N^{0.4} + 0.4\frac{W}{N}\frac{\mathrm{d}N}{\mathrm{d}H}$$

where p is the proportion of pine basal area in the stand. These are differential equations (continuous time t) rather than difference equations (discrete time). Although at first sight difference equations seem simpler, their use is cumbersome when measurement and projection intervals are not uniform and multiples of each other. Rates relative to height increment were independent of site quality (an extension of an old forestry hypothesis known as Eichhorn's law); they can be expressed as conventional time rates multiplying by the first equation.

6. Modules

The model can be extended by interfacing to other environmental or management components through suitable input and output variables. For example, the productivity index q could be driven by a climate model, and altered by genetic improvement. Similarly, nutrient cycling and fertilizing may control foliage and fine roots formation. Carbon cycling and the fate of dead biomass can be modelled further in a separate module.

Figure 7 depicts such a modular structure. The central block is the stand growth model from Figure 6. The other components are shown as simplified examples for illustration only. Additional feedback or feed-forward links, e. g., between soil organic matter and nutrition, are possible but not shown.

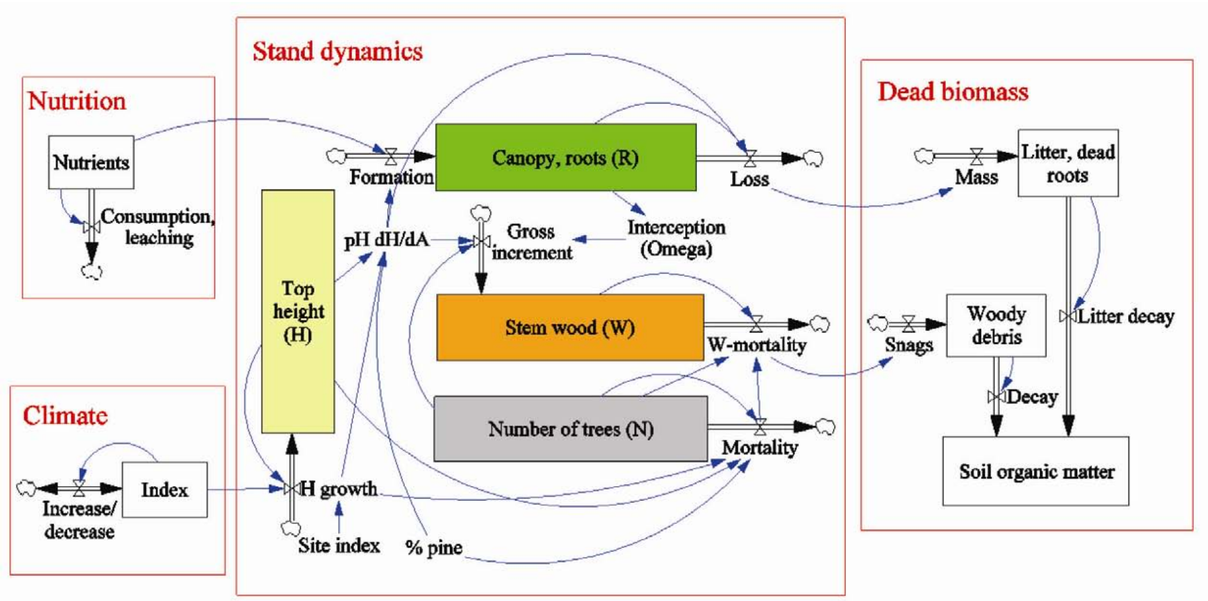

Figure 7. The example stand model linked to blocks representing simplified nutrition, climate, and dead biomass components. Reproduced with permission from García et al. (2011)

It is common for modellers to focus on certain elements depending on their background and interests, at the expense of oversimplifying other components of the system. The result is typically monolithic models with detailed environmental components and simplistic stand dynamics, or vice-versa. A modular approach would facilitate an independent development of sub-models by specialists, and their later “mix and match” into more balanced wholes according to requirements.

Modules can also be interfaced to describe more complex forests. For instance, spruce and aspen models like the one previously described can be coupled through the occupancy variable to build a model for two-storied spruce-aspen mixtures. Individual-based models can be interpreted as a large number of interacting tree dynamic models.

7. Conclusions

Long-term forest planning requires mathematical models, and the principles of Dynamical System Theory provide a solid foundation for these. The state-space approach makes it possible to accommodate disturbances and a varying environment. The concepts were demonstrated with a biologically consistent, parsimonious and robust class of semi-empirical models. Modular strategies can improve balance and efficiency.

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Notes

Note 1. Not to be confused with the “algebraic difference equations” used in forest modelling (Burkhart & Tomé, 2012, Section 7.8). Mathematically, those are neither difference equations nor algebraic.

Note 2. In some models growth is assumed to depend on age, instead of or in addition to height. Although both variables are highly correlated, physiology suggests that the dominant factor is size, not age.