Solving the Multiobjective Two Stage Fuzzy Transportation Problem by Zero Suffix Method

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Abstract

In this paper, Multi objective two stage Fuzzy transportation problem is solved in a feasible method. For this solution zero suffix method is used in which the supplies and demands are trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. Here a numerical example is solved to check the validity of the proposed method.

Mathematics Subject Classifications: 90C08, 90C90

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1. Introduction

Transportation problems are one of the powerful frame works which ensures efficient movement and timely availability of the raw materials and finished goods. This transportation problem is a linear programming problem obtained from a network structure consisting of a defined numbers of nodes and arcs attached to them. Let us consider in which a production is to be transported from m sources to n destinations and their capacities $a_1, a_2, \cdots, a_m$ and $b_1, b_2, \cdots, b_n$ respectively. In addition to this there is a penalty C_{ij} associated with transporting unit of production from source i to destination j this penalty may be either cost or delivery time or safety delivery.

A variable x_{ij} represents the unknown quantity to be shipped from source i destination j. But in general the real life problems are modeled with multi-objective which are measured in difference scales and at the same time in conflict. In some situations due to storage constraints designations are unable to receive the quantity in excess of their minimum demand. After consuming the parts of whole of this initial shipment they are prepared to receive the excess quantity in the second stage. According to Sonia and Rita Malhotra in such situations the product transported to destination has two stages. Just enough of the product is shipped in stage I so that the minimum requirements of the destinations are satisfied and having done this the surplus quantities (if any) at these sources are shipped to destinations according to cost consideration. In both stages the transportations of the product from sources to destinations is done in parallel.

Lot of efficient algorithms had been developed for solving the transportation problems when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in an exact manner. For example the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors. To deal quantitatively with not exact information in making decisions, Bellman and Zadeh and Zadeh introduce the notion of fuzziness. Since the transportation problem is almost a linear programme one straightforward idea is to apply the existing fuzzy linear programming techniques to the fuzzy transportation problems.

Unfortunately most of the existing techniques only provide crisp solutions. The method of Julien and Parraet al is able to find the possibility distribution of the objective values provided all the inequality constraints are of " $\leq$ " type or " $\geq$ " type. However due to the structure of the transportation problem in some cases, their method requires the refinement of the problem parameters to be able to derive bounce of the objective value. There are also some studies discussing the fuzzy transportation problem. Chanas et al, investigates the transportation problems with fuzzy supplies and demands and solve them via the parametric programming techniques in terms of Bellman-Zadhe criterion.

Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree.

Chanas and Kuchta discuss the type the transportation problems with fuzzy cost coefficients and transforms the problem to bicriterial transportation with crisp objective function. Their method is able to determine the efficient solutions of the transformed problem, unless only crisp solutions are provided. Verma et al applied the fuzzy programming technique with hyperbolic and exponential membership function to solve a multi-objective transportation problem, the solution derived is a compromise solution. Similarly to the method Chanas and Kuchta, only crisp solution is provided. Obviously when the cost coefficients or the supply and the demand quantities are fuzzy numbers, the total transportation cost will be fuzzy as usual.

This paper finds the best compromise solution among the set of feasible solution for the multi-objective two stage transportation problem using zero suffix method. To illustrate the proposed method, an example is used. Finally some conclusions are obtained from the discussion.

2. Definitions

2.1 Fuzzy Number

A real fuzzy number $\bar{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\bar{a}}$ satisfying the following conditions

- $\mu_{\bar{a}}$ is continuous from R to the closed interval $[0, 1]$
- $\mu_{\bar{a}}$ is strictly increasing and continuous on $[a_1, a_2]$
- $\mu_{\bar{a}}$ is strictly decreasing and continuous on $[a_2, a_3]$

where a_1, a_2, a_3 and a_4 are real numbers, and the fuzzy denoted by $\bar{a} = [a_1, a_2, a_3, a_4]$ is called fuzzy trapezoidal number.

2.2 Trapezoidal Number

The fuzzy number $\bar{a}$ is a trpezoidal number, denoted by $[a_1, a_2, a_3, a_4]$ its membership function $\mu_{\bar{a}}$ is given by fig 1.

< Figure 1 >

2.3 α-level set

The α-level set of the fuzzy number $\bar{a}$ and $\bar{b}$ is defined as the ordinary set $L_\alpha(\bar{a}, \bar{b})$ for which the degree of their memebership function exceeds the level $\alpha \in [0, 1]$

$$L_\alpha(\bar{a}, \bar{b}) = \{a, b \in R^m / \mu_{\bar{a}}(a_i, b_j) \geq \alpha, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n\}$$

2.4 Compromise Solution

A feasible vector $x^* \in S$ is called a compromise solution of P_1 iff $x^* \in E$ and $F(x^*) \leq_{x \in S}^{\wedge} F(x)$ where $\wedge$ stands for 'minimum' and E is set of feasible solutions. From a practical point of view the knowledge of the set of feasible E is not always necessary. In such a case, a procedure is needed to determine a compromise solution. The purpose of this paper is to present a fuzzy programming approach to find an optimal compromise solution of a transportation problem with several objective in which the quantities are transported in two stages. Numerical example is given to illustrate the approach.

3. Theoretical Development

Let $\bar{b}_j$ be the minimum fuzzy requirement of a homogeneous product at the destination j and $\bar{a}_i$ the fuzzy availability of the same at source i. $F^k(x) = \{F^1(x), F^2(x), \cdots, F^n(x)\}$ is a vector of K objectives and the superscript of both $F^k(x)$ and C_{ij}^k are used to identify the number of objectives functions, without loss of generality it will be assumed in the whole paper that $a_i > 0 \forall i, b_j > 0 \forall j, c_{ij}^k > 0 \forall i, j$ and $\sum_i \bar{a}_i = \sum_j \bar{b}_j$. The Multi-objective Tow-stage Fuzzy Cost Minimization Transportation Problem (MOTSFTP) deals with supplying the destinations their minimum requirement is state-I and the quantity $\sum_i \bar{a}_i - \sum_j \bar{b}_j$ is supplied to the destination is stage-II from the sources which have surplus quantity left after the completion of stage-I, Mathematically stated the stage-I problem is

$$\min_{x \in s_1}[F^k(x)] = \min_{x \in s_1}[\max_{|x|}(C_{ij}^k(x_{ij}))] \tag{1}$$

Where the set S_1 is given by

$$S_1 = \left\{ \begin{array}{l} \sum_{j=1}^{n} x_{ij} \leq \bar{a}_i, i = 1, 2, \cdots, m \\ \sum_{i=1}^{m} x_{ij} = \bar{b}_j, j = 1, 2, \cdots, n \end{array} \right\}$$

$\bar{x}_{ij} \geq 0 \forall (i, j)$ Corresponding to a feasible solution $X = (x_{ij})$ of the stage-I problem, the set

$S_2 = \{\bar{X} = (\bar{x}_{ij})\}$ of feasible solutions of the stage-II problem is given by

$$S_2 = \left\{ \begin{array}{l} \sum\limits_{j=1}^{n} x_{ij} \leq \bar{a}_i, i = 1, 2, \cdots, m \\ \sum\limits_{i=1}^{m} x_{ij} \geq \bar{b}_j, j = 1, 2, \cdots, n \end{array} \right\}$$

$\bar{x}_{ij} \geq 0, \forall (i, j)$ where $\bar{a}_i$ is the quantity available at the ith source on completion so the stage-I, that is $\bar{a}'_{ij} = \bar{a}_i - \sum\limits_j x_{ij}$. Clearly $\sum\limits_i \bar{a}'_i = \sum\limits_i \bar{a}_i - \sum\limits_j \bar{b}_j$. Thus the stage-II problem would be mathematically formulated as:

$$\min_{x \in s_2}[F^k(x)] = \min_{x \in s_2}[\max_{|x|}(C^k_{ij}(x_{ij}))] \tag{2}$$

We aim at finding that schedule $x = (x_{ij})$ of the stage-I problem corresponding to which the optimal cost for stage-II is such that the sum of the shipment if the least. The Multi-objective two stage fuzzy cost minimizing transportation problem can, therefore, be stated as,

$$MinF^k(x) = \min_{x \in s_1}[C^k_1(x) + \min_{x \in s_2}\{C^k_2(x)\}] \tag{3}$$

Also from a feasible solution of the problem (3) can be obtained. Further the problem (3) can be solved by following fuzzy cost minimizing transportation problem.

$$\text{P(1)} : \min_{x \in s_2}[F^k(x')] = \min_{x' \in s_2}[\max_{|x|}(C^k_{ij}(x'_{ij}))] \tag{4}$$

where S_2

$$S_2 = \left\{ \begin{array}{l} \sum\limits_{j=1}^{n} x'_{ij} = \bar{a}_i, i = 1, 2, \cdots, m \\ \sum\limits_{i=1}^{m} x'_{ij} = \bar{b}_j, j = 1, 2, \cdots, n \end{array} \right\}$$

$x'_{ij} \geq 0 \forall (i, j)$, where $\bar{a}_i$ and $\bar{b}_j$ represent fuzzy parameters involved in the constraints with their membership functions for $\mu_{\bar{a}}$ a certain degree α together with the concept of α level set of the fuzzy numbers $\bar{a}_i, \bar{b}_j$. Therefore problem two stage MOFCMTP can be understood as following non fuzzy α-general two stage transportation problem (α-two stage MOFCMTP).

$$S = \left\{ \begin{array}{l} \sum\limits_{j=1}^{n} x_{ij} = \bar{a}_i, i = 1, 2, \cdots, m \\ \sum\limits_{i=1}^{m} x_{ij} = \bar{b}_j, j = 1, 2, \cdots, n \end{array} \right\}$$

$a_i, b_j \in L\alpha(\bar{a}_i, \bar{b}_j)$

Where $a_i, b_j \in L\alpha(\bar{a}_i, \bar{b}_j)$ are the α-level set of the fuzzy number $(\bar{a}_i, \bar{b}_j)$, let $x(\bar{a}_i, \bar{b}_j)$ denote the constraint set of the problem and supposed to be non empty. On the basis of the α-level set of the fuzzy numbers, we give the concept of α-optimal solution in the following definition.

A point $x^* \in x(\bar{a}_i, \bar{b}_j)$ is said to be α-optimal solution (α-two stage FCMTP), if and only if there does not exist another $x, y \in x(a, b), a, b \in L_\alpha(\bar{a}_i, \bar{b}_j)$, such that $c_{ij}x_{ij} \leq c_{ij}x^*_{ij}$ with strict inequality holding for at least one c_{ij} where for corresponding values of paramenters $(\bar{a}_i, \bar{b}_j)$ are called α-level optimal parameters.

The problem (α-two stage MOFCMTP) can be re written in the following equivalent form (α'-two stage MOFCMTP)

$$S = \left\{ \begin{array}{l} \sum\limits_{j=1}^{n} x_{ij} = \bar{a}_i, i = 1, 2, \cdots, m \\ \sum\limits_{i=1}^{m} x_{ij} = \bar{b}_j, j = 1, 2, \cdots, n \end{array} \right\}$$

$h_i^\circ \le a_i \le H_i^\circ, h_j^\circ \le b_j \le H_j^\circ$

$x_{ij} \ge 0 \forall i, j$

It should be noted that the constraint $a_i, b_j \in L\alpha(\bar{a}_i, \bar{b}_j)$ has been replaced by the constraint $h_i^\circ \le a_i \le H_i^\circ$ and $h_j^\circ \le b_j \le H_j^\circ$

Where h_i° and H_i° and h_j° and H_j° are lower and upper bounds and a_i, b_j are constants.

The parametric study of the problem (α'-two stage MOFCMTP) where h_i°,H_i° and h_j°,H_j°, are assumed to be parameters rather than constants and (renamed h_i°,H_i° and h_j°,H_j°) can be understood as follows.

Let $X(h, H)$ denoted the decision space of problem (α'-two stage FCMTP) defined by

$X(h, H) = (X_{ij}, a_i, b_j) \in R^{n(n+1)}$

$a_i - \sum_j x_{ij} \ge 0, b_j - \sum_i x_{ij} \ge 0$

$H_i - a_i \ge 0, H_j - b_j \ge 0$

$a_i - h_i \ge 0, b_j - h_j \ge 0, x_{ij} \ge 0, i \in I, j \in J$

4. Zero Suffix Method

We, now introduce a new method called the zero suffix method for finding an optimal solution to the transportation problem.

The zero suffix method proceeds as follows.

Step 1: Construct the transportation table.

Step 2: Subtract each row entries of the transportation table from the corresponding row minimum after that subtract each column entries of the transportation table from the corresponding column minimum.

Step 3: In the reduced cost matrix there will be atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

$$\text{Therefore S} = \frac{\text{Add the costs of nearest adjacent sides of zero which are greater than zero}}{\text{No. of costs added}}$$

Step 4: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i, b_j\}$ and supply to that demand maximum possible.

Step 5: After the above step, the exhausted demands (column) or supplies(row) to be trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat Step 2.

Step 6: Repeat Step 3 to Step 5 until the optimal solution is obtained.

5. Solution Algorithm

Step 1: Construct the Transportation problem

Step 2: Supply and Demand are fuzzy number (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) in the formulation Problem(two stage MOFCMTP).

Step 3: Convert the problem (α-two stage MOFCMTP) in the form of the(α' two stage MOFCMTP)

Step 4: Formulate the (α'-two stage FCMTP) in the parametric form.

Step 5: Apply the zero suffix method to get optimal value in stage-I and stage-II.

Step 6: The optimal value of the objective function of the problem is Min$(C_1 + C_2)$

6. Numerical Example

Consider the following two stage cost minimizing transportation problem. Here supplies and demands are trapezoidal fuzzy numbers.

$a_1 = (4, 5, 7, 8), a_2 = (6, 7, 8, 9), a_3 = (5, 6, 7, 8), a_4 = (4, 6, 8, 9)$ and $b_1 = (1, 2, 4, 5), b_2(4, 5, 6, 7), b_3 = (3, 4, 5, 7), b_4 = (4, 5, 6, 7), b_5 = (2, 3, 4, 5),$

$b_6 = (3, 4, 5, 6).$

Consider the α-level set to be $\alpha = 0.75$, then we get $4.5 \le a_1 7.5, 6.5 \le a_2 \le 8.5, 5.5 \le a_3 \le 7.5, 5.0 \le a_4 \le 8.5, 1.5 \le$

$b_1 \le 4.5, 4.5 \le b_2 \le 6.5, 3.5 \le b_3 \le 6.0, 4.5 \le b_4 \le 6.5, 2.5 \le b_5 \le 4.5, 3.5 \le b_6 \le 5.5.$

The α-optimal parameters are

$a_1 = 6, a_2 = 8, a_3 = 7, a_4 = 7$

$b_1 = 3, b_2 = 5, b_3 = 5, b_4 = 6, b_5 = 4, b_6 = 5$

Penalties:

$$C^1 = \begin{bmatrix} 2 & 3 & 5 & 11 & 4 & 2 \\ 4 & 7 & 9 & 5 & 10 & 4 \\ 12 & 25 & 9 & 6 & 26 & 12 \\ 8 & 7 & 9 & 24 & 10 & 8 \end{bmatrix} \text{ and } C^2 = \begin{bmatrix} 1 & 2 & 7 & 7 & 4 & 2 \\ 1 & 9 & 3 & 4 & 5 & 8 \\ 8 & 9 & 4 & 6 & 6 & 2 \\ 3 & 4 & 9 & 10 & 5 & 1 \end{bmatrix}$$

Stage - I

We assign $a_1 = 3, a_2 = 4, a_3 = 3, a_4 = 3$

$b_1 = 1, b_2 = 2, b_3 = 3, b_4 = 3, b_5 = 2, b_6 = 2$

With respect to C^1, apply the zero suffix method which gives following allocation

$X_{13} = 1, X_{15} = 2, X_{21} = 1, X_{22} = 1, X_{26} = 2, X_{34} = 3, X_{43} = 3$ and Minimum $Z = 75$

With respect to C^2, apply the zero suffix method which gives following allocation

$X_{12} = 2, X_{15} = 1, X_{21} = 1, X_{24} = 3, X_{33} = 3, X_{45} = 1, X_{46} = 2$ and Minimum $Z = 40$

Stage -II

We assign $a_1 = 3, a_2 = 4, a_3 = 4, a_4 = 4$

$b_1 = 2, b_2 = 3, b_3 = 2, b_4 = 3, b_5 = 2, b_6 = 3$

With respect to C^1, apply the zero suffix method which gives following allocation

$X_{12} = 1, X_{15} = 2, X_{21} = 2, X_{22} = 2, X_{33} = 1, X_{34} = 3, X_{43} = 1, X_{46} = 3$ and Minimum $Z = 93$

With respect to C^2, apply the zero suffix method which gives following allocation

$X_{12} = 3, X_{21} = 2, X_{24} = 2, X_{33} = 2, X_{34} = 1, X_{35} = 1, X_{45} = 1, X_{46} = 3$ and Minimum $Z = 44$

The optimal value of the objective function is obtained function is obtained by combining stage-I and stage-II, therefore Minimum $F^1(x) = 75 + 93 = 168$ and Minimum $F^2(x) = 40 + 44 = 84$

7. Conclusion

Transportation models have wide applications in logistics and supply chain for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. In this paper zero suffix method is used to determine the optimal compromise solution for a multi-objective two stage fuzzy transportation problem, in which supplies, demands are trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. In real world applications, the parameters in transportation problem may not be known precisely due to some uncontrollable factors .If the obtained results are crisp values then it might lose some helpful information. Since the objective value is expressed by membership function rather than by a crisp value, more information is provided for making decisions.

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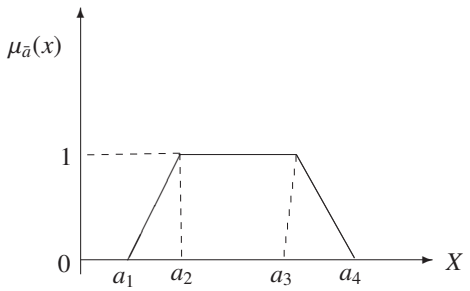

Figure 1. Membership function of a fuzzy number $\bar{a}$