# The Deformation Analysis of the Curved Box Girder Bridges under Different Radius

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# Abstract

This paper investigates the influence of radius on the deformation of curved box girder bridges under the action of pre-stressed tendons. A spiral pre-stressed concrete box girder bridge in Chongqing is analyzed by using nonlinear finite element program ANSYS. The five different curved bridge models under weight and prestressing were created and the sameness and difference among them are gained and compared by numerical simulation. The results showed: 1. In five different situations, the box girder all occurs the vertical, radial and longitudinal deformation, among which vertical deformation is the main. At the same time, the box section occurred reverse and distortion, and the whole body of the beam rolls out. 2. The value of vertical displacement of mid-span of continuous curved box girder bridges relates with the horizontal radius. When the radius between one hundred meters and one hundred fifty meters, the influence of displacement is most sensitive, on the contrary, when the radius greater than two hundred meters, the influence of displacement is almost no effect to mid-span displacement.

Keywords: Curved Beam Bridge, Radius, Numerical Simulation, Deformation

# 1. Introduction

With the development of traffic construction, the curved pre-stressed concrete (PC) box girder bridges have got a wide range of applications. The structure has larger torsion stiffness, beautiful shapes, and it is often used in complex interchange and city viaduct. Relative to the reinforced concrete bridge, curved PC box girder bridges have high strength and small section, and the pre-stressed tendons usually use three-dimensional shape in box girder space, mainly arranged in the webs of box girders. The use of pre-stressing, on one hand, greatly improves force condition of the bridge, thus enhanced its spanning capacity; on the other hand, because of the existence of horizontal curvature, the curved pre-stressed tendons make the girder become more complex, often produce some adverse effects. The curved box girder bridges is different from straight one, we should pay much attention when in design and construction. In this paper, a spiral pre-stressed concrete box girder bridge in Chongqing is analyzed by using nonlinear finite element program ANSYS. The five different curved bridge models under self-weight and prestressing were created and the sameness and difference among them are gained and compared by numerical simulation (Zhou X. H., 2007).

# 2. Related Analysis Methodology

# 2.1 Static Equilibrium Equations

We could take out a micro segment from the curved girder, and then, take the tangential direction of beam as z

axis, the centripetal direction of curve as x axis, and the down direction perpendicular to the curve plane as y axis. Then, a three-dimensional fluid rectangular coordinate system is established. Six possible load types would exist in the beam segment which were distributed-load  $q_x$ ,  $q_y$ ,  $q_z$  along the coordinate axis and moment  $m_x$ ,  $m_y$ ,  $m_z$  around the coordinate axis. The coordinate system and each types of load were shown in Figure 1. Internal forces in the beginning section of micro segment including axial force N, torque force T, shear force  $Q_x$ ,  $Q_y$  and bending moment  $M_x$ ,  $M_y$  were caused by load above. A differential increment came out in the terminal section of micro segment through  $d_y$ , as is shown in Figure 2.

Static equilibrium differential equations of curved girders could be established by six equilibrium conditions of space force system. The equation is as follows:

$$\frac{\partial Q_x}{\partial z} + \frac{N}{r} + q_x = 0 \qquad (1a) \qquad \qquad \frac{\partial Q_y}{\partial z} + q_y = 0 \qquad (1b) \qquad \qquad \frac{\partial N}{\partial z} - \frac{Q_x}{r} + q_z = 0 \qquad (1c)$$

$$\frac{\partial M_x}{\partial z} + \frac{T}{r} - Q_y + m_x = 0 \quad (1d) \qquad \qquad \frac{\partial M_y}{\partial z} + Q_x + m_y = 0 \quad (1e) \qquad \qquad \frac{\partial T}{\partial z} - \frac{M_x}{r} + m_z = 0 \quad (1f)$$

#### 2.2 Geometric Equations

The relationship between inner force and deformation in sections can't be concluded without establishing the geometric deformation relationship of micro segment. For any possible point in the curved axis of the curved beam, u, v, w were taken as the displacement along axis x, y and z, and  $\phi$  is taken as the torsion angle. According to literature (Yao, L. S., 1986), geometric equations of curved beam were found out as follows:

$$\varepsilon_{z} = \frac{du}{dz} - \frac{v}{r}$$
 (2a)  $k_{y} = \frac{d^{2}v}{dz^{2}} + \frac{v}{r^{2}}$  (2b)  $k_{x} = \frac{d^{2}w}{dz^{2}} - \frac{\phi}{r}$  (2c)  $k_{z} = \frac{d\phi}{dz} + \frac{1}{r}\frac{dw}{dz}$  (2d)

In formulas above:  $\varepsilon_z$  is the axial stain;  $k_x$  and  $k_y$  represent to the variation of curvature in axis x and y separately;  $k_z$  is the torsion angle per unit length in the curved beam.

# 2.3 Basic Differential Equations

The inner force and deformation could be connected by physical equations of material mechanics. Equation (3) below is the basic differential equation of curved bridges conclude from Equation (1) and (2), which is called Vlasov differential equation (Liu, 2007; Zhu, 1984; Xiong, 1984).

$$\frac{EI_{\omega}}{R}w^{IV} - \frac{EI_x + GI_d}{R}w^{"'} + EI_{\omega}\varphi^{IV} - GI_d\varphi^{"} + \frac{EI_x}{R^2}\varphi = m_z$$
(3a)

$$(EI_x + \frac{EI_{\omega}}{R^2})w^{IV} - \frac{GI_d}{R^2}w^{"''} + \frac{EI_{\omega}\varphi^{IV}}{R} - (\frac{GI_d + EI_x}{R})\varphi^{"} = q_y + \frac{\partial m_x}{\partial z}$$
(3b)

$$(EI_{y}(v^{\nu} + \frac{2}{R^{2}}v^{m} + \frac{1}{R^{4}}v^{\prime}) = \frac{\partial q_{z}}{\partial z} - \frac{q_{z}}{R} - \frac{\partial^{2}m_{y}}{\partial z^{2}} - \frac{m_{y}}{R^{2}}$$
(3c)

# 3. Numerical Simulation

#### 3.1 Project Overview

The full bridge length is 643.885 m, width is19 m, two-way four-lane. Horizontal curve radius is 55 m and design load is City Class-A. The structure used pre-stressed concrete box girder, of which cross-section is the discrete two-box and each with single-chamber and has big torsion stiffness. The whole spiral bridge is shown as Figure 3. Five different groups of radius which are commonly used in engineering were adopted in simulation. The values are: 50 m, 100 m, 150 m, 200 m and 250 m.

#### 3.2 Finite-Element Modeling

The full bridge model was established use finite element program ANSYS. The bridge has lateral inside one-way cross slope of 2% and use the same cross-section. The thickened weight of the box girder at the bearing is replaced with a vertical uniform line load. The concrete box girder unit use solid65 and pre-stressed steel with link8 unit in ANSYS. The Full-bridge finite element models of radius fifty shown as Figure 4 and Figure 5.

# 3.3 Calculation Results

The paper analyzed displacement characteristic in five different box-girder radius, the result as Figure 6. From it we can concluded that:

(1) The continuous curved box girder bridge occurred deformation in the vertical, transverse and along the bridge under the combined load of weight and pre-stressing. And the vertical, transverse displacement curve are axisymmetric along the centerline of the bridge; the displacement curve along the bridge is centrosymmetric (due to the provisions of second coordinates component direction of the cylindrical coordinate).

(2) When the radius is between 50 m and 250 m, vertical deformation and deformation along the bridge is major, and the deformation along the bridge in both ends is greater than the maximum of anti-arch vertical deformation. The radial deformation is small, which can be neglected. The vertical displacement of side span is greater than the middle span, the ratio between 1.5 to 2.0. (Wang X. D. & Ding H. S., 2006; JIANG J. J., 2002).

From Figure 7 to Figure 8, we can concluded that:

(1) Figure 7 shows that the vertical displacement in middle of continuous curved box girder bridges is related to horizontal radius. When the radius is less than 170 m, as the radius increasing, the maximum of the vertical displacement caused by pre-stressed steel increases. Especially when the radius is between 100 m and 150 m, displacement increases more rapidly. When the radius is greatter than 200 m, the curve gradually tends to level, that is horizontal radius no longer affect the value of vertical anti-arch significantly, at this time, force characteristics same as the straight beam bridge.

(2) In Figure 8, vertical displacement of the outer box girder is less than the inside, which shows that box girder cross-section produces a clockwise rotation deformation around the center of curvature under weight and pre-stressed loads, and both inside and outside vertical displacement are not linear. The cross-section occurred distortion. With the radius increasing, the difference of vertical displacement between the outer edge and the inner edge in box girder cross-section decreases and vertical displacement distribution curve gradually tends to level. In other words, when the radius increases gradually, torque decreases and the "bending - twisting" coupling effect of the curved bridge weakens, the force characteristics is same as the straight bridge.

# 3. Conclusion

(1) Continuous curved box girder bridges occur deformation in the vertical, Transverse and along the bridge under the combined load of weight and pre-stressing, vertical deformation is the main. The box girder cross-section produces reverse and distortion and the structure rolls out.

(2) The vertical displacement of continuous curved box girder bridges in mid-span is related to horizontal radius. When the radius is between 100 m and 150 m, displacement increases more rapidly, when the radius is more than 200 m, displacement curve gradually tends to level, the force characteristics is the same as straight bridge.

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Figure 1. The three-dimensional mobile Cartesian coordinate and load components



Figure 2. The section internal force of the micro-curved beam



Figure 3. The whole spiral bridge model



Figure 4. The finite element model



Figure 6. (a)- (e) Separately represents the outer-bottom edge of the box girder displacement curve when the radius in the case of 50 m, 100 m, 150 m, 200 m and 250 m



Figure 7. The mid-span vertical displacement-radius relationship curve



Figure 8. Mid-span vertical displacement distribution curve of box girder along the Transversal direction