On Acoustic Radiation by Rotating Body Sources in Frequency Domain

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Abstract

For the complexity of the sound generated mechanism of rotating body source, this paper describes a frequency-domain numerical method for predicting sound radiation of rotating body sources based on the Kirchhoff integral equation with analytical Green's function of rotating monopole and dipole source in free space. The sound radiation model is established in free space and the relationship between characteristics of sound field and acoustic nature frequency of source, angular frequency and its harmonics can be revealed by the mathematical solution. Numerical simulation shows that sound field has a strong directivity, fundamental frequency transmitting in the rotary shaft direction and harmonics spreading along radial direction and frequency shift phenomena appearing clearly in higher rotating speed of source. The method has a theoretical significance for exploring the low-noise rotating machinery.

Keywords: Rotating source, Acoustic propagation, Far-field sound pressure

1. Introduction

Research about sound radiation of moving source originates from Lighthill. The first integral approach for acoustic propagation is the acoustic analogy. In the acoustic analogy, the governing Navier-Stokes equations are rearranged to be in wave-type form. There is some question as to which terms should be identified as part of the sound source and retained in the right-hand side of the equation and which terms should be in the left-hand side as part of the operator. The far-field sound pressure is then given in terms of a volume integral over the domain containing the sound source. However the major difficulty with the acoustic analogy is that the sound source is not compact in supersonic flows. Several modifications to Lighthill's original theory have been proposed to account for the moving sound source or other effects. The Ffowcs Williams and Hawkings (FW-H) equation was introduced to extend acoustic analogy in the case of solid surfaces. However, when acoustic sources are present in the flowfield a volume integration is needed. This volume integration of the quadrupole source term is difficult to compute and is usually neglected in most acoustic analogy codes. Errors could be encountered in calculating the sound field. Another alternative is the Kirchhoff method which assumes that the sound transmission is governed by the simple wave equation. Kirchhoff's method consists of the calculation of the nonlinear near- and mid-field, usually numerically, with the far-field solutions found from a linear Kirchhoff formulation evaluated on a control surface surrounding the nonlinear-field. The control surface is assumed to enclose all the nonlinear flow effects and noise sources. The sound pressure can be obtained in terms of a surface integral of the surface pressure and its normal and time derivatives. This approach has the potential to overcome some of the difficulties associated with the traditional acoustic analogy approach. The method is simple and accurate and accounts for the nonlinear quadrupole noise in the far-field. Full diffraction and focusing effects are included while eliminating the propagation of the reactive near-field. Kirchhoff's formula, published in 1882, is used in the theory of diffraction of light and in other electromaqnetic problems. It also has many applications to problems of wave in acoustics. One of the novel uses of this formula was proposed by Hawkinqs for predicting the noise of high speed propellers and helicopter rotors. The theoretical method above laid the foundation for sound radiation of moving source, but there are some problems needed to explore .In the paper, sound radiation of rotating body source is illustrated. The analytical sound radiation predictive model of the rotating body source by Kirchhoff's integral equation with analytical Green function is presented in section2.The results are presented in section3.The analytical method provided important theoretical value for the research of sound radiation and noise control of moving source.

2. Sound Radiation Predication of rotating body sound sources

2.1. Model of Sound Radiation Prediction

The difficulty in describing the rotating source in the frequency domain is overcome by introducing a fixed Kirchhoff surface around the rotating source. Acoustic pressures generated from a rotating dipole source, a rotating point source, on the Kirchhoff surface are calculated using prediction model deduced in section 2.Normal derivatives on the Kirchhoff surface of acoustic pressures in the Helmholtz integral equation are derived analytically in this paper.Figure.12 shows a schematic diagram of the Kirchhoff source.

Several methods can be used for calculating the acoustic field of a stationary sources. This paper introduces a Kirchhoff source to represent rotating sources as a stationary source. The Kirchhoff source modeling of the original rotating sources. The Kirchhoff source is usually represented as a closed surface called the Kirchhoff surface. The Kirchhoff surface is a virtual surface that encloses all rotating sources, as shown in Figure.1. The Kirchhoff surface contains all the information of the acoustic sources. The characteristics of various acoustic sources are replaced by pressure and its derivative on the Kirchhoff surface.

Pressure at an observer position P can be calculated by the surface integration on the Kirchhoff surface, as follows:

$$p(P)=\frac{1}{4\pi}\iint_S\left(p(Q)\frac{\partial G(p,Q)}{\partial n_S}-\frac{\partial p(Q)}{\partial n_S}G(p,Q)\right)dS(Q) \tag{1}$$

where $p(P)$ is the acoustic pressure at $P\notin S$, S is the Kirchhoff surface, $Q\in S$ is a source position on S , $p(Q)$ and $\partial p(Q)/\partial n$ are acoustic pressure and its normal derivative on the Kirchhoff surface S . G and $\partial G/\partial n$ are the free field Green function and its normal derivative on the surface. The formulation of variables, respectively, are given. The $\partial G/\partial n$ and $p(Q)$ are obtained as below:

$$\frac{\partial G}{\partial n_s}=\frac{\partial G}{\partial R}\frac{\partial R}{\partial n_s}=-\left(ik+\frac{1}{R}\right)G\frac{\partial R}{\partial n_s} \tag{2}$$

$$p(Q)=\frac{1}{2\pi}\iint_k p_0dK+\iint_k p_1dK+\iint_k p_2dK \tag{3}$$

Where p_1 , p_2 and p_3 are pressures of monopole, horizontal dipole and vertical dipole. While the shape of the Kirchhoff surface can be arbitrary in general, the spherical and cylindrical Kirchhoff surface is usually selected for its convenient application to the rotating sources. For the irregular surface the calculation of normal derivative of surface become difficulty. In this paper, the formula used to compute the normal derivative on the arbitrary Kirchhoff surface is provided. The formula of coordinate transform between orthogonal and spherical coordinate system is shown as follows:

$$\begin{cases}\dfrac{\partial}{\partial x}=\sin\theta\cos\varphi\dfrac{\partial}{\partial r}+\cos\theta\cos\varphi\dfrac{1}{r}\dfrac{\partial}{\partial\theta}-\dfrac{\sin\varphi}{r\sin\theta}\dfrac{\partial}{\partial\varphi}\\ \dfrac{\partial}{\partial y}=\sin\theta\sin\varphi\dfrac{\partial}{\partial r}+\cos\theta\sin\varphi\dfrac{1}{r}\dfrac{\partial}{\partial\theta}+\dfrac{\cos\varphi}{r\sin\theta}\dfrac{\partial}{\partial\varphi}\\ \dfrac{\partial}{\partial z}=\cos\theta\dfrac{\partial}{\partial r}-\dfrac{\sin\theta}{r}\dfrac{\partial}{\partial\theta}\end{cases} \tag{4}$$

The directional derivatives in spherical coordinate system:

$$\begin{cases} \dfrac{\partial r}{\partial x} = \sin\theta\cos\varphi \\ \dfrac{\partial \theta}{\partial x} = \dfrac{\cos\theta\cos\varphi}{r} \\ \dfrac{\partial \varphi}{\partial x} = -\dfrac{\sin\varphi}{r\sin\theta} \end{cases} \begin{cases} \dfrac{\partial r}{\partial y} = \sin\theta\sin\varphi \\ \dfrac{\partial \theta}{\partial y} = \dfrac{\cos\theta\sin\varphi}{r} \\ \dfrac{\partial \varphi}{\partial y} = \dfrac{\cos\varphi}{r\sin\theta} \end{cases} \begin{cases} \dfrac{\partial r}{\partial z} = \cos\theta \\ \dfrac{\partial \theta}{\partial z} = -\dfrac{\sin\theta}{r} \\ \dfrac{\partial \varphi}{\partial z} = 0 \end{cases} \quad (5)$$

The directional cosines of arbitrary orientation is assumed $(\cos\alpha, \cos\beta, \cos\gamma)$, then the derivatives of arbitrary orientation is obtained as below:

$$\begin{aligned} \frac{\partial p}{\partial l} &= \frac{\partial p}{\partial x}\cos\alpha + \frac{\partial p}{\partial y}\cos\beta + \frac{\partial p}{\partial z}\cos\gamma \\ &= \frac{\partial p}{\partial r}\left(\frac{\partial r}{\partial x}\cos\alpha + \frac{\partial r}{\partial y}\cos\beta + \frac{\partial r}{\partial z}\cos\gamma\right) + \frac{\partial p}{\partial \theta}\left(\frac{\partial \theta}{\partial x}\cos\alpha + \frac{\partial \theta}{\partial y}\cos\beta + \frac{\partial \theta}{\partial z}\cos\gamma\right) \\ &+ \frac{\partial p}{\partial \varphi}\left(\frac{\partial \varphi}{\partial x}\cos\alpha + \frac{\partial \varphi}{\partial y}\cos\beta\right) \end{aligned} \quad (6)$$

2.2. Sound Field Characteristics

For the present simulation, the sound pressures of fan rotating blades are calculated using prediction model. The radiation sound field of rotating blades consists of thickness noise and loading noise. In the paper, the pressures of rotating monopole and dipole sources instead of thickness noise and loading noise.By sound radiation prediction model, the pressure of rotating blades of fan can be given as below:

$$P(X,\omega) = \iint QGd\tau dv + \int_0^{2\pi}\int_0^a f_z G_Z r dr d\varphi + \int_0^a f_\varphi G_\varphi 2\pi r dr$$

Where a is the length of fan blades, f_z and f_φ are forces along z and φ direction respectively. The sound field calculated of rotating blades is at rotate speed 996rpm and fundamental frequency of blades is 200Hz.The order of harmonics is assumed $k = -6 \cdots 6$. Figure.2 shows sound pressures' spectrums for a rotating blades source at a selected observer angle $\theta = 0^\circ, 10^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ with the same rotate speed 996rpm.

Figure.2（a）shows the harmonics distribution on the azimuth axis z. There is only fundamental frequency with obvious directivity.Figure.2（f）shows the harmonics distribution on the x-y plane. Abundant orders of harmonics distribute on the plane direction. From (b) to (e), the order of harmonics become plenty and the fundamental frequency presents at small observer angle with the observer angle increasing.

Figure.3 shows the change of harmonics' orders with the increasing of angular frequencies. From (a) to (d) harmonics' orders are more abundant in the lower angular frequencies than the higher. The higher harmonics' orders become outstanding in the higher angular frequencies as compared with the lower. The change of angular frequencies has a impact on the harmonics distribution to a great extent.

Figure.4 shows the directivity of rotating blades with different harmonics' orders. The directivity of body source is very intensive, especially, sound power focuses on the range from $\pi/6$ to $\pi/2$ at higher orders k and on the range from 0 to $\pi/3$ at lower orders k.

3. Conclusions

A frequency-domain method has been developed for prediction sound field of rotating source. The method is based on the acoustic solution of rotating compact sources and Kirchhoff's integral equation in frequency-domain with analytical Green's function. Solutions are carried out employing an explicit sound pressure prediction model and the characteristics of sound field of moving source. By numerical simulations the sound field frequency characteristics and directivity are presented. Especially, the method is applied to investigation of acoustic radiation of rotating body source. The method and results have important theoretical significance on the moving source sound field characteristic analysis and exploring the low-noise design of rotary machine.

References

Farassat, F., & Myers, M. K. (1987). Extension of kirchhoff's Formula to Radiation From Moving surfaces. *NASA Technical Memorandum*.

Ffcows, Williams, J. E., & Hawkings, D. L. (1969) Sound generation by turbulence and surface in arbitrary motion. *Phil.Trans.RoySoc*, 264A, 321-342.

Hawkinqs, D. L. (1979). Noise Generation by Transonic Open Rotors. *Westland Research Paper,* 599.

Jeon, W. H. & Lee, D. J. (2000). An analysis of the flow and aerodynamic acoustic sources of a centrifugal impeller. *Journal of Sound and Vibration*. 222, 505-511. http://dx.doi.org/10.1006/jsvi.1998.1983

Lighthill, M. J. (1952). On sound generated aerodynamically.I.General Theroy. *Proc.Roy. LondonSoc*, 211 A (1107), 564-587.

Lowson, M. V. (1965). The sound field for singularities in motion . *Proceeding of the Royal Society in London*, Series A. 286, 559-572.

Myers, M. K., & Hausmann, J. S. (1990). On the Application of the Kirchhoff Formula for Moving Surfaces, *Journal of Sound and Vibration*. 139, 174–178. http://dx.doi.org/10.1016/0022-460X(90)90784-W

Morgans. R. P. (1930). The Kirchhoff Formula Extended to a Moving Surface. *Philosophical Magazine*, 9, 141–161.

Morse, P. M., & Ingard, K. U. (1968). Theoretical Acoustics, McGraw-Hill. New York.

Tyler, J. M., & Sofrin, T. G. (1962).Axial flow compressor noise studies. *Society of Automotive Engineers Transactions*, 70, 309–332.

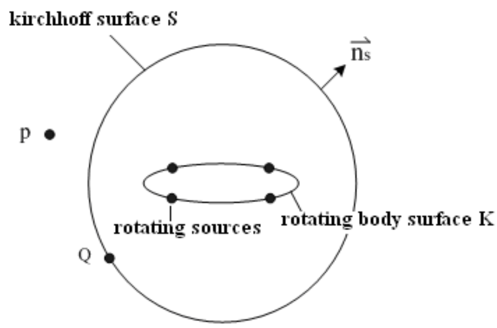
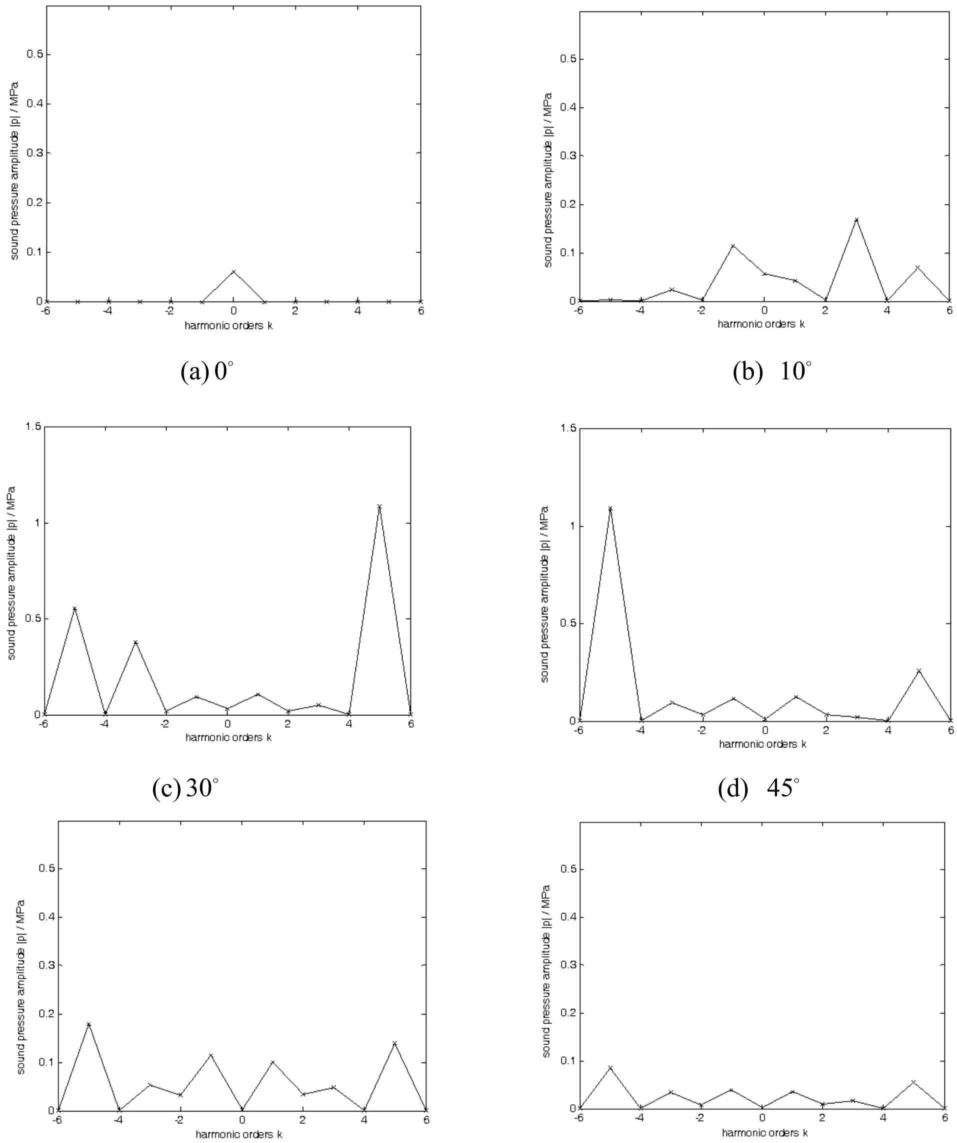


Figure 1. A schematic diagram of rotating sources enclosed by the Kirchhoff surface

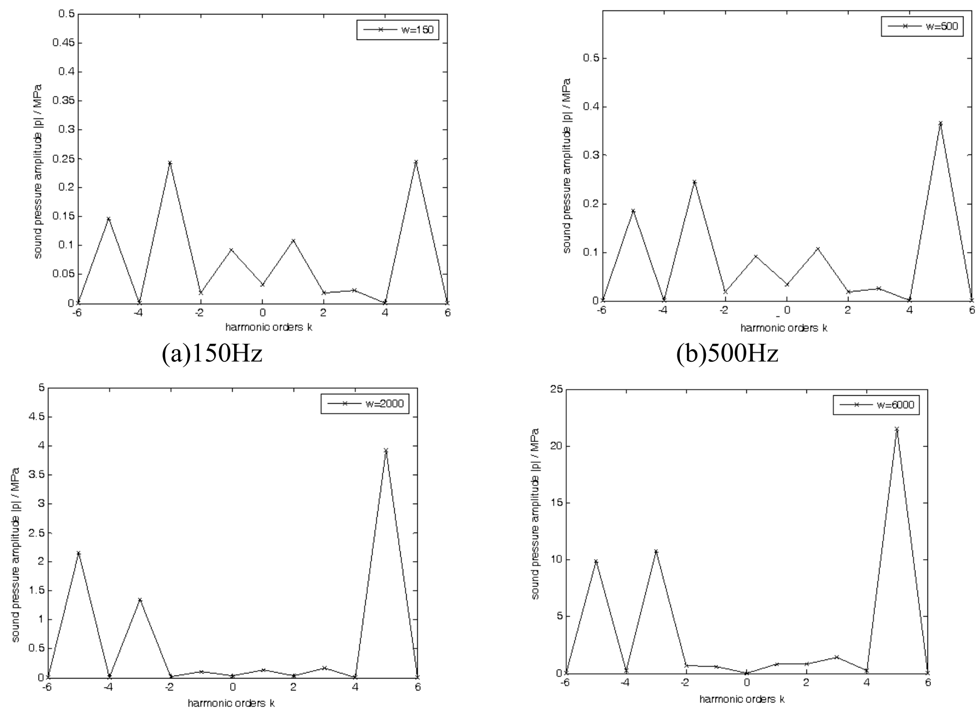


(a) 0° (b) 10°

(c) 30° (d) 45°

(e) 60° (f) 90°

Figure 2. Harmonics distribution at different observer angle with the same nature and angular frequencies



(a)150Hz (b)500Hz

(c)2000Hz (d)6000Hz

Figure 3. Harmonics distribution at same observer angle 60° with different angular frequencies

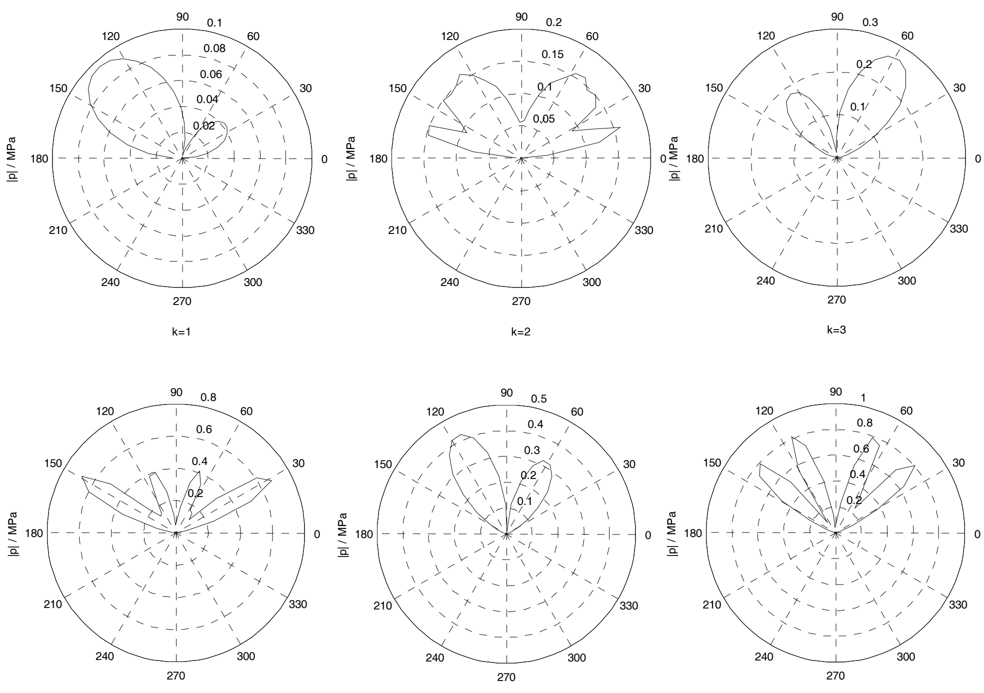


Figure 4. The directivity of rotating body source with different harmonics' order k