Fuzzy Reliability of Two Units of the Cold Storing System

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Abstract
This paper which adopts Probability statistics fuzzy mathematical principles and methods gives the Fuzzy reliability index of the cold storing system with two different units when the switch is completely reliable and the switch is not completely reliable (Switch life 0-1 and Exponential distribution). And this paper gives a new kind of Failure mode, that is: system will be immediately failure if the switch is failure, meanwhile, it gives the new mode’s Fuzzy reliability index.

Keywords: Cold storing system, Reliability, Fuzzy reliability

1. Prior knowledge
Knows C by literature [1] to express in the classical reliable definition “the product in... maintains its stipulation function” this clear event, $C_1, C_2, ... C_n$ expressed separately each fuzzy function represent fuzzy event. Obviously C separately belongs to $\tilde{C}_1, \tilde{C}_2, ... \tilde{C}_n$ in varying degrees. $\tilde{C}$ expresses the system breakdown, $C_i$ expressed “the ist unit is working”, $\tilde{C}_i$ expressed the fuzzy function subset which we discussed.

By fuzzy conditional probability definition we obtain:

$$P(C \Delta C_i) = P(C_i / C) \cdot P(C)$$  \hspace{1cm} (1)

$$P(C_i \Delta C) = P(C / C_i) \cdot P(C_i)$$  \hspace{1cm} (2)

According to the fuzzy reliable theory and the ordinary reliable theory we have:

$$P(C \Delta C_i) = R_i$$  \hspace{1cm} $P(C_i \Delta C) = R_i$

$$P(C_i / C) = u_c (R_i)$$  \hspace{1cm} $P(C / C_i) = u_c (R_i)$  \hspace{1cm} (3)

Substitutes (3) into (1), (2) we have:

$$R_i = u_c (R_i) \cdot R_i$$  \hspace{1cm} (4)
\[ R_i = u c_j(R_i) \cdot R_i \]  

(5)

Based on the literature [1], [4] knowledge, the relations between every unit fuzzy failure rate \( \lambda_i \) and the ordinary failure rate \( \lambda_i \) is:

\[ \lambda_i = \lambda_i - \frac{d u c_j(R_i) \cdot dt}{u c_j(R_i) dt} = \lambda_i - u' c_j(R_i) \]  

(6)

Where \( u' c_j(R_i) \) is the relative rate of \( u' c_j(R_i) \).

2. Fuzzy Reliability analysis

**Theorem 1** Suppose the system is the cold storing system with two different units and the switch is completely reliable, its life respectively is \( x_1, x_2 \), also obeys separately exponential distribution \( \lambda_1, \lambda_2 \), mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

\[ R = u c_j(R_i) \left( \frac{\lambda_2 + u' c_j(R_i)}{\lambda_2 - \lambda_1 + u' c_j(R_i) - u' c_j(R_i)} R_i + \frac{\lambda_1 + u' c_j(R_i)}{\lambda_1 - \lambda_2 + u' c_j(R_i) - u' c_j(R_i)} R_2 \right) \]

\[ \text{MTTF} = u c_j(R_i)_m \left( \frac{\lambda_2 + u' c_j(R_i)}{\lambda_2 - \lambda_1 + u' c_j(R_i) - u' c_j(R_i)} R_i + \frac{\lambda_1 + u' c_j(R_i)}{\lambda_1 - \lambda_2 + u' c_j(R_i) - u' c_j(R_i)} R_2 \right) \]

**Proof:** Known two unit life distributions respectively are \( F_1 = 1 - e^{-\lambda_1 t} \), \( F_2 = 1 - e^{-\lambda_2 t} \), also knows the system by literature [2] the reliability is:

\[ R_i = \frac{\lambda_2}{\lambda_2 - \lambda_1} R_i + \frac{\lambda_1}{\lambda_1 - \lambda_2} R_2 \]  

(7)

So substitutes (5) (6) (7) into (4) we obtain the fuzzy reliability

\[ R = u c_j(R_i) \left( \frac{\lambda_2 + u' c_j(R_i)}{\lambda_2 - \lambda_1 + u' c_j(R_i) - u' c_j(R_i)} R_i + \frac{\lambda_1 + u' c_j(R_i)}{\lambda_1 - \lambda_2 + u' c_j(R_i) - u' c_j(R_i)} R_2 \right) \]

Result of \( R_i = \frac{\lambda_2}{\lambda_2 - \lambda_1} R_i + \frac{\lambda_1}{\lambda_1 - \lambda_2} R_2 \), obtain easily:

\[ \text{MTTF} = \int R_i \, dt = \int u c_j(R_i) \cdot R_i \cdot dt = u c_j(R_i)_m \cdot \int R_i \cdot dt \]

\[ = u c_j(R_i)_m \left[ \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} \right] dt \]

\[ = u c_j(R_i)_m \left[ \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right] \]  

(8)

Therefore substitute (6) into equation (8), we have:

\[ \text{MTTF} = u c_j(R_i)_m \frac{\lambda_1 + \lambda_2 + u' c_j(R_i) + u' c_j(R_2)}{[\lambda_1 + u' c_j(R_i)][\lambda_2 + u' c_j(R_2)]} \]  

(9)

Where, \( u c_j(R_i)_m \) is an average value which is in operating time sector \([0, \infty)\), and it is a constant.
Theorem 2 Suppose the system is the cold storing system with two different units and the switch is not completely reliable, its life respectively is \( x_1, x_2 \). Also obeys separately exponential distribution \( \lambda_1, \lambda_2 \), mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

\[
R_s = u_c(R_s) \left[ \frac{R_1}{u_c(R_1)} + \frac{P(\lambda_1 + u'c_j(R_1))}{\lambda_1 - \lambda_2 + u'c_j(R_1) - u'c_j(R_2)} \right] \left[ \frac{R_2}{u_c(R_2)} - \frac{R_1}{u_c(R_1)} \right]
\]

\[
MTTF = u_c(R_s) \left[ \frac{1}{\lambda_1 + u'c_j(R_1)} + p \cdot \frac{1}{\lambda_2 + u'c_j(R_2)} \right]
\]

Proof: Introduces a random variable \( v \), we have:

\[
p\{v = j\} = \begin{cases} \theta^{(\gamma_j=1)} & \text{if } j = 1 \\ \rho^{(\gamma_j=2)} & \text{if } j = 2 \end{cases}
\]

The reliability of system is:

\[
R = P\left( \bigcap_{j=1}^n \{ x_j > t \} \right) = q \cdot P\{x_1 > t\} + p \cdot P\{x_1 + x_2 > t\}
\]

\[
= e^{-\lambda_1 t} + \frac{p \lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t}) = R_1 + \frac{p \lambda_1}{\lambda_1 - \lambda_2} (R_2 - R_1)
\]  
(10)

Substituting (10) (5) (6) into (4) entails that:

\[
r_s = u_c(R_s) \cdot R_s
\]

\[
= u_c(R_s) \left[ \frac{R_1}{u_c(R_1)} + \frac{P(\lambda_1 + u'c_j(R_1))}{\lambda_1 - \lambda_2 + u'c_j(R_1) - u'c_j(R_2)} \right] \left[ \frac{R_2}{u_c(R_2)} - \frac{R_1}{u_c(R_1)} \right]
\]

Because the mean life of system is:

\[
MTTF = \int R_s dt = \int_0^\infty e^{-\lambda_1 t} + \frac{p \lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t}) dt = \frac{1}{\lambda_1} + p \frac{1}{\lambda_2}
\]

So the \( MTTF \) is:

\[
MTTF = \int u_c(R_s) \cdot dt = \int u_c(R_s) \cdot R_s \cdot dt = u_c(R_s) \left[ \frac{1}{\lambda_1} + p \frac{1}{\lambda_2} \right]
\]

\[
= u_c(R_s) \left[ \frac{1}{\lambda_1 + u'c_j(R_1)} + p \cdot \frac{1}{\lambda_2 + u'c_j(R_2)} \right]
\]  
(11)

Theorem 3 The system not immediately expires when the switch is not working, the life of two different units is \( x_1, x_2 \), the life of switch is \( x_k \). Obeys the exponential distribution separately and the parameter is \( \lambda_1, \lambda_2 \) and \( \lambda_k \), mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

\[
r_s = u_c(R_s) \left[ \frac{R_1}{u_c(R_1)} + \frac{\lambda_1 + u'c_j(R_1)}{\lambda_k + \lambda_1 - \lambda_2 + u'c_j(R_k) - u'c_j(R_2)} \right] \left[ \frac{R_2}{u_c(R_2)} - \frac{R_1}{u_c(R_1)} \right] \left[ \frac{R_k}{u_c(R_k)} - \frac{R_1}{u_c(R_1)} \right]
\]

\[
MTTF = u_c(R_s) \cdot R_s
\]
\[ \begin{align*}
\lambda_1 + u' c_j(R_i) \\
\lambda_1 + u' c_j(R_i) + [\lambda_1 + u' c_j(R_i) \times [\lambda_1 + u' c_j(R_i) + \lambda_K + u' c_j(R_K)]]
\end{align*} \]

**Proof:** From literature [2] we know

\[ R = e^{-\lambda t} + \frac{\lambda_1}{\lambda_K + \lambda_1 - \lambda_2} \left[ e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2) t} \right] \]  

(12)

\[ MTTF = \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_2 (\lambda_1 + \lambda_K)} \]  

(13)

Substituting (12) (5) (6) into (4) entails that

\[ R_i = u c_j(R_i) \cdot R, \]

\[ = u c_j(R_i) \cdot \frac{R_1}{u c_j(R_1)} + \frac{\lambda_1 + u' c_j(R_i)}{\lambda_2 + \lambda_1 - \lambda_2 + u' c_j(R_K) + u' c_j(R_i) - u' c_j(R_i)} \]

\[ \left[ \begin{array}{ccc}
R_1 \\
R_2 \\
R_K
\end{array} \right] = \left[ \begin{array}{ccc}
R_1 \\
R_2 \\
R_K
\end{array} \right] \]

Because also

\[ MTTF = \int R_i dt = \int u c_j(R_i) \cdot R_i dt \]

\[ = u c_j(R_i) \cdot \int R_i \cdot dt = u c_j(R_i) \cdot \left[ \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_2 (\lambda_1 + \lambda_K)} \right] \]

(14)

Substituting (6) into (14) entails that

\[ MTTF = u c_j(R_i) \cdot \left[ \begin{array}{ccc}
\lambda_1 + u' c_j(R_i) \\
\lambda_1 + u' c_j(R_i) + [\lambda_1 + u' c_j(R_i) \times [\lambda_1 + u' c_j(R_i) + \lambda_K + u' c_j(R_K)]]
\end{array} \right] \]

(13)

**Theorem 4** The system immediately expires when the switch is not working, the life of two different units is \( x_1, x_2 \), the life of switch is \( x_K \), Obey the exponential distribution separately and the parameter is \( \lambda_1, \lambda_2 \) and \( \lambda_K \), mutually independent, so the fuzzy reliability is:

\[ R_i = u c_j(R_i) \cdot \left[ \begin{array}{ccc}
R_1 \\
R_2 \\
R_K
\end{array} \right] = \left[ \begin{array}{ccc}
R_1 \\
R_2 \\
R_K
\end{array} \right] \]

\[ \left[ \begin{array}{ccc}
\lambda_1 + u' c_j(R_i) \\
\lambda_1 + u' c_j(R_i) + [\lambda_1 + u' c_j(R_i) \times [\lambda_1 + u' c_j(R_i) + \lambda_K + u' c_j(R_K)]]
\end{array} \right] \]

\[ \left[ \begin{array}{ccc}
\lambda_1 + u' c_j(R_i) \\
\lambda_1 + u' c_j(R_i) + [\lambda_1 + u' c_j(R_i) \times [\lambda_1 + u' c_j(R_i) + \lambda_K + u' c_j(R_K)]]
\end{array} \right] \]

\[ \left[ \begin{array}{ccc}
\lambda_1 + u' c_j(R_i) \\
\lambda_1 + u' c_j(R_i) + [\lambda_1 + u' c_j(R_i) \times [\lambda_1 + u' c_j(R_i) + \lambda_K + u' c_j(R_K)]]
\end{array} \right] \]
Proof: the switch is not expire when the unit \( x_1 \) is not working, \( x_K > x_1 \), unit \( x_1 \) is replaced by storing unit \( x_2 \), the life of system is \( x_K \) when unit \( x_2 \) is not expire.

Because the life distribution of system is:

\[
1 - R_s = \int_{t_1}^{t_2} \lambda_1 \lambda_K e^{-\lambda_1 t_1} e^{-\lambda_K t_2} dt_1 dt_2 + \\
\int_{t_1+t_2}^{t_1+t_2} \lambda_1 \lambda_2 \lambda_K e^{-\lambda_1 t_1} e^{-\lambda_2 t_2} e^{-\lambda_K t_2} dt_1 dt_2 + \\
\int_{t_1+t_2}^{t_1+t_2} \lambda_1 \lambda_2 \lambda_K e^{-\lambda_1 t_1} e^{-\lambda_2 t_2} e^{-\lambda_K t_2} dt_1 dt_2 \\
= 1 - e^{-\lambda t} - \frac{\lambda_1}{\lambda_K + \lambda_1 - \lambda_2} e^{-\lambda t} + e^{-\lambda t} \\
+ \frac{\lambda_K \lambda_1}{(\lambda_K + \lambda_1)(\lambda_2 - \lambda_1)} [1 - e^{-\lambda t} + e^{-\lambda t}] + \frac{\lambda_K \lambda_2}{(\lambda_K + \lambda_2)(\lambda_2 - \lambda_1)} [1 - e^{-\lambda t} + e^{-\lambda t}]
\]

So the reliability of system is:

\[
R_s = e^{-\lambda t} + \frac{\lambda_1}{\lambda_K + \lambda_1 - \lambda_2} e^{-\lambda t} - \frac{\lambda_K \lambda_1}{(\lambda_K + \lambda_1)(\lambda_2 - \lambda_1)} [1 - e^{-\lambda t} + e^{-\lambda t}] - \frac{\lambda_K \lambda_2}{(\lambda_K + \lambda_2)(\lambda_2 - \lambda_1)} [1 - e^{-\lambda t} + e^{-\lambda t}]
\]

Substituting (15) (5) (6) into (4) we obtain:

\[
R_s = u c_j(R_1) \cdot R_s
\]

References


