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The Statistics Properties of Orthogonal Coherent State Interacting with Two-level Atom

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Abstract

In this paper, the statistics properties of the statistics properties of orthogonal coherent state Interacting with two-level atom have been discussed, the statistics properties of both atom and field have been discussed. A set of conclusion have been gotten that atom inversion exit the collapse,calm,revival and chaotic behavior. The filed exit several unclassical effect: Sub-poisson photon distribution, Antibanching effect and $(\Delta x)^2$ squeezing effect.

Keywords: Orthogonal coherent state, Antibanching effect, Sub-poisson photon distribution, Squeezing effect

1. Introduction

Jayness-Cummings model(JCM)(F.W.CUMMINGS, 1965) has been studied many times because the relatively realistic way that it represents the quantum physics of a resonant interact ion .It is the simplest fully quantized model of quantum optics, quantum electronics, and resonance physics .From this model one hopes to know more properties of single mode filed interaction with a two level atom. For example, the coherent–state Jaynes-cummings model has been studied by J.H.Eberly, *et al* (J.H.Eberly, N.B.Narozhny And J.J.Sanchez-Mondragon, 1980).a set of equations characterizing the Jaynes-Cummings model ,which can be used to describe the dynamic and statistical aspect of the system ,have presented by Qiao.Gu(QIAO QU, 2003).

In general, when people study the statistics properties of JCM ,both atom and field should be discussed.

(1)The atomic inversion is given by expectation value of atomic inversion operator σ_z . $\langle \sigma_z \rangle$ which its scale between -1 and +1 represents the degree of excitation of a two-level system.

$$\langle \sigma_z \rangle = \begin{cases} = -1 & \text{(downer level)} \\ = 1 & \text{(upper level)} \end{cases} \tag{1.1}$$

(2) Squeezed characteristic statistics

Based on the definition the quadrature operators x and p

$$x = a^+ + a \tag{1.2a}$$

$$p = i(a^+ - a) \tag{1.2b}$$

The fluctuations of the radiation field are represented by (QIAO GU, J.ZHANG, 1989).

$$\begin{pmatrix}(\Delta x)^2\\(\Delta p)^2\end{pmatrix}=1+2\langle a^+a\rangle\pm 2\mathrm{Re}\langle a^2\rangle-4\begin{pmatrix}\mathrm{Re}^2\langle a\rangle\\\mathrm{Im}^2\langle a\rangle\end{pmatrix} \tag{1.3}$$

If $(\Delta x)^2$ or $(\Delta p)^2<1$, the field has squeezed effect,which it is unclassical effect.

(3) Photon statistical properties.

The photon statistical properties of the filed if characterized by the Mandel factor can be written by

$$Q=\frac{(\Delta N)^2-\langle N\rangle}{\langle N\rangle} \tag{1.4}$$

where $(\Delta N)^2=\langle N^2\rangle-\langle N\rangle^2 . \langle N\rangle=\langle a^+a\rangle$.

$$Q=\begin{cases}>0 & \text{(supper - poisson)}\\=0 & \text{(non - poisson)}\\<0 & \text{(sub - poisson)}\end{cases} \tag{1.5}$$

Sub-poisson is a unclassical effect.

(4) Banching characteristic statistics.

The Second-order Coherence degree is defined below:

$$g^2_{(0)}=\frac{\langle N^2\rangle-\langle N\rangle}{\langle N\rangle^2} \tag{1.6}$$

$$g^2_{(0)}=\begin{cases}>1 & \text{(banching photon)}\\=1 & \text{(coherent state photon)}\\<1 & \text{(antibanching photon)}\end{cases} \tag{1.7}$$

Antibanching -effect is uncalssical effect.

the properity of the coherent state and the squeezing state filed has been discussed(QIAO QU, 2003). The Orthogonal Coherent state (PENG SHI AN, GUO GUANG CAN, 1990) interacting two-level atom has never been heart to report. we will discuss its statistics properties .

The arragement of this paper is as follows. We first introduced general solution of the Jaynes-Cummings Model (QIAO QU, 2003), then we discussed the statistical properties of the orthogonal coherent state interacting with two-level atom

2. General Solution of the Jaynes-Cummings Model

A two–level atom description is valid in Fig.1. if the two atomic levels involved are resonant or nearly resnant with driving field ,while all other levels are highly detuned.

The two-level atom is characterized by the ground state $|b\rangle$ and an excited state $|a\rangle$.

The Hamiltonian system of Jaynes-Cummings model is

$$H=\hbar\omega a^+a+\frac{1}{2}\hbar\omega_0\sigma_z+\hbar g(a\sigma^++a^+\sigma^-) \tag{2.1}$$

where a and a^+ are annihilation and creation operators of the field mode , σ_z is the atomic inversion operator and $\sigma^\pm$ are the atomic raising and lowering operators . ω and ω_0 are the frequencies of the field mode and of the two level atom, repectively, g is a coupling constant ,and $\hbar$ is Planck's constant divied by 2π .

where

$$\sigma_z=\sigma^+\sigma-\sigma\sigma^+ \tag{2.2a}$$

$$\sigma=|b\rangle\langle a| \tag{2.2b}$$

$$\sigma^+=|a\rangle\langle b| \tag{2.2c}$$

$$\sigma\sigma^+=|b\rangle\langle b| \tag{2.2d}$$

$$\sigma^+\sigma=|a\rangle\langle a| \tag{2.2e}$$

We can solve the eigenequation of H:

$$H|\phi\rangle = E|\phi\rangle \tag{2.3}$$

where E and $|\phi\rangle$ represent the eigenvalue and the corresponding eigenstate. We assume that $|\phi\rangle$ be written as a linear combination of the states $|n,a\rangle$ and $|n+1,b\rangle$, in the form

$$|\phi\rangle = A|n,a\rangle + B|n+1,b\rangle \tag{2.4}$$

Where $|n,a\rangle$ corresponds to n photons in the field atom at the upper state $|a\rangle$,and $|n+1,b\rangle$ corresponds to n+1 photons in the field with atom at the downer $|b\rangle$ state .A and B are the probability amplitudes of finding the system in the states $|n,a\rangle$ and $|n+1,b\rangle$ repectively, we obtain

$$[\hbar\omega n + \frac{1}{2}\hbar\omega_0 - E]A + \hbar g\sqrt{n+1}B = 0 \tag{2.5 a}$$

$$[\hbar\omega(n+1) - \frac{1}{2}\hbar\omega_0 - E]B + \hbar g\sqrt{n+1}A = 0 \tag{2.5 b}$$

Eq.(2.5) display a set of the linear and homogeneous equations on A and B,and the necessary and sufficient condition for occurring of the non-zero solution is that the coefficient determinant is equal to zero, its results is the eigenvalues

$$E^{\pm} = \hbar[\omega(n+\frac{1}{2}) \pm \Omega_n] \tag{2.6a}$$

where

$$\Omega_n = \sqrt{(\frac{\Delta}{2})^2 + g^2(n+1)} \tag{1.6b}$$

with $\Delta = \omega - \omega_0$ being the detuning. substituting (2.6)into(2.5)and using(2.4),we obtain the coefficients

$$A^+ = \sin\theta_n, B^+ = \cos\theta_n \quad ; \tag{2.7}$$

$$A^- = \cos\theta_n, B^- = -\sin\theta_n \quad , \tag{2.8}$$

where θ_n is defined by

$$\tan 2\theta_n = \frac{g\sqrt{n+1}}{\Delta/2}(0 \le 2\theta_n \le \pi) \tag{2.9}$$

We have then the eigenstates

$$|\phi_n^{\pm}\rangle = \begin{pmatrix} \sin\theta_n \\ \cos\theta_n \end{pmatrix}|n,a\rangle + \begin{pmatrix} \cos\theta_n \\ \sin\theta_n \end{pmatrix}|n+1,b\rangle \tag{2.10}$$

Which are usually called “dresses states”. we observe that

$$\langle\phi_n^+|\phi_n^-\rangle = 0 \tag{2.11 a}$$

$$\langle\phi_n^+|\phi_n^+\rangle = \langle\phi_n^-|\phi_n^-\rangle = 1 \tag{2.11 b}$$

There is another states that is not included,namely. no photon in the field with atom at the downer state

$$|\phi_g\rangle = |0,b\rangle \tag{2.12}$$

It is the lowest energy and is called ground state. The corresponding eigenvalue E_g is calculated from the eigenequation $H|\phi_g\rangle = E_g|\phi_g\rangle$, result in

$$E_g = -\frac{1}{2}\hbar\omega_0 \tag{2.13}$$

Each of the dressed states $|\phi_n^{\pm}\rangle$ is orthogonal to the ground state $|\phi_g\rangle$,and all of them are complete so that

$$\sum_{n=0}^{\infty}(|\phi_n^+\rangle\langle\phi_n^+| + |\phi_n^-\rangle\langle\phi_n^-|) + |\phi_g\rangle\langle\phi_g| = 1 \tag{2.14}$$

We now consider an arbitrary field-atom system described by a time dependent state vector

$$\frac{\partial}{\partial t}\left|\Phi(t)\right\rangle = -\frac{i}{\hbar}H\left|\Phi(t)\right\rangle \tag{2.15}$$

Assume that the system is initially in state $\left|\Phi(0)\right\rangle$, which can be expended by using the completeness relation (2.14)

$$\left|\Phi(0)\right\rangle = \sum_{n=0}^{\infty}(\alpha_n\left|\phi_n^+\right\rangle + \beta_n\left|\phi_n^-\right\rangle) + \gamma_g\left|\phi_g\right\rangle \tag{2.16}$$

where

$$\alpha_n = \left\langle\phi_n^+\middle|\Phi(0)\right\rangle, \tag{2.17 a}$$

$$\beta_n = \left\langle\phi_n^-\middle|\Phi(0)\right\rangle, \tag{2.17b}$$

$$\gamma_g = \left\langle\phi_g\middle|\Phi(0)\right\rangle. \tag{2.17c}$$

(2.17) are the expanding coefficients ,which satisfy the normalized condition

$$\sum_{n=0}^{\infty}(|\alpha_n|^2 + |\beta_n|^2) + |\gamma_g|^2 = 1 \tag{2.18}$$

Since the Hamiltonian (2.1) does not contain the time explicitly, we can write the solution of the Schrödinger equation (2.15) as

$$\left|\Phi(t)\right\rangle = \exp(-\frac{i}{\hbar}Ht)\left|\Phi(0)\right\rangle \tag{2.19}$$

Substituting (2.16)into (2.19)and using the eigenequation(1.2),we have

$$\left|\Phi(t)\right\rangle = \sum_{n=0}^{\infty}\begin{pmatrix}\alpha_n\left|\phi_n^+\right\rangle\exp(-\frac{i}{\hbar}E_n^+t) + \\ \beta_n\left|\phi_n^-\right\rangle\exp(-\frac{i}{\hbar}E_n^-t)\end{pmatrix} + \gamma_g\left|\phi_g\right\rangle\exp(-\frac{i}{\hbar}E_g t) \tag{2.20}$$

Eg. (2.20) displays a general solution [5]of the Jaynes_Cummings model in terms of the dressed states $\left|\phi_n^{\pm}\right\rangle$ and the ground state $\left|\phi_g\right\rangle$.

3. Orthogonal Coherent State Interact with Two-Level Atom

For convenient our work, we briefly review the orthogonal coherent state .the orthogonal coherent state is one of the eigenstates of the operator of square of annihilation a^2 (PENG SHI AN, GUO GUANG CAN, 1990).

Because coherent state $\left|\alpha\right\rangle$ and the $\left|-\alpha\right\rangle$ are not orthogonal .So Peng Shi An *et al* (WILLIAM H.LOUISELLl, 1982) by use the Gram-Schmidt method, gained a new state $\left|\alpha_{\perp}\right\rangle$ which it is orthogonal to $\left|\alpha\right\rangle$.

So

$$\left|\alpha\right\rangle_{\perp} = \frac{-\exp(-|\alpha|^2)\left|\alpha\right\rangle + \exp(|\alpha|^2)\left|-\alpha\right\rangle}{2S^{1/2}C^{1/2}} \tag{3.1a}$$

$$S = \sinh|\alpha|^2, C = \cosh|\alpha|^2 \tag{3.1 b}$$

By representation of number state we can get that:

$$\left|\alpha_{\perp}\right\rangle = \frac{[-\exp(-\frac{3}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{(\alpha)^n}{\sqrt{n!}} + \exp(\frac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{\sqrt{n!}}]}{2S^{1/2}C^{1/2}}\left|n\right\rangle \tag{3.2}$$

We now consider a system with initial state that atom is in the upper level $\left|a\right\rangle$ .and the field is orthogonal coherent state describing by (3.2).so the initial state can be written as:

$$\left|\Phi(0)\right\rangle = \sum_{n=0}^{\infty} p_n \left|n,a\right\rangle \tag{3.3}$$

where

$$p_n = \frac{[-\exp(-\frac{3}{2}|\alpha|^2)\frac{(\alpha)^n}{\sqrt{n!}} + \exp(\frac{1}{2}|\alpha|^2)\frac{(-\alpha)^n}{\sqrt{n!}}]}{2S^{1/2}C^{1/2}} \tag{3.4}$$

is the probability amplitude of finding n photons in thermal equilibrium state.

After expending $\left|n,a\right\rangle$ in term of the dress states (2.10), (3.2) takes the form as (2.16) with the coefficients

$$\alpha_n = p_n \sin\theta_n, \beta_n = p_n \cos\theta_n, \gamma_g = 0\,. \tag{3.5}$$

Under the resonant condition, substitute (3.5) into (2.20) .we have wave function of the Orthogonal Coherent State interacting with the two level atom which can be represent as

$$\left|\Phi(t)\right\rangle = \sum_{n=0}^{\infty} \begin{pmatrix} p_n \sin\theta_n \left|\phi_n^+\right\rangle \exp(-\frac{i}{\hbar}E_n^+ t) + \\ p_n \cos\theta_n \left|\phi_n^-\right\rangle \exp(-\frac{i}{\hbar}E_n^- t) \end{pmatrix} \tag{3.6}$$

So we can get:

$$W(t) = \left\langle \sigma_z \right\rangle = \left\langle \Phi(t) \right| \sigma_z \left| \Phi(t) \right\rangle \tag{3.7}$$

Substitute (3.15) into (3.16), using (2.2).

We get the result (QIAO QU, 2003):

$$W(t) = \sum_{n=0}^{\infty} \left|p_n\right|^2 \cos(2\sqrt{n+1}gt)\ , \tag{3.8}$$

As the same process, we can get:

$$\left\langle N \right\rangle = \left\langle a^+ a \right\rangle = \sum_{n=0}^{\infty} \left|p_n\right|^2 [n + \sin^2(\sqrt{n+1}gt)] \tag{3.9}$$

$$\begin{aligned} \left\langle N^2 \right\rangle &= \left\langle (a^+ a)^2 \right\rangle \\ &= \sum_{n=0}^{\infty} \left|p_n\right|^2 [n^2 + (2n+1)\sin^2(\sqrt{n+1}gt)] \end{aligned} \tag{3.10}$$

$$\begin{aligned} Q(t) &= \frac{(\Delta N)^2 - \left\langle N \right\rangle}{\left\langle N \right\rangle} \\ &\frac{\sum_{n=0}^{\infty} \left|p_n\right|^2 [n^2 + (2n+1)\sin^2(\sqrt{n+1}gt)]}{\sum_{n=0}^{\infty} \left|p_n\right|^2 [n + \sin^2(\sqrt{n+1}gt)]} \\ &- \sum_{n=0}^{\infty} \left|p_n\right|^2 [n + \sin 2(\sqrt{n+1}gt)] - 1. \end{aligned} \tag{3.11}$$

$$\begin{aligned} g_{(0)}^2(t) &= \frac{\left\langle N^2 \right\rangle - \left\langle N \right\rangle}{\left\langle N \right\rangle^2} \\ &= 1 - \frac{\sum_{n=0}^{\infty} \left|p_n\right|^2 [n + \sin^2(\sqrt{n+1}gt)]}{\sum_{n=0}^{\infty} \left|p_n\right|^2 [n^2 + (2n+1)\sin^2(\sqrt{n+1}gt)} \end{aligned} \tag{3.12}$$

From (3), and deriving as the same method, we get:

$$\begin{pmatrix}(\Delta x)^2(t)\\(\Delta p)^2(t)\end{pmatrix}=1+2\sum_{n=0}^{\infty}|p_n|^2[n+\sin^2(\sqrt{n+1}gt)]-$$
$$\left\{\begin{pmatrix}\mathrm{Re}\\\mathrm{Im}\end{pmatrix}\sum_{n=0}^{\infty}p_n^*p_{n+1}\begin{bmatrix}(\sqrt{n+1}+\sqrt{n+2})\cos(\sqrt{n+1}-\sqrt{n+2})gt\\+(\sqrt{n+1}-\sqrt{n+2})\cos(\sqrt{n+1}+\sqrt{n+2})gt\end{bmatrix}\right\}^2 \tag{3.13}$$
$$\pm\mathrm{Re}\sum_{n=0}^{\infty}p_n^*p_{n+2}\begin{bmatrix}(\sqrt{(n+1)(n+2)}+\sqrt{(n+2)(n+3)})\cos(\sqrt{n+1}-\sqrt{n+3})gt\\+(\sqrt{(n+1)(n+2)}-\sqrt{(n+2)(n+3)})\cos(\sqrt{n+1}-\sqrt{n+2})gt\end{bmatrix}$$

4. Discussing the results

We plot expression (3.8) of *W(t),*show in Fig.2 ,Fig.3,

When $\alpha=1$, it exhibits chaotic behaviour of the atom inversion .however ,when $\alpha=3$,it exhibits an ordered behaviour, this is so-called the quantum "collapse and revival"(PENG SHI AN, GUO GUANG CAN, 1990),and it also exhibits chaotic behaviour after a longer time.

Display (3.11) of *Q(t,)*show in Fig.4

The time dependence of Mandel parameter is shown in Fig.4, where the collapse ,calms, revivals, and chaos occur again ,similarly to *W(t)*. we also found that there are main part of the data under zero, form(5),it imply that there exit a sub-Poisson photon statistics in the system.

Plot expression (3.12) of $g^2_{(0)}$ (t),show in Fig.5

The $g^2_{(0)}$ (t) of the system is shown in Fig.5, we found that there are different shape with vary α .We also simple calculate that $g^2_{(0)}$ (t)=0.when $\alpha=0,\text{or}\ \alpha\ge 5$.

From Fig.5, we know that $g^2_{(0)}\le 1$ for all time and for all α , from (1.7) we can get that there exit Antibanching effect in its system.

We plot expression (3.13)of $(\Delta x)^2$and $(\Delta\mathrm{p})^2$,show in Fig.6 and Fig.7

From the Fig.6 show that: in first valley exit $(\Delta x)^2\le 1$. from this we can get that the fluctuation $(\Delta x)^2$ have squeezing effect .but it last a very little time .it is a unclassical effect.

But from Fig.7, we find that $(\Delta p)^2\ge 1$ for all time, so there do not exit squeezing effect.

5. Conclusion

In our paper, we discussed the Orthogonal Coherent state interacting two-level atom, the statistics properties of both atom and field will be discussed. There are several results after we study:

5.1 Atom inversion exit the collapse, calm, revival and chaotic behaviou r.

5.2 The field of the system have uncalssical effect.

The photon distribution exit sub-Poisson statistics; there exits Antibanching effect .The fluctuation of $(\Delta x)^2$ have squeezing effect, at the mean time, the fluctuation of $(\Delta p)^2$ do not exit squeezing effect.

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References

F.W.Cummings. (1965). *Phys.Rev*.140, A1051.

J.H.Eerly, N.B.Narozhny and J.J.Sanchez-Mondragon. (1980). *Phys.Rev.lett*.44, 20.

Peng Shi An, Guo Guang Can. (1990). Generated a None-Classical Filed by Orthogonal Symmetry method. *Science Report*. 1990(8):579~583.

Qiao Gu, J.Zhang. (1989). A possible way for generating sub-poissonisn field,*Acta Optical Sinica* 9,478-480.

Qiao Qu. (2003). *Radiation and Bioinformation*,BeiJing:Scince Press, 2003.10.

Willam, H. Louiselll, (1982). *Quantum statistical Properties of Radiation*. BeiJing:Scince Press,1982.9.

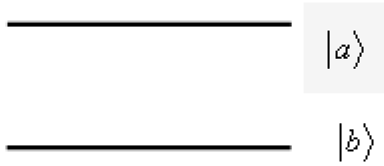

Figure 1. a two-level atom model

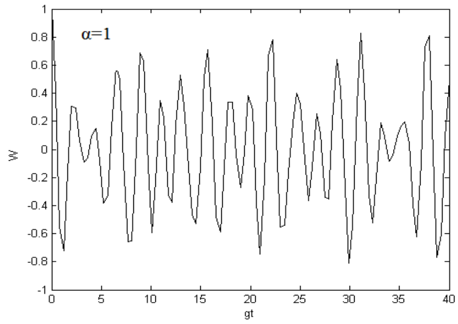

Figure 2. the chaotic behavior of *W(t)*as $\alpha = 1$

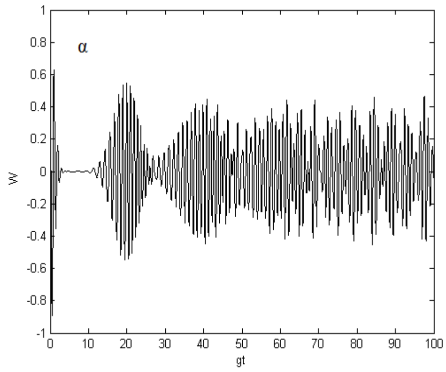

Figure 3. the collapse, calm, revival and chaotic behavior of *W(t)*as $\alpha = 3$

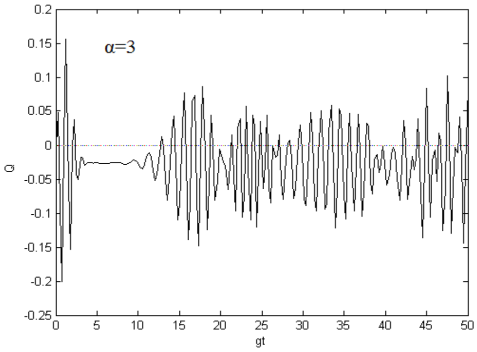

Figure 4. the collapse, calm, revival and chaotic behavior of *Q(t)*as $\alpha = 3$

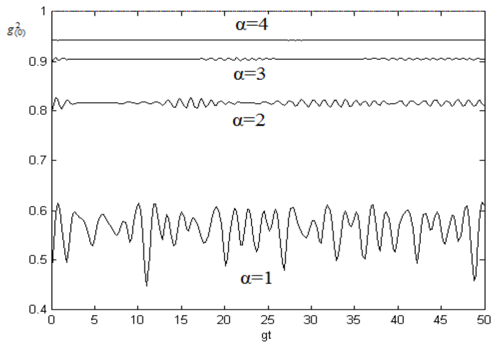

Figure 5. second order coherence degree when $\alpha = 1 \cdot\cdot 4$

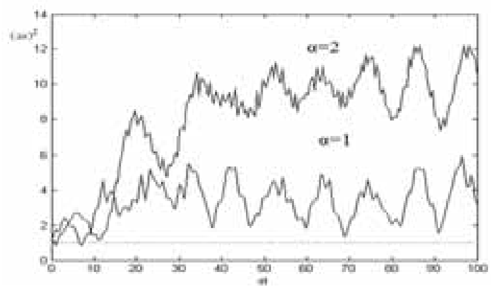

Figure. 6 $(\Delta x)^2$ when $\alpha = 1$ and 2

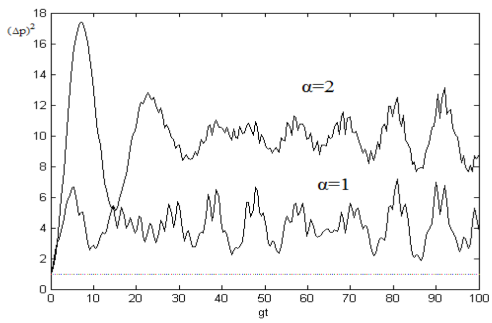

Figure 7. $(\Delta p)^2$ when $\alpha = 1$ and 2