An Improvement for the Locally One-Dimensional Finite-Difference Time-Domain Method

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Abstract

To reduce the memory usage of computing, the locally one-dimensional reduced finite-difference time-domain method is proposed. It is proven that the divergence relationship of electric-field and magnetic-field is non-zero even in charge-free regions, when the electric-field and magnetic-field are calculated with locally one-dimensional finite-difference time-domain (LOD-FDTD) method, and the concrete expression of the divergence relationship is derived. Based on the non-zero divergence relationship, the LOD-FDTD method is combined with the reduced finite-difference time-domain (R-FDTD) method. In the proposed method, the memory requirement of LOD-R-FDTD is reduced by1/6 (3D case) of the memory requirement of LOD-FDTD averagely. The formulation is presented and the accuracy and efficiency of the proposed method is verified by comparing the results with the conventional results.

Keywords: FDTD, LOD-FDTD, R-FDTD, CFL

1. Introduction

The finite-difference time-domain (FDTD) method has been widely utilized in the electromagnetic calculations (Singh. G, Eng Leong Tan, Zhi Ning Chen, 2010). However, its time-step size is constrained by the Courant-Friedrichs-Lewy (CFL) condition. And its requirement of computer memory can become a limitation for electrically large bodies. To eliminate the restriction of the CFL, some unconditionally stable techniques, such as the alternating-direction implicit FDTD (ADI-FDTD) method (Takefumi Namiki, 1999)(Eng Leong Tan, 2010), and more recently, the locally-one-dimensional FDTD(LOD-FDTD) (F. Zheng, Z. Chen, and J. Zhang, 1999)(Rouf H. K, Costen F, Garcia D. G. 2010), have been proposed. The advantage of the LOD-FDTD is fewer requirements of arithmetic operations than ADI-FDTD while providing comparable accuracy. But the LOD method needs more computer memory than the original FDTD also, because of the iteration process.

Reduced finite-difference time-domain (R-FDTD) is a method, using the divergence relationship which links the Maxwell curl equations, to reduce the number of required field components (B. Liu, B. Q. Gao, W. Tan, and W. Ren, 2002). In the three-dimensional(or two-dimensional) case, it makes use of a 2-D array(or 1-D array) to repeat storing a 3-D(or 2-D) field component array instead of storing this field component in the whole domain.

As a result, a 1/3 memory reduction is achieved average (G. D. Kondylis, F. D. Flaviis, G. J. Pottie, T. Itoh, 2001)(B. Liu, B. Q. Gao, W. Tan, and W. Ren, 2002).

In this paper, it is proven that the divergence relationship of electric-field and magnetic-field is non-zero even in charge-free regions, when the electric-field and magnetic-field are calculated with the LOD-FDTD method. The concrete expression of it is derived. Based on the relationship, the LOD-R-FDTD method is proposed, which is combined the unconditionally stable LOD-FDTD method with the R-FDTD method. The 3-D equations are derived. In the proposed method, the merit of LOD-FDTD, e.g. increasing time-step size and decreasing calculation time, is kept. At the same time, the memory requirement is reduced by 1/3(2-D case) or 1/6(3-D case) of the memory requirement of LOD-FDTD. Numerical experiment is presented to verify the efficiency of the presented method.

2. The LOD-R-FDTD Method

A) Non-zero divergence relationship for LOD-FDTD

Without loss of generality, the 3-D case is considered. The 3-D LOD-FDTD equations proposed in(Iftikhar Ahmed, Eng-Kee Chua, Er-Ping Li, and Zhizhang Chen, 2008) are complex relatively, because of three updating steps. In (E. L. Tan, 2007), the 3-D method is a two-step approach, but is complicated in its formulations. In this paper, we use the method of (Liu Guo-sheng, Zhang Guo-ji, 2010), in which equations are simpler and unconditionally stable. We assume the medium in which the wave propagates is a vacuum. The solution marching from the *n*th time step to the (*n*+1)th time step is broken up into two sub-steps. The 3-D LOD-FDTD equations are as follows:

Sub-step 1: $n \rightarrow n+1/2$

$$E_x^{n+1/2} = E_x^n + \frac{\Delta t}{2\varepsilon}\partial_y H_z^{n+1/2} + \frac{\Delta t}{2\varepsilon}\partial_y H_z^n \tag{1}$$

$$E_y^{n+1/2} = E_y^n + \frac{\Delta t}{2\varepsilon}\partial_z H_x^{n+1/2} + \frac{\Delta t}{2\varepsilon}\partial_z H_x^n \tag{2}$$

$$E_z^{n+1/2} = E_z^n + \frac{\Delta t}{2\varepsilon}\partial_x H_y^{n+1/2} + \frac{\Delta t}{2\varepsilon}\partial_x H_y^n \tag{3}$$

$$H_x^{n+1/2} = H_x^n + \frac{\Delta t}{2\mu}\partial_z E_y^{n+1/2} + \frac{\Delta t}{2\mu}\partial_z E_y^n \tag{4}$$

$$H_y^{n+1/2} = H_y^n + \frac{\Delta t}{2\mu}\partial_x E_z^{n+1/2} + \frac{\Delta t}{2\mu}\partial_x E_z^n \tag{5}$$

$$H_z^{n+1/2} = H_z^n + \frac{\Delta t}{2\mu}\partial_y E_x^{n+1/2} + \frac{\Delta t}{2\mu}\partial_y E_x^n \tag{6}$$

Sub-step 2: $n+1/2 \rightarrow n+1$

$$E_x^{n+1} = E_x^{n+1/2} - \frac{\Delta t}{2\varepsilon}\partial_z H_y^{n+1} - \frac{\Delta t}{2\varepsilon}\partial_z H_y^{n+1/2} \tag{7}$$

$$E_y^{n+1} = E_y^{n+1/2} - \frac{\Delta t}{2\varepsilon}\partial_x H_z^{n+1} - \frac{\Delta t}{2\varepsilon}\partial_x H_z^{n+1/2} \tag{8}$$

$$E_z^{n+1} = E_z^{n+1/2} - \frac{\Delta t}{2\varepsilon}\partial_y H_x^{n+1} - \frac{\Delta t}{2\varepsilon}\partial_y H_x^{n+1/2} \tag{9}$$

$$H_x^{n+1} = H_x^{n+1/2} - \frac{\Delta t}{2\mu}\partial_y E_z^{n+1} - \frac{\Delta t}{2\mu}\partial_y E_z^{n+1/2} \tag{10}$$

$$H_y^{n+1} = H_y^{n+1/2} - \frac{\Delta t}{2\mu}\partial_z E_x^{n+1} - \frac{\Delta t}{2\mu}\partial_z E_x^{n+1/2} \tag{11}$$

$$H_z^{n+1} = H_z^{n+1/2} - \frac{\Delta t}{2\mu}\partial_x E_y^{n+1} - \frac{\Delta t}{2\mu}\partial_x E_y^{n+1/2} \tag{12}$$

Based on the classical electromagnetic field theory, it can be derived that the divergence is zero in the charge-free regions (G. D. Kondylis, F. D. Flaviis, G. J. Pottie, T. Itoh, 2001). The equations of E and H are as follows:

$$\nabla \cdot E = 0 \tag{13}$$

$$\nabla \cdot H = 0 \tag{14}$$

It can be proven that the zero divergence of E and H is not existent when the components of electromagnetic field being calculated by the LOD-FDTD method, by bringing the components of electric-field and magnetic-field of standard Yee algorithm into (13) and (14). Its equations derive as follows:

At time $n+1/2$, the divergence of $E^{n+1/2}$ is

$$\nabla \cdot E^{n+1/2} = \partial_x E_x^{n+1/2} + \partial_y E_y^{n+1/2} + \partial_z E_z^{n+1/2} \tag{15}$$

Bring (1)-(3) into (15)

$$\nabla \cdot E^{n+1/2} = \nabla \cdot E^{n} + \frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n+1/2} + \partial_z \partial_x H_y^{n+1/2} + \partial_x \partial_y H_z^{n+1/2}\right) + \frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n} + \partial_z \partial_x H_y^{n} + \partial_x \partial_y H_z^{n}\right) \tag{16}$$

Assuming that initially at time t=0($n = 0$), all the field components are zero over the whole computational domain, we obtain

$$\nabla \cdot E^{n+1/2} = \frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n+1/2} + \partial_z \partial_x H_y^{n+1/2} + \partial_x \partial_y H_z^{n+1/2}\right) \tag{17}$$

In the magnetic-field, we obtain the divergence of $H^{n+1/2}$ as

$$\nabla \cdot H^{n+1/2} = \frac{\Delta t}{2\mu}\left(\partial_y \partial_z E_x^{n+1/2} + \partial_z \partial_x E_y^{n+1/2} + \partial_x \partial_y E_z^{n+1/2}\right) \tag{18}$$

At time $n+1$, the divergence of E^{n+1} is

$$\nabla \cdot E^{n+1} = \partial_x E_x^{n+1} + \partial_y E_y^{n+1} + \partial_z E_z^{n+1} \tag{19}$$

Bring (7)-(9) into (19)

$$\nabla \cdot E^{n+1} = \nabla \cdot E^{n+1/2} - \frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n+1} + \partial_x \partial_z H_y^{n+1} + \partial_y \partial_x H_z^{n+1}\right) - \frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n+1/2} + \partial_x \partial_z H_y^{n+1/2} + \partial_y \partial_x H_z^{n+1/2}\right) \tag{20}$$

Assuming that initially at time t=0($n = 0$), all the field components are zero over the whole computational domain, and bring (17) into (20), we obtain the divergence of E^{n+1} as

$$\nabla \cdot E^{n+1} = -\frac{\Delta t}{2\varepsilon}\left(\partial_y \partial_z H_x^{n+1} + \partial_x \partial_z H_y^{n+1} + \partial_y \partial_x H_z^{n+1}\right) \tag{21}$$

In the magnetic-field, the divergence of H^{n+1} is

$$\nabla \cdot H^{n+1} = -\frac{\Delta t}{2\mu}\left(\partial_y \partial_z E_x^{n+1} + \partial_x \partial_z E_y^{n+1} + \partial_y \partial_x E_z^{n+1}\right) \tag{22}$$

Equations (17) (22) and (18) (21) demonstrate a spatial dependence among the field components. We can use it to link a field component to the other five at each time step, and this way, we reduce the number of field components needed in LOD-FDTD from twelve to ten in the 3-D case. Similarly, in the 2-D case, we reduce the number of components from six to four.

B) Combining LOD-FDTD with R-FDTD

We assume the magnetic permeability coincides and the electric conductance coincides with that of free space ($\varepsilon = \varepsilon_0$, $\mu = \mu_0$). Consider the 3-D case, where the size of whole computational domain is $N_x \times N_y \times N_z$, i. e., at time $n+1/2$, the field components are E_x , E_y , E_z , H_x , H_y and H_z . Equation (17) can be used to link one of them to the others, and equation (22) can be used to establish another link at time $n+1$. Due to spatial dependence of the electric and magnetic fields through these two equations, the total number of variables for the 3-D formulations is reduced from twelve required to ten. In principle, one can independently choose the components of E and H, which will be only locally updated and not stored, but it will simplify calculations if we choose components of the same direction for E and H. Without loss of generality, consider the case where $E_y^{n+1/2}$ and H_z^{n+1} are the components that are only locally calculated, and are not stored in the whole domain. The update equations are as follows:

$$\begin{aligned}
E_y^{n+1/2}(i,j,k) &= E_y^{n+1/2}(i,j-1,k) \\
&- \frac{\Delta y}{\Delta x}\left[E_x^{n+1/2}(i,j,k) - E_x^{n+1/2}(i-1,j,k)\right] - \frac{\Delta y}{\Delta z}\left[E_z^{n+1/2}(i,j,k)\right. \\
&\left. - E_z^{n+1/2}(i,j,k-1)\right] + \frac{\Delta t}{2\varepsilon\Delta z}\left[H_x^{n+1/2}(i,j,k) - H_x^{n+1/2}(i,j,k-1)\right. \\
&\left. - H_x^{n+1/2}(i,j-1,k) + H_x^{n+1/2}(i,j-1,k-1)\right] + \frac{\Delta t\Delta y}{2\varepsilon\Delta x\Delta z} \\
&\cdot\left[H_y^{n+1/2}(i,j,k) - H_y^{n+1/2}(i-1,j,k) - H_y^{n+1/2}(i,j,k-1)\right. \\
&\left. + H_y^{n+1/2}(i-1,j,k-1)\right] + \frac{\Delta t}{2\varepsilon\Delta x}\left[H_z^{n+1/2}(i,j,k)\right. \\
&\left. - H_z^{n+1/2}(i,j-1,k) - H_z^{n+1/2}(i-1,j,k) + H_z^{n+1/2}(i-1,j-1,k)\right]
\end{aligned} \tag{23}$$

$$\begin{aligned}
H_z^{n+1}(i,j,k) &= H_z^{n+1}(i,j,k-1) - \frac{\Delta z}{\Delta x}\left[H_x^{n+1}(i,j,k) - H_x^{n+1}(i-1,j,k)\right] - \frac{\Delta z}{\Delta y}\left[H_y^{n+1}(i,j,k)\right. \\
&\left. - H_y^{n+1}(i,j-1,k)\right] - \frac{\Delta t}{2\mu\Delta y}\left[E_x^{n+1}(i,j,k) - E_x^{n+1}(i,j,k-1)\right. \\
&\left. - E_x^{n+1}(i,j-1,k) + E_x^{n+1}(i,j-1,k-1)\right] - \frac{\Delta t}{2\mu\Delta x}\left[E_y^{n+1}(i,j,k)\right. \\
&\left. - E_y^{n+1}(i,j,k-1) - E_y^{n+1}(i-1,j,k) + E_y^{n+1}(i-1,j,k-1)\right] \\
&- \frac{\Delta t\Delta z}{2\mu\Delta x\Delta y}\left[E_z^{n+1}(i,j,k) - E_z^{n+1}(i,j-1,k) - E_z^{n+1}(i-1,j-1,k) + E_z^{n+1}(i-1,j-1,k)\right]
\end{aligned} \tag{24}$$

Beginning with the updates of $H_z^{n+1/2}$, one realizes that these can be done one k=constant plane at a time, with the prior spatial update of H_z^n through (24). In the same way, the updates of E_y^{n+1} can also be done one j=constant plane at a time, with the prior spatial update of $E_y^{n+1/2}$ through (23).

For the 3-D case, a two-dimensional array having size of $N_x \times N_z$ must be used for removed component $E_y^{n+1/2}$. For the removed component H_z^{n+1}, a two-dimensional array having size of $N_x \times N_y$ is used. The extra memory requirement for these arrays is not significant, and since the choice of which components to remove is arbitrary, we can always choose to remove the field components in the direction for which the corresponding arrays are minimized. Note the innovative feature of the formulation: e. g., for store $E_y^{n+1/2}$, one j=constant plane at a time and is stored in the same memory locations as for the previous j. Correspondingly, for H_z^{n+1}, also one k=constant plane at a time and is stored in the memory locations for the previous k. The pseudocode for the 3-D case is shown at the end of this paper.

3. Numerical Results

To assess the LOD-R-FDTD in terms of accuracy and efficiency, we analyze numerical example as a benchmark. The 3-D problem is considered.

We consider a $2cm \times 1cm \times 2cm$ rectangular cavity, the space is vacuum and charge-free ($\varepsilon = \varepsilon_0$). We set

$\Delta x = \Delta y = \Delta z = 1mm$, and use a problem space of $20 \times 10 \times 20$ cells. The time step of LOD-R-FDTD is set as $dt_{LOD-R} = 5.8dt_{CFL}$. The absorbing boundary condition is PML medium having 8 cells along x and y direction, which is set on the surface of the cavity. The source region will be treated as (G. D. Kondylis, F. D. Flaviis, G. J. Pottie, T. Itoh, 2001). Fig. 1 shows the time-domain electric-field waveforms obtained by the traditional FDTD method, the LOD-FDTD method and the LOD-R-FDTD method.

From Fig. 1(a), one can see that the LOD-R-FDTD method is unconditionally stable. In Fig. 1(b), the results of the proposed method and the LOD-FDTD method are in good agreement. In this problem, the computer memory requirements of LOD-R-FDTD are reduced to 86.3% of the LOD-FDTD requirements.

4. Conclusion

A 3-D LOD-R-FDTD method is formulated in this paper. And it is proven that the divergence relationship of electric-field and magnetic-field is non-zero even in charge-free regions, when the electric-field and magnetic-field are calculated with the LOD-FDTD method. Based on the non-zero divergence, we propose the LOD-R-FDTD method. This method not only eliminates the restriction of the CFL, but also reduced the memory requirements by 1/3(2-D case) or 1/6(3-D case) of the memory requirements of LOD-FDTD. The numerical results prove the proposed method is valid.

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(1) sub-step $n \to n+1/2$:

$H_{zc}^{n} = H_{z1}^{n}$

Do $k=1,N_z$

Substitute (1) to (6)

$$H_z^{n+1/2}(i,j,k) = H_{zc}^{n}(i,j) + \frac{\Delta t}{\mu\Delta y}\left[E_x^{n}(i,j,k) - E_x^{n}(i,j-1,k)\right] + \frac{\Delta t^2}{4\varepsilon\mu\Delta y^2}\left[H_z^{n+1/2}(i,j+1,k) - 2H_z^{n+1/2}(i,j,k) + H_z^{n+1/2}(i,j-1,k)\right] + \frac{\Delta t^2}{4\varepsilon\mu\Delta y^2}\left[H_{zc}^{n}(i,j+1) - 2H_{zc}^{n}(i,j) + H_{zc}^{n}(i,j-1)\right]$$

Update H_{zc}^{n} using (24)

Substitute (2) to (4), calculate $H_x^{n+1/2}$
Do $k=1,N_z$
Do $i=1,N_x$
Substitute (2) to (4), calculate $E_y^{n+1/2}(i,1,k)$
End do
End do

Do $k=1,N_z$
Do $j=1,N_y$
Do $i=1,N_z$
Utilize (3) and (5), calculate $E_z^{n+1/2}$ $H_y^{n+1/2}$
End do
End do
End do

$E_{y1}^{n+1/2} = E_y^{n+1/2}(i,1,k)$

(2) sub-step $n+1/2 \to n+1$:

$E_{yc}^{n+1/2} = E_{y1}^{n+1/2}$

Do $k=1,N_z$
Do $j=1,N_y$
Do $i=1,N_z$
Utilize (9) and (10), calculate H_x^{n+1} E_z^{n+1}
Utilize (7) and (11), calculate H_x^{n+1} E_y^{n+1}
End do
End do
End do

Do $j=1,N_y$

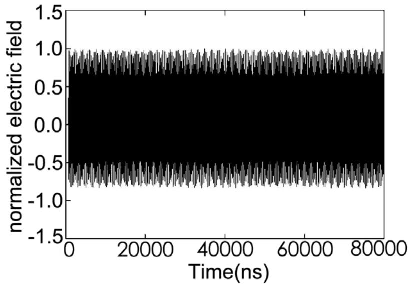
Substitute (12) to (8)

$$E_y^{n+1}(i,j,k) = E_{yc}^{n+1/2}(i,j) - \frac{\Delta t}{\varepsilon\Delta x}\left[H_z^{n+1/2}(i,j,k) - H_x^{n+1/2}(i-1,j,k)\right] + \frac{\Delta t^2}{4\varepsilon\mu\Delta x^2}\left[E_y^{n+1}(i+1,j,k) - 2E_y^{n+1}(i,j,k) + E_y^{n+1}(i-1,j,k)\right] + \frac{\Delta t^2}{4\varepsilon\mu\Delta x^2}\left[E_{yc}^{n+1/2}(i+1,k) - 2E_{yc}^{n+1/2}(i,k) + E_{yc}^{n+1/2}(i-1,k)\right]$$

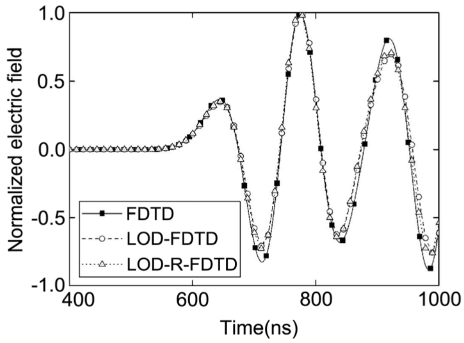
Update $E_{yc}^{n+1/2}$ using (23)

Do $j=1,N_y$
Do $i=1,N_x$
Substitute (8) to (12), calculate $H_z^{n+1}(i,j,1)$
End do
End do

$H_{z1}^{n+1} = H_z^{n+1}(i,j,1)$



(a)



(b)

Figure 1. Time-domain electric-field waveform

(a) LOD-R-FDTD method. (b) Comparison among these three methods.