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An N-Component Series Repairable System with Repairman Doing Other Work and Priority in Repair

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Abstract

In this paper, we discussed the reliability of an N-unit series repairable system with the repairman doing other work and priority in repair. In this system, it is assumed that the working time distributions of the n components and the arrival interval time distributions of the customers which are out of the system are both exponential and the components in system is given priority in repair. It is also assumed that the repair time distributions of the n components and the service time distributions of the customers are both general continuous distributions. After repair, components are "as good as new". Under these assumptions, using a supplementary variable technique and Laplace transform technique, some important reliability indices such as the system availability, the idle probability of the repairman and the rate of service for customers are derived. Our problem is to determine whether or not given the priority to components in repairing such that the benefit of the system is maximized.

Keywords: Reliability, Supplementary technique, Generalized Markov process

1. Introduction

In order to improve the interest of the system, the supervisor always arranges the repairman service for the customers out of system. Under this condition and assumptions that the working time distributions are exponential and the repair time distributions and the time distribution of the repairman doing other work are both general continuous distributions, by using supplementary technique and vector Markov process, the reliability indices have been obtained and the benefits of the model discussed (Su, B.H., 1994, pp.34-39). Later the number of the customers has been introduced. LIU R.B. and TANG Y.H. (2005, pp.493-496) assumed that the working time and the arrival interval time have exponential distribution while others to be general continuous distributions. By using the supplementary variable method, the vector Markov process and the tool of the Laplace transform, some reliability indexes of the system have been derived and the benefit of the system discussed. HU L.M., WU J.B. and TIAN R.L. (2007, pp.47-50) have been introduced the assumption that each unit had two types of failure. They have obtained the reliability indices of the system. Zhang Y.L. and Wang G. J.(2006,pp.278－295) introduced the priority in use in a deteriorating cold standby repairable system.

While it isn't always maximizing the interest of system that repairman is servicing for customers out of system, the repair work of the components maybe delayed for the repairman doing other work. The purpose of this paper is to apply the priority repair model to an n-component series repairable system with the repairman doing other work. Now we may assume that the units after repair is "as good as new" and the units have priority in repair. Furthermore, we assume that the working time of the components and the arrival interval time customers are both exponentially distributed and others generally distributed.

2. Model

We study an n-component series repairable system with repairman doing other work and priority in repair by making the following assumptions:

Assumption 1. Initially, the n components are all new and work in double harness while the repairman is idle.

Assumption 2. When one component is on repairing, the others will stop working and not go wrong. The system is failing now.

Assumption 3. If and only if when the n components are all working and the customer doesn't arrive, the repairman is idle. If and only if when the n components are all working, the customers will be likely to arrival. The customer won't arrive, when one component is repaired or the repairman is serving for one customer and another customer is waiting for service. When one customer is waiting for service and one component goes wrong, the customer will leave. And the component is repaired immediately, while the customer served will be waiting for service. The customer will be served after the repair over and the service before is valid.

Let X_i and Y_i be, respectively, the working time and the repair time of the component $i, i=1,2,\cdots n$. We assume that the distributions of X_i and Y_i ($i=1,2,\cdots n$) are, respectively,

$$F_i(t)=1-\exp(-\lambda_i t),$$

$$G_i(t)=\int_0^t g_i(x)dx=1-\exp(-\int_0^t \mu_i(x)dx),$$

Where $t\geq 0, \lambda_i>0, \mu_i>0; i=1,2,\cdots n$.

Let H be the service time of the customer. We assume that the distribution of H is

$$H(t)=\int_0^t h(x)dx=1-\exp(-\int_0^t d(x)dx).$$

Let V be the time between repairman start idle and the arrival of the first customer and between the arrival of the first customer and the second one. We assume that the distribution of V is

$$V(x)=1-\exp(-ct).$$

Where $t\geq 0, c>0$.

Assumption 4. Assume that X_i, Y_i ($i=1,2,\cdots n$), H and V are independent.

3. The system analysis

Let $\{S(t), t\geq 0\}$ be a stochastic process characterized by the following mutually exclusive events:

$\{S(t)=0\}$: n components are all working and the repairman is idle.

$\{S(t)=1i\}$: The component i is repaired, $i=1,2,\cdots n$.

$\{S(t)=2\}$: n components are all working, the repairman is serving for the customer.

$\{S(t)=3\}$: n components are all working, the repairman is serving for one customer and the anther waiting for service.

$\{S(t)=4i\}$: the component i is repaired, the customer is waiting for service, $i=1,2,\cdots n$.

Then $\{S(t), t\geq 0\}$ is a stochastic process with state space $\Omega=\{0,1i,2,3,4i; i=1,2,\cdots n\}$. The set of working states is $W=\{0,2,3\}$ and the set of failure states is $F=\{1i,4i; i=1,2,\cdots n\}$. According to the model assumptions, $\{S(t), t\geq 0\}$ is not a Markov process. However, it can be extended to a generalized Markov process by introducing a supplementary variable. Let $X_i(t)$ be the repair time of the component i used at time t, $i=1,2,\cdots n$. Let $Y(t)$ be the service time of the customer used. Then $\{S(t), X_i(t), Y(t), t\geq 0; i=1,2,\cdots n\}$ constitutes a generalized Markov process.

The state marginal probability of the system at time t are defined by

$P_0(t) = P\{S(t) = 0\}$;

$P_{1i}(t,x)dx = P\{S(t) = 1i, x \le X_{1i}(t) < x + dx\}, i = 1,2,\cdots n$;

$P_2(t,y)dy = P\{S(t) = 2, y \le Y(t) < y + dy\}$;

$P_3(t,y)dy = P\{S(t) = 3, y \le Y(t) < y + dy\}$;

$P_{4i}(t,x,y)dx = P\{S(t) = 4i, x \le X_i(t) < x + dx, Y(t) = y\}, i = 1,2,\cdots n.$

According to the model assumptions and the supplementary variable technique, we can obtain the following differential equations for the system. By straightforward probability arguments, for example, we have

$P_0(t+\Delta t) = P_0(t)(1 - \Lambda\Delta t - c\Delta t) + \sum_{i=1}^{n}\int_0^\infty \mu_i(x)\Delta t P_{1i}(t,x)dx + \int_0^\infty d(y)\Delta t P_{1i}(t,y)dy + o(\Delta t)$ where $\Lambda = \sum_{i=1}^{n}\lambda_i$, Letting Δt tend to zero, we can get

$$\left(\frac{d}{dt} + \Lambda + c\right)P_0(t) = \sum_{i=1}^{n}\int_0^\infty \mu_i(x)P_{1i}(t,x)dx + \int_0^\infty d(y)P_2(t,y)dy\,; \quad (1)$$

In the same way, we have

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_i(x)\right)P_{1i}(t,x) = 0\,, i = 1,2,\cdots n\,; \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \Lambda + c + d(y)\right)P_2(t,y) = \sum_{i=1}^{n}\int_0^\infty \mu_i(x)P_{4i}(t,x,y)dx\,; \quad (3)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \Lambda + d(y)\right)P_3(t,y) = cP_2(t,y)\,; \quad (4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_i(x)\right)P_{4i}(t,x,y) = 0\,, i = 1,2,\cdots n\,; \quad (5)$$

The boundary conditions are

$$P_{1i}(t,0) = \lambda_i P_0(t)\,, i = 1,2,\cdots n\,; \quad (6)$$

$$P_2(t,0) = cP_0(t) + \int_0^\infty d(y)P_3(t,y)dy\,; \quad (7)$$

$$P_3(t,0) = 0\,; \quad (8)$$

$$P_{4i}(t,0,y) = \lambda_i P_2(t,y) + \lambda_i P_3(t,y)\,; \quad (9)$$

The initial conditions are

$P_0(0) = 1, P_{1i}(0) = 0, i = 1,2,\cdots n, P_2(0) = 0, P_3(0) = 0, P_{4i}(0), = 0, i = 1,2,\cdots n$.

The system differential equations using Laplace transforms are obtained as follows:

$$(s + \Lambda + c)P_0^*(s) = P_0^*(0) + \sum_{i=1}^{n}\int_0^\infty \mu_i(x)P_{1i}^*(s,x)dx + \int_0^\infty d(y)P_2^*(s,y)dy\,; \quad (10)$$

$$\left(\frac{d}{dx} + s + \mu_i(x)\right)P_{1i}^*(s,x) = 0\,, i = 1,2,\cdots n\,; \quad (11)$$

$$\left(\frac{d}{dy} + s + \Lambda + c + d(y)\right)P_2^*(s,y) = \sum_{i=1}^{n}\int_0^\infty \mu_i(x)P_{4i}^*(s,x,y)dx\,; \quad (12)$$

$$\left(\frac{d}{dy} + s + \Lambda + d(y)\right)P_3^*(s,y) = cP_2^*(s,y)\,; \quad (13)$$

$$\left(\frac{d}{dx}+s+\mu_i(x)\right)P_{4i}^*(s,x,y)=0\,,i=1,2,\cdots n\,; \tag{14}$$

Together with the borderline conditions and the initial conditions, this system of equations can be solved to yield:

$$P_0^*(s)=\frac{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\left[1-\hat{H}(w)+\hat{H}(s+\Lambda+c)\right]}{(w+c)\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)-c(s+\Lambda+c)\hat{H}(w)+cw\hat{H}(s+\Lambda+c)}\,; \tag{15}$$

$$P_{1i}^*(t,x)=\lambda_i P_0^*(s)e^{-sx}\overline{G}_i(x)\,,i=1,2,\cdots n\,; \tag{16}$$

$$P_2^*(s,y)=P_2^*(s,0)\frac{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)}{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c}\left(e^{\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)y}+\frac{c}{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)}\right)e^{-(s+\Lambda+c)y}\overline{H}(y)\,; \tag{17}$$

$$P_3^*(s,y)=P_2^*(s,0)\frac{c}{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c}\left(e^{\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)y}-1\right)e^{-(s+\Lambda+c)y}\overline{H}(y)\,; \tag{18}$$

$$P_{4i}^*(s,x,y)=P_2^*(s,0)e^{-Sy}\overline{H}(y)e^{-sx}\overline{G}_i(x)\,,i=1,2,\cdots n\,; \tag{19}$$

Where
$$P_2^*(s,0)=\frac{c\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)}{w\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)-c(s+\Lambda+c)\hat{H}(w)+cw\hat{H}(s+\Lambda+c)}\,; \tag{20}$$

$$w=s+\Lambda-\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)\,.$$

4. Reliability indices

4.1 system availability and system rate of occurrence of failure

By the definition, the system availability at t is given by

A(t) = P (the system is working at time t) = P(N(t) $\in$ W) = $P_0(t)+\sum_{k=2}^{3}\int_0^\infty P_k(t,y)dy\,.$

The Laplace-transform of A(t) is

$$A^*(s)=P_0^*(s)+\int_0^\infty P_2^*(s,y)dy+\int_0^\infty P_3^*(s,y)dy$$

Where, according to (15), (17), (18) and (20), thus

$$A^*(s)=\frac{\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c-c\hat{H}(w)+c\hat{H}(s+\Lambda+c)\right)+\frac{c}{w}(s+\Lambda+c)\left(1-\hat{H}(w)\right)}{(w+c)(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c)-c(s+\Lambda+c)\hat{H}(w)+cw\hat{H}(s+\Lambda+c)} \tag{21}$$

Where $w=s+\Lambda-\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)$. Using Tauberian theorem, the stay state availability or the limiting availability of the system is given by

$$A=\lim_{t\to+\infty}A(t)=\lim_{s\to 0^+}sA^*(s)=\frac{1}{1+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}} \tag{22}$$

Let $W_f(t)$ be the rate of occurrence of failure or the failure frequency of the system at time t.

Hence

$$M_f(t)=\int_0^t W_f(x)dx\cdot$$

According to Lam [6], we have

$$W_f(t)=\sum_{i=1}^{n}\left[\lambda_i P_0(t)+\int_0^\infty \lambda_i P_2(t,y)dy+\int_0^\infty \lambda_i P_3(t,y)dy\right]=\Lambda A(t)\cdot$$

The Laplace transform of $W_f(t)$ is given by

$$W_f^*(s)=\sum_{i=1}^{n}\left[\lambda_i P_0^*(s)+\int_0^\infty \lambda_i P_2^*(s,y)dy+\int_0^\infty \lambda_i P_3^*(s,y)dy\right]=\Lambda A^*(s)\,. \tag{23}$$

Using Tauberian theorem, the stay state failure frequency of the system is given by

$$W_f=\lim_{t\to+\infty}W_f(t)=\lim_{s\to 0^+}sW_f^*(s)=\frac{\Lambda}{1+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}\cdot \tag{24}$$

4.2 The idle time probability of the repairman

Clearly, the repairman will be idle if and only if the components are all working and no customer is arrival. Thus, the idle time probability of the repairman at time t is given by

$$I(t)=P_0(t)$$

The Laplace transform of $I(t)$ is $I^*(s)=P_0^*(s)$.

Using Tauberian theorem, the stay state failure frequency of the system is given by

$$I=\lim_{t\to+\infty}I(t)=\lim_{s\to 0^+}sI^*(s)=\frac{\Lambda+c\hat{H}(\Lambda+c)}{\left(1+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\right)\left(\Lambda+c\hat{H}(\Lambda+c)\right)+\frac{c}{d}(\Lambda+c)\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}}\,. \tag{25}$$

4.3 The rate of occurrence of the repairman serving for customer

Let $N(t)$ be the number of the customer served during a given time (0,t], then according Lam[6], we have

$$N(t)=\int_0^t D_s(x)dx$$

Where $D_s(t)$ is the rate of occurrence of the repairman serving for customer or the serving frequency of the repairman at time t.

According to Lam [6], we have

$$D_s(t)=\int_0^\infty d(y)P_2(t,y)dy+\int_0^\infty d(y)P_3(t,y)dy$$

The Laplace transform of $D_s(t)$ is given by

$$D_s^*(s)=\int_0^\infty d(y)P_2^*(s,y)dy+\int_0^\infty d(y)P_3^*(s,y)dy$$

$$=\frac{c\left(\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c\right)\hat{H}(s+\Lambda-\sum_{i=1}^{n}\lambda_i\hat{G}_i(s))P_0^*(s)}{\sum_{i=1}^{n}\lambda_i\hat{G}_i(s)+c-c\left[\hat{H}(s+\Lambda-\sum_{i=1}^{n}\lambda_i\hat{G}_i(s))-\hat{H}(s+\Lambda+c)\right]}\,.$$

Using Tauberian theorem, the stay state number of the customer served by the repairman per unit time is given by

$$N=\lim_{t\to+\infty}D_s(t)=\lim_{s\to 0^+}sD_s^*(s)=\frac{c(\Lambda+c)}{\left(1+\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i}\right)\left[\Lambda+\frac{c}{d}(\Lambda+c)+c\hat{H}(\Lambda+c)\right]}\,. \tag{26}$$

5. The system benefit analysis

In this section, our objective is to determine whether or not given the priority to components in repairing such that the benefit of the system is maximized. Let x_1' be the working reward per unit time of the system, x_2' be the average cost each time of the system, and x_3' be the average reward of serving for one customer. Based on the assumptions, the stay state average reward per unit time of the system is

$$y_1 = Ax_1' - W_f x_2' + Dx_3' = \frac{(x_1' - \Lambda x_2')\left[\Lambda + \frac{c}{d}(\Lambda + c) + \hat{H}(\Lambda + c)\right] + c(\Lambda + c)x_3'}{\left(1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\right)\left[\Lambda + \frac{c}{d}(\Lambda + c) + \hat{H}(\Lambda + c)\right]}, \quad (27)$$

In order to solve the our problem, it is necessary to introduce some conclusion and assumption that LIU,Ren – bin, TANG,Ying – hui &LUO Chuan – yi(2005) have studied. In this paper, let x_1 be the working reward per unit time of the system, x_2 be the average cost each time failure of the system, and x_3 be the average reward of serving for one customer. Clearly, for one system, the x_1' and x_3' will be, respectively, same to the x_1 and x_3 while the x_2' will be different from x_2, because the x_2 include the cost produced by waiting repair. So the equation will be changed to

$$y_1 = Ax_1 - W_f x_2' + Dx_3 = \frac{(x_1 - \Lambda x_2')\left[\Lambda + \frac{c}{d}(\Lambda + c) + \hat{H}(\Lambda + c)\right] + c(\Lambda + c)x_3}{\left(1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\right)\left[\Lambda + \frac{c}{d}(\Lambda + c) + \hat{H}(\Lambda + c)\right]}$$

The stay state average reward per unit time of the system in Lam [2] is

$$y_0 = \frac{(x_1 - \Lambda x_2)\left[1 + \frac{c}{\Lambda} \cdot \frac{1 - \hat{H}(\Lambda)}{1 - \hat{H}(\Lambda) + \hat{H}(\Lambda + c)}\right] + \frac{cx_3}{1 - \hat{H}(\Lambda) + \hat{H}(\Lambda + c)}}{1 + \frac{c}{d} \cdot \frac{1}{1 - \hat{H}(\Lambda) + \hat{H}(\Lambda + c)} + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i} \cdot \frac{c}{\Lambda} \cdot \frac{1 - \hat{H}(\Lambda)}{1 - \hat{H}(\Lambda) + \hat{H}(\Lambda + c)}}.$$

whether or not the components given the priority in repairing will depend on the result of the equation $y_1 - y_0$: the component will be given priority in repair when the result of equation $y_1 - y_0$ is positive number, while the component will not be given priority in repair when the result of equation $y_1 - y_0$ is negative. In this way the benefit of the system will be maximized.

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