Digital Topology Optimization Design and Manufacturing Based on the Level Set Method

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Abstract

This paper presents a unified process of digital structural topological optimization and digital manufacturing. Based on a level set method and an augmented Lagrange multiplier method, the cloud data of the level-set surface for the optimized result are obtained. A new triangular facet approximation method to approach the level-set surface is presented to extract the zero-level-set cloud data of the topologically optimized structure. By using a slice method and the distance formula between two points, the extracted cloud data of the boundaries of the optimized structures are classified into different boundary curves. Along with the standards of the digital manufacturing, the sorted points are translated into a processing program for digital manufacturing of the optimized structures by using an automatic programming technology. The Wire-cut Electrical Discharge Machining is employed to manufacture digitally the work-piece with the optimized design. The digital structural optimization and manufacturing of a 2-D work-piece validate the method and process presented in this paper.

Keywords: Data extraction, Digital manufacturing, Digital topology optimization, Triangular facet method

1. Introduction

Topology optimization is identified as one of the most challenging tasks in structural design, and has been applied in various industries, especially aeronautical, spaceship and civil industries. Generally the processes of topology optimization are: 1) firstly, a mathematical optimization model of a practical problem is established according to the specific requirements for the problem, 2) secondly, optimization theory and optimization algorithms are used together with programming software so as to obtain the optimized design for the structure, and 3) finally, based on the specified requirements, the structure is optimized and then a work-piece with the optimized structure is manufactured.

Quite a few studies on optimization techniques and approaches have been reported. By using various optimization algorithms along with the finite element method, many structures were optimized to satisfy different designing requirements, such as the requirements for less energy consumption, less materials, superior mechanical properties and structural stability (Rajkumar et al. 2008 and Akihiro et al. 2010). The optimization techniques and approaches were developed include the ground structure method (Rozvany 1979), the homogenization method (Bendsøe and Kikuchi 1988), the solid isotropic material with penalization approach (Zhou and Rozvany1991), barrier methods (Khot et al. 1993), the penalty function method, the genetic algorithm (Pruettha and Konlakarn 2001), the level-set method (Wang et al. 2003), and the Lagrange multiplier method (Chen et al. 2006). Some other methods were also proposed, such as the current flexible building block method (Kim et al. 2006), the unit cell approach (Wang 2004), and the radial-basis-function (RBF) level-set method (Wang and Wang 2006). The essence of all these methods is to transform the problem into an optimal material distribution problem so that the designed configuration can be measured quantitatively by an objective function.

The intention of topological optimization is to make the full use of the material performance and meet the needs of an objective function under the condition of the given constrain. So this structure is very sensitivity to geometric structure. If we do not strictly manufacture a work-piece according to the topological optimization design result, failure of the work-piece will be happened. According to the authors' investigation, although a variety of numerical methods for structural optimization have been developed, very few studies on turning the level set topology optimization result into a practical work-piece by digital manufacturing have been carried out. In this paper, the whole processes connecting the digital topology optimization design and digital manufacturing are presented in detail. The level set function and the Augmented Lagrangian method are employed for digital topology optimization design, and the cloud data of the level set model is extracted by using a new triangular facet approximation method to approach the level set surface of the optimized result. In this method, the parametric equations of the straight lines surrounding the triangular facets are employed to obtain the boundary data of the optimized structure, and the extracted cloud data based on the optimized structure boundaries are classified into different boundary curves by employing a slice method and the distance formula. According to the digital manufacturing technology standards, the sorted points are then translated into the processing program for digital manufacturing on the basis of an automatic programming technology. The Wire-cut Electrical Discharge Machining is employed to complete the digital manufacturing.

2. Digital topology optimization designs

Among the various structural optimization methods, the level set method has been applied widely in the field of structural topology optimization research for many years, which was originally developed by Sethian (1999) as well as by Osher & Fedkiw (2003) to solve the problems such as tracking, computer vision, crack propagation in solid material, image processing, and so on. In the level set method, topology optimization is regarded as a dynamic evolution process of the level-set function varying with pseudo time *t*. The embedded function allows its surface to move up and down on a fixed coordinate system without changing its surface topology structure, and the optimization shape embedded in high-dimensional level-set function can automatically modify the topology structure by boundary merging and breaking. The topological change of the structure can be tracked by checking the high-dimensional level-set surface. This process can be completed by solving the Hamilton-Jacobi Partial Differential Equation (PDE) with the evolution velocity V_n and a set of initial values. The Hamilton-Jacobi PDE is generally solved by using the upwind based on the fixed Eulerian grids. At the same time, the level set function can digitally records the whole evolution information in optimization process. As it is very difficult to obtain a reasonable optimal result with a fixed Lagrangian multiplier, so in this paper we employed the level-set function and Augmented Lagrangian method to achieve a digital topology optimization design. The whole process is shown as follows.

Let us define a closed subset $\Omega \subseteq R^{\mathrm{d}}(d=2 \quad or \quad 3)$ as a design domain including the whole admissible shapes. The closed boundary $\Gamma = \partial\Omega \subseteq D$ is described by zero level-set which is Lipschits continuous. An embedded function $\phi(x(t),t)$ is introduced to denote the different parts of the design domain as shown in Fig. 1.

$$\begin{cases} \phi(x(t),t) > 0 & \forall x \in \Omega \backslash \partial\Omega \\ \phi(x(t),t) = 0 & \forall x \in \partial\Omega \cap D \\ \phi(x(t),t) < 0 & \forall x \in D \backslash \Omega \end{cases} \tag{1}$$

In the level set method, the design boundary is implicit in the zero-level set of the high-dimensional level set surface. The movement of high-dimensional function is governed by the Hamilton-Jacobi PDE, the evolution velocity and a set of initial values. The Hamilton-Jacobi equation is given by differentiating equation $\phi(x(t),t)=0$ with respect to t as

$$\phi_{\mathrm{t}} + \mathbf{V}.(\nabla\phi)^T = 0 \tag{2}$$

where $\mathbf{V}$ is the evolution velocity of the objective function, and ∇ is the gradient operator. In the three-dimensional space, $\mathbf{V}$ is given by

$$\mathrm{V} = V_{\mathrm{n}}\mathrm{S}_{\mathrm{n}} + V_t\mathrm{S}_{\mathrm{t}} \tag{3}$$

where V_n and V_t are the normal and tangent components of the velocity $\mathbf{V}$, and S_{n} and S_{t} are unit vectors in the normal direction and the tangent direction, respectively. Substituting equation (3) into equation (2), we have

$$\phi_t + V_n(\nabla\phi)^T.S_n = 0 \tag{4}$$

Equation (4) demonstrates that only the normal velocity can contribute for the boundary evolution. In this way, a topology optimization process can be seen as a dynamic evolutionary process of high-dimensional level set function varying with the pseudo-time. As long as the evolution velocity of objective function is known, the design boundary embedded into the high-dimensional level set function can update its topology automatically in the iteration process until an optimized result is obtained. So in the following sections, the material derivative (Azegami and Wu 1996) and Augmented Lagrange multipliers are used to obtain the evolution velocity of objective function.

Set $f(x)$ as the target function, $g_j(x) \le 0$ is the inequality constraints, and the initial optimization model could be described as

$$\begin{aligned} &\underset{x}{Min} f(x) \\ &\text{Subjected to} \quad g_i(x) \le 0, \quad (i = 1, 2, \ldots m) \end{aligned} \tag{5}$$

By using the augmented Lagrange multipliers and the material derivative formulae, the evolution speed of the optimization model is derived (Wang et al. 2003 and Chen 2006) and written in equation (6).

$$V_n = -\beta - \max[\lambda - g_i(x)/\mu, 0] \tag{6}$$

where V_n is the normal evolution velocity, β is the shape gradient density, and μ is a penalty factor. Once the velocity V_n in the Hamilton-Jacobi PDE equation (2) is obtained, the finite difference method with respect to t can be used and the following equation can be obtained.

$$\frac{\phi^{n+1} - \phi^n}{\Delta t} + V_n^n.(\nabla\phi^n)^T = 0 \tag{7}$$

where V_n^n is the velocity at time t^n, and ∇ denotes the gradient operator. For a 3-D case, the formula can be written as

$$\frac{\phi^{n+1} - \phi^n}{\Delta t} + \alpha^n\phi_x{}^n + \beta^n\phi_y{}^n + \gamma^n\phi_z^n = 0 \tag{8}$$

where $V_n^n = \alpha^n i + \beta^n j + \gamma^n k$ and $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$.

By solving equation (8) with the evolution velocity V_n and a set of initial values, the digital optimization structure on a fixed mesh can be obtained by using the upwind method (Wang et al. 2003).

3. A triangular facet approximation method

Considering the excellent performance of MATLAB on matrix data processing, the cloud data of the high-dimensional level-set surface for the optimized structure are stored in the form of a matrix. The optimized boundary data will then be extracted and classified according to the digital manufacturing requirements.

3.1 Data extraction of the optimized boundary

The optimized boundary is implicit in the zero-level set function of the high-dimensional level set surface. When the convergence conditions are satisfied, the cloud data of the high-dimensional level-set function for the optimized structure are stored in a form of $[x, y, \phi]$ for a 2-D structure and in a form of $[x, y, z, \phi]$ for a 3-D structure, where x, y and z are coordinates of the node in the finite element analysis, and ϕ is the final value of the level set function with respect to node coordinates. In the topological optimization of a 2-D structure, the level set function of the final optimization result is a three-dimensional surface, and the optimized structure with respect to the zero-level-set function is a 2-D structure. In a 3-D optimization structure, the level set function of the final optimization result is a 4-D surface, and the optimized structure with respect to the zero-level-set function is a 3-D structure.

A new triangular facet approximation method using triangular facets to approach the level set surface as shown in Fig. 2 is presented in this paper. The smaller the element is, the more accurate the result will be. In order to obtain the boundary data of the optimized structure, the intersecting point of the straight lines surrounding these facets with zero-level-set function is employed to replace the intersecting line of high-dimensional level set function with zero-level-set. In Fig. 2, the number 1, 2, 3, 4, 5, 6 represents the element number in finite element analysis, and triangular facets are made up of a series of small triangles such as triangles ABD, BCD, ABC and ACD which are formed by straight lines AB, BC, CD, DA, AC and BD. Let us assume that the end point coordinates of a straight line are (x0, y0, z0) and (x1, y1, z1), with respect to level set function values ϕ_0 and ϕ_1, respectively, and that the coordinate of any point along the straight line is denoted as (x, y, z), which is with respect to the level set function value ϕ. In a 3-D topological optimization structure, the parametric equations of these lines are written as

$$\frac{x_0 - x}{x_0 - x_1} = \frac{y_0 - y}{y_0 - y_1} = \frac{z_0 - z}{z_0 - z_1} = \frac{\phi_0 - \phi}{\phi_0 - \phi_1} = k \tag{9}$$

where k is a parametrical variable, and if $0 \le k \le 1$, then $\phi_0 \le \phi \le \phi_1$ ($\phi_1 \ge \phi_0$). As the topological optimization result is implicit in zero-level-set, the intersecting points of these straight lines with the zero-level-set function can be derived. Let $\phi = 0$, and we have

$$k = \frac{\phi_0}{\phi_0 - \phi_1} \tag{10}$$

By substituting equation (10) into equation (9), the coordinates of intersecting point x, y and z can be obtained, which must be a boundary point of the optimized result. Otherwise, the high-dimensional level set function with respect to element has no intersection point with zero-level-set. In a 2-D case, the process with the geometrical configuration is indicated in Fig. 3.

Intersecting points of the straight lines AB, BC, CD, DA, AC and BD forming boundaries of these triangular facets with zero-level-set function can be obtained. This process is repeated until the whole intersecting points with zero-level-set function are calculated and stored in a matrix A. In section 3.2, these intersections will be classified according to the different boundary curves.

3.2 Sorting boundary points

In a 2-D case, the level set surface is 3-D, and the topological optimization result is a plane structure. In a 3-D case, the level set function is 4-D, and topological optimization structure is a 3-D structure. These two cases are investigated respectively in this paper.

3.2.1 Two-dimensional case

As the boundary data extracted are in disorder, these data should be classified according to the digital manufacturing requirements. In order to obtain data in the different boundary curves, the bubble sort method (Jagdish and Richard1987) is used herein. Since the distance between two adjacent points in the same boundary curve is the shortest, the points in the same boundary curve can be sorted in order. In this process, every boundary points are rearranged according to different boundary curves. In order to sort the data in matrix A, the distance formula between any two points is used here.

$$D = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{11}$$

where i, j=1,2,3,…k' ($j \ge i$), and k'is the last number in matrix A. The flowchart for data sorting is developed and shown in Fig. 4.

Using this flowchart, the boundary coordinate points forming optimization structure can be rearranged according to different curves.

3.2.2 Three-dimensional case

Although the different machining method has different processing, most Numerical Control (NC) machine tools generally change 3-D surface into 2-D structure in programming. By combination with 2-D tracks of the machine cutter, a 3-D surface can be machined. In mechanical machining process, there must be a distance between the adjacent track centers of machine cutter due to the dimension of diameter of the cutting tool. Of course, the smaller the distance is, the more accurate and smooth the machined surface will be. Let us slice up the 3-D optimization structure into a series of thin flakes, every flake can be regarded as a 2-D structure, and all coordinate points can be considered as in the same plane because the thickness of flake can be set very thin about 50μm~500μm for rapid prototyping (RP) technology. Let us assume that every slice has the same coordinate in the z direction. After obtaining the cloud data of the 3-D optimization result according to equation (9), these cloud data along the z coordinate direction is sliced up. The distance of the adjacent slices can be determined according to machine tools employed and work-piece precision machined. In every slice, it is assumed that the points in the optimization boundaries are of the same coordinate in the z direction. Fig. 4 is employed to sort the boundary curves for every slice, and the process is repeated until the boundary curves for all slices are sorted out. The flowchart is presented as follows.

4. Digital manufacturing

After the coordinates of boundary points are extracted, sorted and stored in different forms, the sorted data matrix will be changed into a processing program according to the requirements of different machine tools. As the sorted data points are too many to be fitted, the G01 code of Wire-cut Electrical Discharge Machining is used directly for a 2-D case. As for a 3-D case, the tool-path generated is much more complex than that in a 2-D structure, because the cutter location data contains both XYZ coordinate points of the tool-path and associated tool-orientation vector at each point. The boundary data will be converted into a machining program (such as G code) using a machine-specific post-processor (Yang 2005). This machining program is finally output to Computer Numerical Control (CNC) machine tool or Layered Manufacturing equipment that generates motion commands to control tool movements. After the data matrix is programmed according to the manufacturing standard, the digital manufacturing can be realized. The NC viewer can be used before processing, which is a simulation software reading G code and can display the tool-path of the final result. The entire digital processing flowchart is given in Fig. 6.

This process connects the digital topology optimization design and digital processing innovatively and seamlessly.

5. Numerical validation

In this section, a 2-D cantilevered beam as shown in Fig. 7 is employed as the topological optimization example so as to validate the method presented in this paper. Young's modulus and Poisson's ratio is 6.897×10^{10} Pa and 0.3, respectively and the length, width, and thickness of design domain are 60 mm, 30 mm and 1 mm, respectively. The objective function is the smallest displacement at the point F where a concentrated force is applied. The initial optimization structure and the initial level set function are shown in Fig. 8 and Fig. 9, respectively.

A self-compiled program was coded for the optimization. The final topology optimization result and level set function are shown in Fig. 10 and Fig. 11, respectively. In the optimized structure, the black part represents material area and white part stands for the empty area. By using the presented method in this paper, the boundary curves consisting of intersections is shown in Fig. 12. After exported G code to the Wire-cut Electrical Discharge Machining, the optimized structure can be manufactured and the manufactured work-piece is shown in Fig. 13. The digital topology optimization design is connected with digital processing in this process.

6. Conclusion

An innovative process, which connects the digital topology optimization design and digital manufacturing process, has been developed in this paper. The level set function and the Augmented Lagrangian method are employed for digital topology optimization design, and the cloud data of the level set model is saved in a database. An innovative triangular facet approximation method using triangular facets to approach the level set surface of the optimized result is presented. The parametric equations of the straight lines surrounding these facets are employed to extract the boundary data of the optimized structure. Then the slice method and bubble sort method are used to sort different boundary points in different boundary curves. The Wire-cut Electrical Discharge Machining using G code is employed to manufacture the optimized structure digitally. A 2-D plane example including digital optimization design and manufacturing demonstrates the effectiveness of the proposed method.

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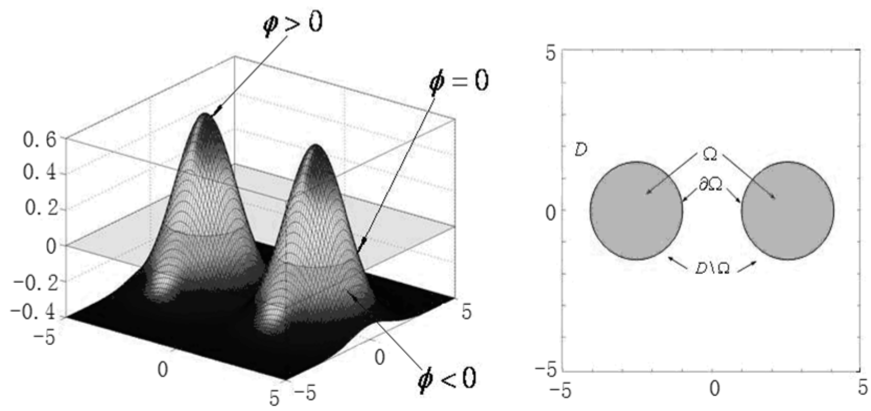

Figure 1. design domain and level set function

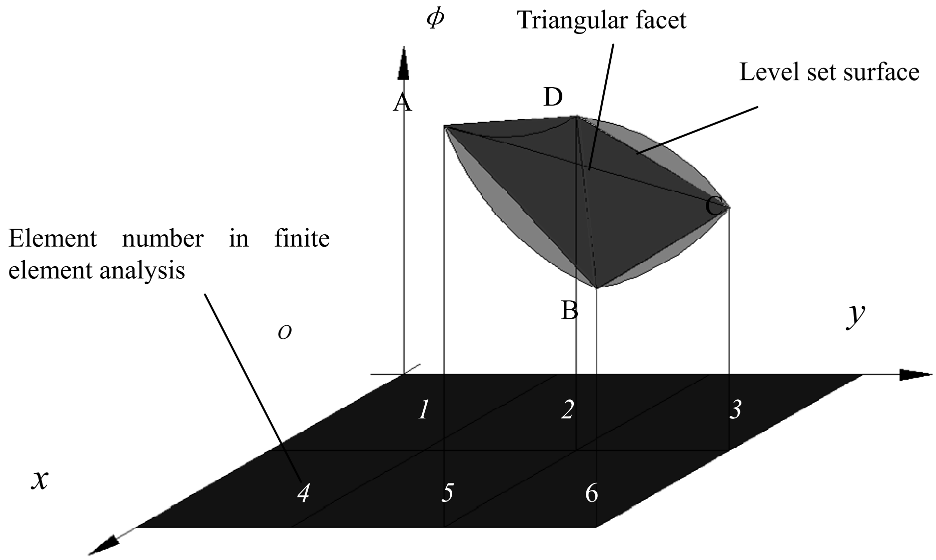

Figure 2. Triangular facets to approach the level set surface (in 2-D case)

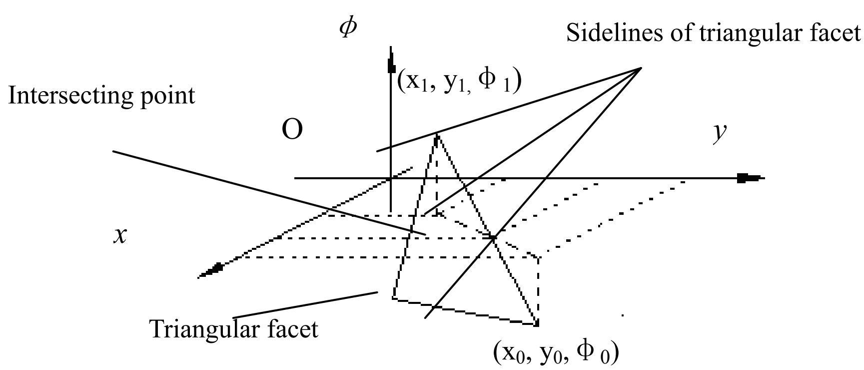

Figure 3. Intersecting point of straight line surrounding triangular facet with zero-level-set function

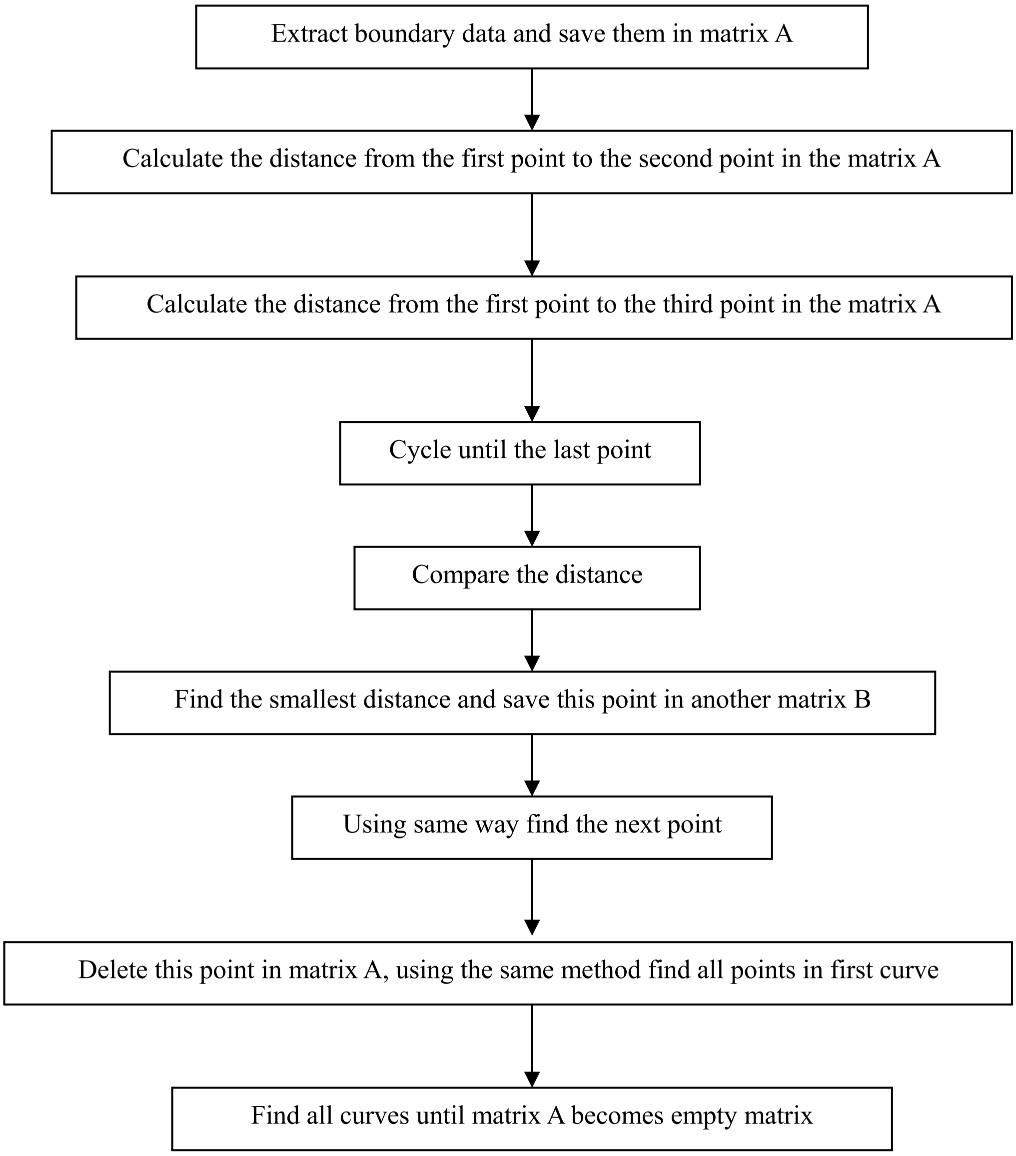

Figure 4. Flowchart of sorting boundary point in a 2-D case

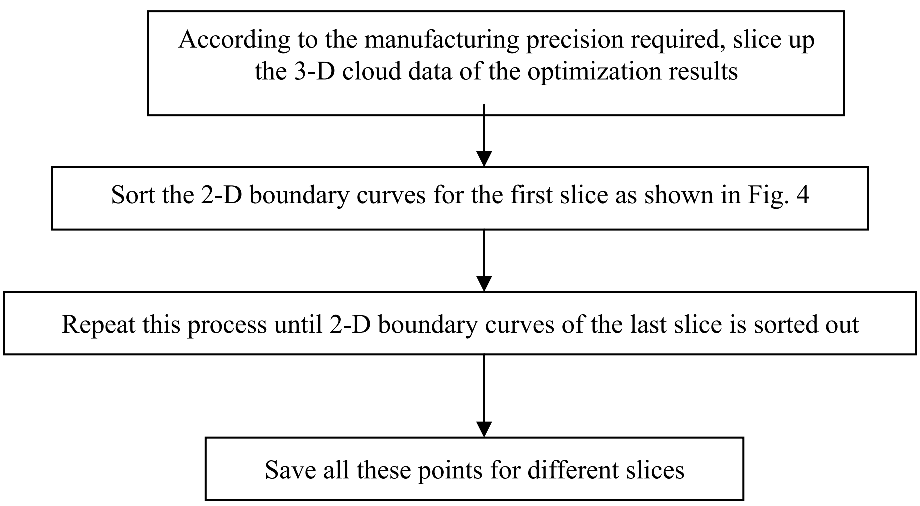

Figure 5. Flowchart for sorting boundary point in a 3-D case

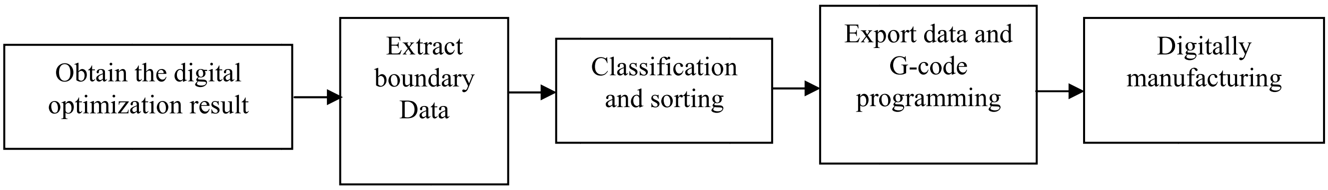

Figure 6. Digital processing flowchart of digital topology optimization design

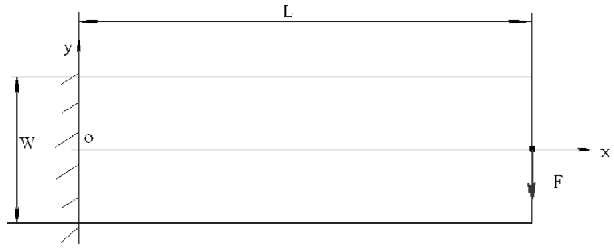

Figure 7. The geometric configuration and boundary conditions of a 2-D structure

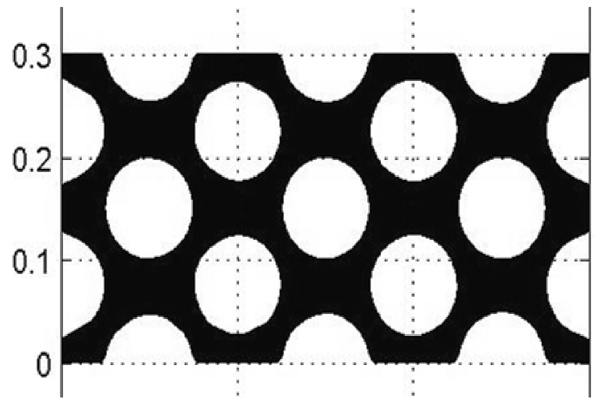

Figure 8. Initial optimized structure

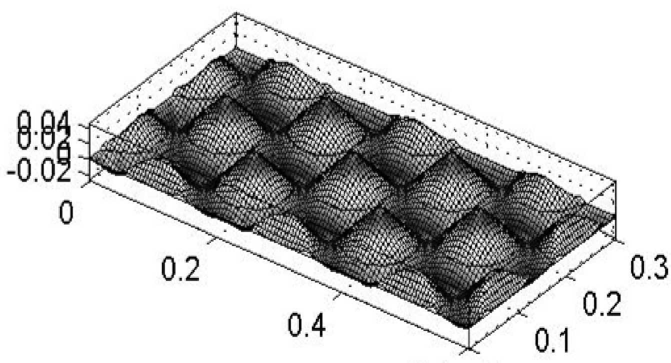

Figure 9. Initial level set function.

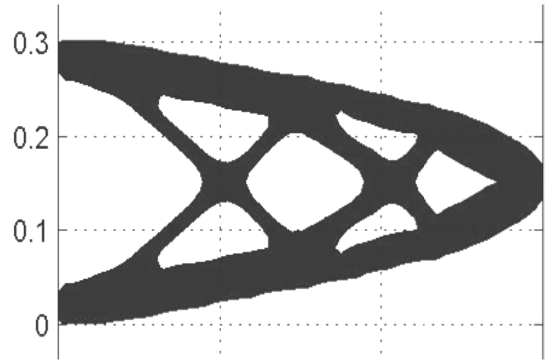

Figure 10. Final topology optimization result

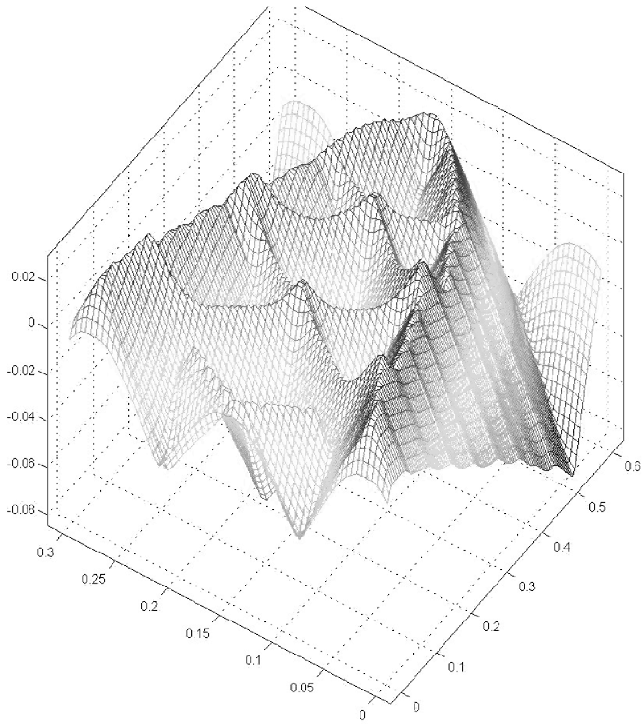

Figure 11. Final level set function

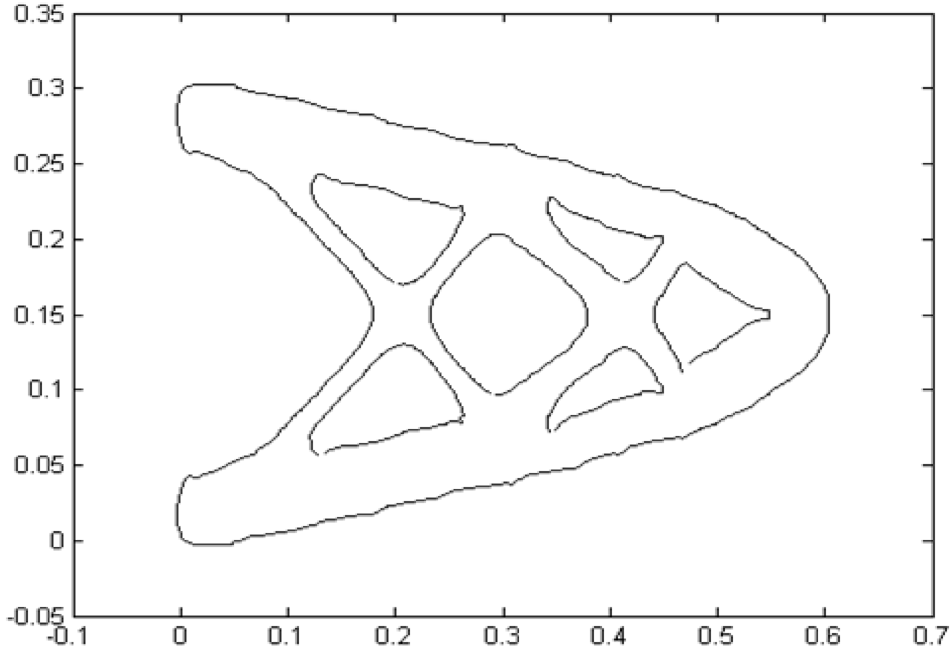

Figure 12. The boundary cloud data of the optimization result

Figure 13. The manufactured work-piece for the optimized structure