Effect of Paired Stenosis on Blood Flow through Small Artery

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Received: January 4, 2011 Accepted: January 21, 2011 doi:10.5539/jmr.v3n2p224

Abstract

In this paper the mathematical models have been developed to study the effect of paired stenosis on blood flow, where the blood flow is assumed to behave like a couple stress fluid, peripheral layer plasma (Newtonian fluid) and core layer of suspension of erythrocytes (Non- Newtonian fluid). The study predicts that wall shear stress increases with the increase of the two heights of stenosis. Also wall shear stress has proportional direction relation with the separation factor, the maximum value of wall shear stress occurs at the peak separation of paired stenoses.

Keywords: Paired stenosis, Blood flow of two layers, Wall shear stress, Newtonian fluid, Cossan fluid

1. Introduction

The blood flow through small arteries or arteries with stenosis is considered non-Newtonian Chan *et al* 2005, Mann and Tarbe 1990, Shukla *et al* 1980 and Nakamura and Sawada 1988. Non -Newtonian flow is the flow that does not obey the Newtonian relationship between shear stress and shear rate. In non Newtonian flow the slope of shear stress versus shear rate curve is not constant and the viscosity of fluid decreases with increasing shear rate. Wall shear rate of non Newtonian is considered less than 100 s-1 Tu and Delwille 1996, Misra *et al* 1993, Shukla *et al* 1990 and Huang *et al* 1987. The simple example of a liquid exhibiting non-Newtonian flow is paints and printing inks. The simple constitutive equation is Cossan which is based on a structure model of the interactive behavior of solid and liquid phases of two-phase suspension. The model describes the flow of viscoplastic fluids that can be mathematically described as flow. $\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{k.\gamma}$, where τ is shear stress, γ is shear rate and k is a Cossan model constant. The abnormal and unnatural growth in the arterial wall thickness that develops at various location of the cardiovascular system under diseased conditions is called arteriosclerosis or stenosis, this can cause serious circulatory disorders by reducing of occluding the blood supply. For instance stenosis in the arteries supplying blood to brain can bring about cerebral strokes, likewise in coronary arteries it can cause myocardial infarction leading to the heart failure. The actual causes of the stenosis are not well known but it has been suggested that the deposits of the cholesterol on the arterial wall and the proliferation of connective tissues may be responsible. Ponalagusamy 2007 and Srivastava 2003 have assumed blood flow through stenosis to behave like a couple stress fluid, peripheral layer plasma Newtonian fluid which acceptable for high shear rate flow (γ > 100 sec-1) in layer arteries and with viscosity coefficients μ_p, and a core region of suspension of erythrocytes non-Newtonian fluid which acceptable for low shear rate flow (γ <100 sec-1) through small diameter arteries and with viscosity coefficients μ_c, Sanker and Ahmmed 2009 have assumed the non-Newtonian fluid in core layer as Casson fluid and indicated that Casson fluid model holds for blood flow in tubes of diameter 0.013 - 0.13 cm also estimated the range of wss 1.677 - 2.290 for hieght stenosis range 0.025 - 0.125.

The initiation and localization of arteriosclerosis is closely related to local hemodynamic factor (such as wall shear stress, hydrostatic pressure and dynamic pressure).Wall shear stress is an important factor in the study of blood flow. Accurate predictions of the distribution of the wall shear stress are particularly useful for the understanding of the effect of blood flow on endothelial cells. Musad and Khan 2010 have developed mathematical models to study wall shear stress in larg and small artery with single stenosis. Thus in this paper we extend the models to study blood flow through artery with paired stenosis. Johnston and Kilpatrick 1990 have considered a mathematical model of paired stenosis. The two velocities of both layers have reported by Kapur 1985.

2. The Model Devloped

Consider an axially symmetric flow of blood which assumed to be incompressible in the two directions through a rigid walled artery with an axially symmetric paired stenosis. Also we consider the flow of a plasma layer and a core layer with viscosity coefficient μ_p, and μ_c respectively, we also assume that the plasma layer is of uniform thickness and δ_1, δ_2 are the hight of stenosis 1 and stenosis 2 respectively

The velocity of plasma layer (Newtonian fluid) and the geometry equation of the stenosis are given by

$$v_p = \frac{G}{4\mu_p}(R^2(z) - R^2),\ 0 < R < R(z) \tag{1}$$

$$R(z) = \begin{cases} 1 - \frac{\delta}{2R_0}(1 + cos\pi\frac{z}{z_1})), ... - z_1 < z < z_1 \\ 1 - \frac{\delta}{2R_0}(1 + cos\pi\frac{z-(z_0+z_1+z_2)}{z_2}), z_0 + z_1 \le z \le z_0 + z_1 + 2z_2 \\ 1, ...z_0 + z_1 + 2z_2 < z < -z_1 \end{cases} \tag{2}$$

Also the velocity of core layer (non-Newtonian fluid) is given by

$$v_c = \frac{G}{4\mu_c}[R^2(z) - r^2] + \frac{G}{\mu_c}[R^2(z) - R_1^2(z)][\frac{\mu_c}{\mu_p} - 1],\ 0 < r \le R_1(z) \tag{3}$$

While the geometry equation of core layer is $R_1(z) = \beta R(z)$, where $(0 < \beta < 1)$

Since blood flow in peripheral is Newtonian, then wall shear stress is given by the equation

$$\tau_p = -\mu_p\frac{\partial v}{\partial r} \tag{4}$$

Where τ is the shear stress, μ_p is the viscosity of plasma, $\partial v/\partial r$ is the velocity gradient or rate of deformation (shear rate), v is the velocity, G is the pressure gradient and R(z) is the radius of artery in the stenosis part and R_0 is the radius of normal part of artery then

Derivative(1), then

$$(\frac{\partial v}{\partial r})_{r=r(z)} = \frac{\partial v_p}{\partial R} = \frac{\partial v_z}{\partial R} = \frac{G}{2\mu_p}r(z) \tag{5}$$

From equations (4) and (5) we get

$$\tau_p = -\frac{G}{2}[\int_{-z_1}^{z_1}(1 - \frac{\delta_1}{2R_0}(1 + cos\pi\frac{z}{z_1}))dz + \int_{z_0+z_1}^{z_0+z_1+2z_2}(1 - \frac{\delta_2}{2R_0}(1 + cos\pi\frac{z - (z_0 + z_1 + z_2)}{z_2})dz + \int_{z_0+z_1+2z_2}^{-z_1} dz] \tag{6}$$

By integrating(6) we get

$$\tau_p = \frac{G}{2}(z_0 + \frac{\delta_1}{R_0}z_1 + \frac{\delta_2}{R_0}z_2) \tag{7}$$

Equation (7) is the equation of wall shear stress of artery with paired stenoses in the case of blood is Newtonian fluid.

Similarly blood flow in core is non- Newtonian, then wall shear stress is given by the equation.

$$\sqrt{\tau_c} = \sqrt{\tau_y} + \sqrt{-\mu_c\frac{\partial v}{\partial r}} \tag{8}$$

The velocity of blood flow in core region is given in equation (3), let $\frac{\mu_c}{\mu_p} - 1 = \mu_m$ then equation (3) reduce to

$$v_c = \frac{G}{4\mu_c}(R^2(z) - R_0^2) + \frac{G}{\mu_c}(R^2(z) - R_1^2(z))(\mu_m). \tag{9}$$

Derivative(9), then

$$(\frac{\partial v_c}{\partial r})_{r=R(z)} = \frac{G}{2\mu_c}[(1 + 4\mu_m)R(Z) - 4\mu_m R_1(z)] \tag{10}$$

from eq(8) and eq(10) we get

$$\sqrt{\tau_c} = \sqrt{-\frac{G}{4}[(2 + 8\mu_m)\int R(z)dz - 8\mu_m\int R_1(z)dz]} + \sqrt{\tau_y} \tag{11}$$

But $R_1(z) = \beta R(z)$, $(0 < \beta < 1)$ then

$$(\sqrt{\tau}_c = \sqrt{-\frac{G}{4}[(2+8\mu_m)\int R(z)dz - 8\mu_m\int \beta R(z)dz]} + \sqrt{\tau_y})$$

$$(\sqrt{\tau}_c = \sqrt{-\frac{G}{4}(2+8\mu_m - 8\mu_m\beta)\int R(z)dz} + \sqrt{\tau_y})$$

$$\sqrt{\tau}_c = \sqrt{-\frac{G}{2}(1+4\mu_m - 4\mu_m\beta)[\int_{-z_1}^{z_1}(1-\frac{\delta_1}{2R_0}(1+cos\pi\frac{z}{z_1}))dz + \int_{z_0+z_1}^{z_0+z_1+2z_2}(1-\frac{\delta_2}{2R_0}(1+cos\pi\frac{z-(z_0+z_1+z_2)}{z_2})dz + \int_{z_0+z_1+2z_2}^{-z_1}dz]} + \sqrt{\tau_y} \quad (12)$$

$$\sqrt{\tau}_c = \sqrt{\frac{G}{2}(z_0 + \frac{\delta_1}{R_0}z_1 + \frac{\delta_2}{R_0}z_2)(1+4\mu_m - 4\mu_m\beta)} + \sqrt{\tau_y} \quad (13)$$

put β=0.95 and substitute $\mu_m$in eq(13)where μ_m=($\frac{\mu_c}{\mu_p}$ − 1) then we get

$$\sqrt{\tau}_c = \sqrt{\frac{G}{2}(z_0 + \frac{\delta_1}{R_0}z_1 + \frac{\delta_2}{R_0}z_2)(0.8 - 0.2\frac{\mu_C}{\mu_p})} + \sqrt{\tau_y} \quad (14)$$

Equation (14) is the equation of wall shear stress of artery with paired stenoses in the case of blood is non-Newtonian fluid

3. Boundary Conditions

1) $\tau \to \infty$ at $R_0 = 0$

2) $\tau_c = \tau_p$ when $\mu_c = \mu_p$.

3) when $z_0 = z_1 + z_2$ then

$$\tau_p = \frac{G}{2}((z_1 + \frac{\delta_1}{R_0}z_1) + (z_2 + \frac{\delta_2}{R_0}z_2))$$

$$\tau_p = \frac{G}{2}(z_1 + \frac{\delta_1}{R_0}z_1) + \frac{G}{2}(z_2 + \frac{\delta_2}{R_0}z_2).$$

$\tau_p = \tau_1 + \tau_2$

while

$$\sqrt{\tau}_c = \sqrt{\tau_1+\tau_2} + \sqrt{\tau}_y \neq (\sqrt{\tau}_1 + \sqrt{\tau}_2) + \sqrt{\tau}_y.$$

4. Results and Conclusion

The results based on the numerical solution of equation (14) by using mathematical software(Microsoft Mathematics 3.0) for $z_0 = 0, z_0 = z_1 = z_2$ and $z_0 = z_1 + z_2$ the values of (δ_1/R_0) and (δ_2/R_0) varied from 0.1 to 0.9 and values of (μ_c/μ_p) varied from 1 to 2.5, observed that, wall shear stress has direct relation with the separation factor z_0 and wall shear stress is maximum at peak separation $z_0 = z_1 + z_2$. Also wall shear stress increases with the increase of one or both heights of stenosis. Mathematically, we see the peak separation in the case of paired stenoses lead to peak separation of wall shear stress in plasma layer (Newtonian fluid), this mean the vessel of paired stenosis or multiple stenosis have wall shear Stress equals the sum of total stresses for each stenosis, while this does not happen in core layer (non-Newtonian fluid), the viscosity of the blood and the yield stress are not doubled by a doubling of stenosis, but change in a certain range. High stress leads to aggregate blood platelets, which in turn lead to the blocked of vessels.

The results listed in the (Tables 1 and 2) and curves (Figures 1 - 3) show that the wall shear stress rang is (0.1 to 0.52 pa), pressure gradient rang is (200 to 300 pa /m), viscosity rang is (0.004 to 0,009 pa.-s) and yield stress rang is (0.01 to 0.03 pa.).

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Table 1. Data on wall shear stress (τ_c) of core layer in the paired stenosis artery for different values of $\frac{\delta_1}{R_0}$, $\frac{\delta_2}{R_0}$, (μ_c/μ_p) 1 - 2.5 and separation factor with respect to the length of both stenosis. $z_1 = z_2 = 0.0002m$, m and $R_0 = 0.0002$m

$\frac{\delta_1}{R_0}=\frac{\delta_2}{R_0}$	WSS(pa) at $z_0 = 0$	WSS (pa) at$z_0 = z_1 = z_2$	WSS (pa) at $z_0 = z_1 + z_2$
0.100	0.088867	0.243046	0.345009
0.188	0.118931	0.264184	0.364651
0.277	0.145444	0.284900	0.383994
0.366	0.170049	0.305259	0.403150
0.455	0.193414	0.325312	0.422136
0.544	0.215889	0.345099	0.440969
0.633	0.237685	0.364651	0.459660
0.722	0.258942	0.383994	0.478222
0.811	0.279757	0.403150	0.496664
0.900	0.300200	0.422136	0.514995

$$1pa. = N/m^2 = 10dyn/cm^2$$

Table 2. Data on wall shear stress for different values of viscosity and different yield shear stress with peak separation and max. pressure gradient, $R_0 = z_1 = z_2 = 0.0002m$ and $\delta_1 = \delta_2 = 0.0001m$.

Viscosity (pa. s) $\times 10^{-3}$	yield stress (pa) $\times 10^{-2}$	WSS (pa)
4.0	1.0	0.275
4.5	1.2	0.292
5.1	1.4	0.309
5.6	1.6	0.326
6.2	1.9	0.342
6.7	2.1	0.358
7.0	2.3	0.373
7.8	2.5	0.389
8.4	2.7	0.404
9.0	3.0	0.419

$$1pa. = N/m^2 = 10dyn/cm^2$$

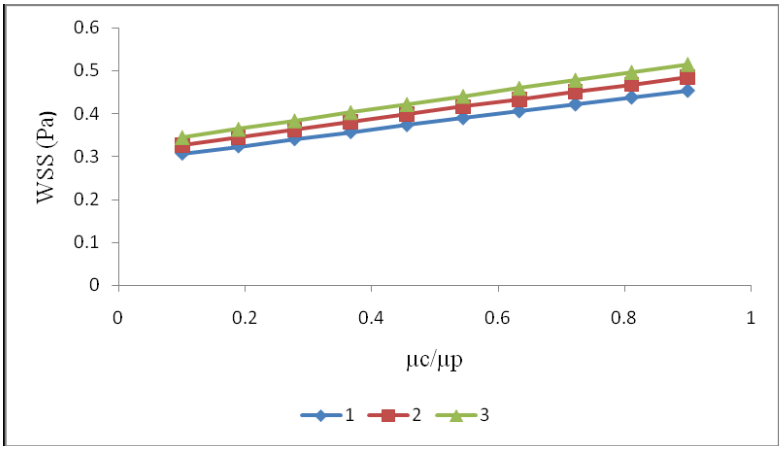

Figure 1. Effect of height stenosis on wss with (1) $\mu c/\mu p$ =1, (2) $\mu c/\mu p$=1.5, $\mu c/\mu p$=2 an(3) $\mu c/\mu p$=2.5

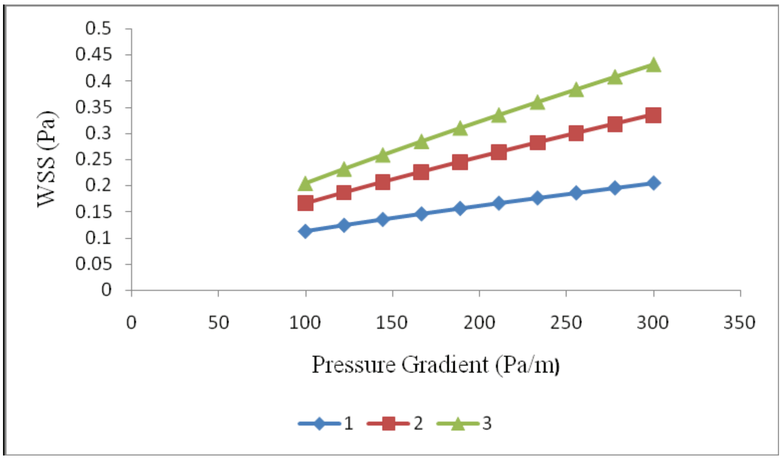

Figure 2. Effect of pressure gradient on wss with different values of seperation factor, z_0= 0.0002m, z_0 =0.0003m and z_0 =0.0004

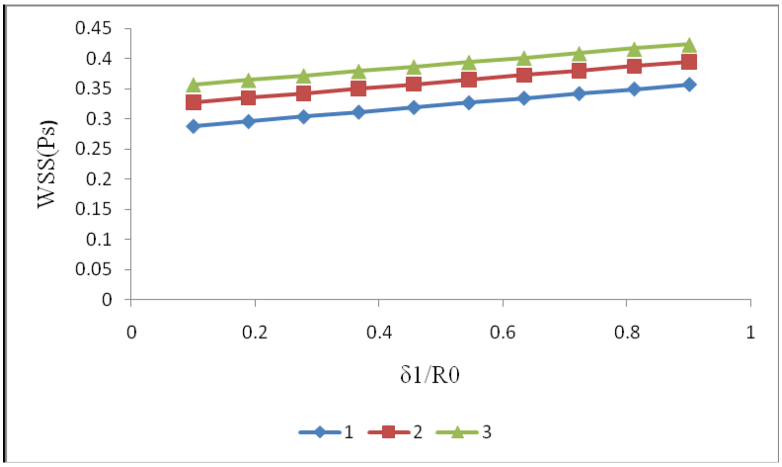

Figure 3. Effect of δ_1/R_0 on wss with deferent values of δ_2/R_0, (δ_2/R_0 =0.1, $\delta 2/R_0$= 0.5 and δ_2/R_0=0.9) at beg separation $z_0 = z_1 + z_2$

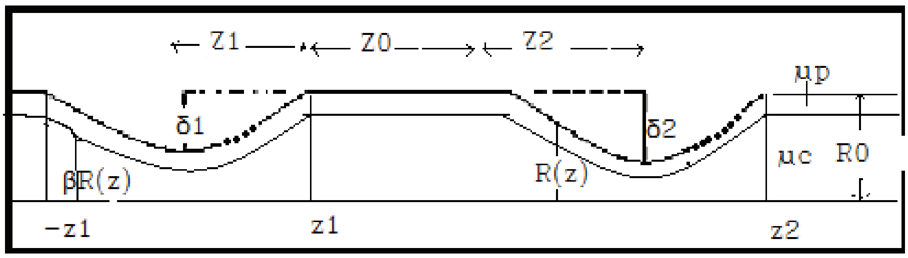

Figure 4. Physical Model and Coordinates System of Paired Stenosis