On Tape Graphs

M. El-Ghoul

Mathematics Department, Faculty of Science

Tanta University, Tanta, Egypt

Tel: 104-684-812 E-mail: m.elghoul@msn.com

H. El-Zohny

Mathematics Department, Faculty of Science

Al-Azahar University, Cairo, Egypt

Tel: 168-308-166 E-mail: elzohny_7@yahoo.com

M. M. Khalil

Mathematics Department, Faculty of Science

Al-Azahar University, Cairo, Egypt

Tel: 106-518-009 E-mail: mona.khalil@msn.com

Abstract

In this paper we will introduce new type of graphs, when vertices of these graphs are appearance like line and edges of these graphs are appearance like tape or ribbon. We introduce types of representation of the new graph by the adjacent and the incidence matrices and we will discuss their transformations.

Keywords: Graphs, Transformations

1. Definitions and Background

(1) Abstract graphs: An abstract graphs G is a diagram consisting of a finite non empty set of the elements, called "vertices" denoted by $V(G)$ together with a set of unordered pairs of these elements, called "edges" denoted by $E(G)$. The set of vertices of the graph G is called "the vertex –set of G" and the list of edges is called "the edge –list of G" (Gibbons A.,1985), (Giblin P.J.,1977).

(2) Adjacency and incidence: let v and w be vertices of a graph. If v and w are joined by an edge e. then v and w are said to be adjacent. Moreover, v and w are said to be incident with e,and e is said to be incident with v and w (Wilson R.J.,1972).

(3) The adjacency matrix: let G be a graph without loops, with n-vertices labeled 1,2,3,. . . ., n. The adjacency matrix $A(G)$ is the n x n matrix in which the entry in row i and column j is the number of edges joining the vertices i and j (Wilson R.J.,1972).

(4) The incidence matrix: let G be a graph without loops, with n-vertices labeled 1,2,3,. . . n and m edges labeled 1,2,3,. . . ., m. the incidence matrix $I(G)$ is the n x n matrix in which the entry in row i and column j is 1 if vertex i is incident with edge j and 0 otherwise (Gross J.L.,Tucker T.W. , 1987), (Wilson R.J., Watkins J.J. ,1990).

(5) Isometric folding: let M and N be smooth connected Riemannian manifolds of dimensions m and n respectively such that $m < n$. A map $f: M \longrightarrow N$ is said to be an isometric folding of M into N iff for every piecewise geodesic path $\gamma: J \rightarrow M$ the induced path $f \circ \gamma : J \rightarrow N$ is a piecewise geodesic and of the same length as γ (Robertson S.A.,1977).

(6) Folding and unfolding of graph:

(a) Let $f : G \rightarrow \overline{G}$ be a map between any two graphs G and (not necessary to be simple) such that if $(u,v) \in G,(f(u), f(v)) \in \overline{G}$. Then f is called a "topological" of G to provided that $d(f(u), f(v)) < d(u, v)$ (Giblin P.J., 1977).

(b) Let $g : G \rightarrow \overline{G}$ be a map between any two graphs G and (not necessary to be simple)such that if (u,v)$\in G$,$(g,(u), g(v)) \in \overline{G}$ .Then g is called a "topological unfolding" of to$\overline{G}$ provided that $d(g(u), g(v)) > d(u, v)$ (El-Ghoul M., 2007).

(7) Retracts: A subset A of a topological space X is called a "retract" of A if there exists a continuous map $r:X \rightarrow A$ (called a retraction) such that $r(a) = a\ \forall a \in A$, where A is closed and X is open. In other words, a retraction is a continuous map of a space onto a subspace leaving each point of the subspace fixed (El-Ghoul M.,El-Ahmady A., Rafat H., 2004).

2. Main Results

Now we will define and discuss the tape graph and some transformations on this new graph, and that will be represented by matrices.

2.1 Definitions

2.1.1 The tape graph G is a diagram consisting of a finite non empty set of the elements with "line or curve" shape called "vertices" denoted by *V(G)* together with elements, with "tape" shape called "edges" denoted by *E(G)*.

The matrix representation of geometric tape graph considers the geometric graph G (v^0v^1) see Fig. (1).

< Figure 1 >

It's adjacent and incidence matrices are:

$$A(G) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2, \ I(G) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_2.$$

2.1.2 Loop and Multiple Edges

A loop is an edge joining a vertex to itself see Fig. (2-a).We says that the tape graph has multiple edges if in the tape graph two or more edges joining the same pair of vertices see Fig.(2-b).

< Figure 2-a >, < Figure 2-b >

2.1.3 Connected tape graph

We can combine two tape graphs to make a larger graph. If the two tape graphs are $G_1 = (V(G_1), E(G_1))$ and $G_2 = (V(G_2), E(G_2))$, where $V(G_1)$ and $V(G_2)$are disjoint,then their union $G_1 \cup G_2$ is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge family $E(G_1) \cup E(G_2)$ see Fig. (3). A tape graph G is connected if it cannot be expressed as the union of two graphs, and disconnected otherwise.i.e a tape graph G is connected if there is a path in G between any given pair of vertices, otherwise it is disconnected. Every disconnected graph can be expressed as the union of connected graphs, each of which is a component of$G$. For example; a graph with three components is shown in Fig (4).i.e. a tape graph is connected if and only if there is a path between each pair of vertices.

< Figure 3 >, < Figure 4 >

2.1.4 Complete tape graphs

A simple tape graph in which each pair of distinct vertices are adjacent is a complete tape graph. We denote the complete tape graph on n vertices k, k has $n\,(n$-1)/2 edges see Fig .(5).

< Figure 5 >

2.1.5 Tree tape graphs

A tree tape graph is connected tape graph with only one path between each pair of vertices containing no cycles see Fig. (6).

< Figure 6 >

It's adjacent and incidence matrices are:

$$A(G)=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}_2, I(G) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_2.$$

2.1.6 Tape graph in higher dimension

We can represent the tape graph in higher dimension by matrices as the following:

1- In dimension two: $A(G)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2, I(G) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_2.$

2-In dimension three: $A(G)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_3, I(G) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_3.$

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3- In dimension four: $A(G)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_4$, $I(G)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}_4$.

4- In dimension n : $A(G)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_n$, $I(G)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}_n$.

2.2 Folding of geometric tape graph

Theorem 2.2.1

The limit of foldings of tape graph G into itself is a tape graph or a simple graph.

Proof:

case(1): let $f_1 : G \to G$,such that f (tape)=tape then $f_2 : f_1(G) \to f_1(G)$ is a tape graph,..., $f_n : f_{n-1}(f_{n-i}(G)) \to f_{n-1}(f_{n-i}(G))$ is a tape graph, $\lim_{n \to \infty} f_n(G)=K$, K=one tape see Fig.(7).

< Figure 7 >

Case (2): Let $\overline{f_1}: G \to (G)$, such that $\overline{f_1}$ (any tape)= tape of less than area. Then $\overline{f_2}(T)=T^1$, $\overline{f_3}(T^1)=T^2$,..., $\overline{f_n}(T^{n-2})=T^{n-1}$,..., $\lim_{n\to\infty} \overline{f_n}(T^{n-2})$=graph of 1-dimension see Fig.(8).

< Figure 8 >

Theorem 2.2.2

The end of the limits of foldings of a tape graph G into itself coincides with the simple graph.

Proof

Consider the geometric tape graph $G(V, E)$ where $V(G)=\{V^0, V^1\}$ and $E(G)=\{e^1\}$.Now let the folding as, $f_1 : G \to G_1$, $f_2 : G_1 \to G_2$, $f_3 : G_2 \to G_3$, ..., $\lim_{n\to\infty} f_n(G_{n-1})=G_n$ which is the usual loop graph see Fig.(9).

Also by matrix:

$$\underset{A}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \cdots\cdots \xrightarrow{\lim_{n\to\infty} f_n} \underset{A_n}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}},$$

And for incidence:

$$\underset{A}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \cdots\cdots\cdots \xrightarrow{\lim_{n\to\infty} f_n} \underset{A_n}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}.$$

In higher dimensions:

$$\underset{A}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_n} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_n} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_n} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_n} \cdots\cdots \xrightarrow{\lim_{n\to\infty} f_n} \underset{A_n}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{n-1}},$$

And for incidence:

$$\underset{A}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \cdots\cdots\cdots \xrightarrow{\lim_{n\to\infty} f_n} \underset{A_n}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{n-1}}.$$

< Figure 9 >

Theorem 2.2.3

The end of limits of the curvature foldings f_{k_n} of a tape graph G is the new graph,(circles, tubes).

Proof:

Let $f : G \to G$ such that f restricted on the curvature k , $f_{k_1} : G \to G$, $f(G)$ such that curvature $f(G)>$ curvature G see Fig. (10).

< Figure 10 >

we will arrive to another new graph it is tube graph with circles vertices and tubes edges.

Also by matrix:

$$\underset{A}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_2} \ldots\ldots \underset{A_m}{\xrightarrow{\lim_{n\to\infty} f_{k_m}}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_3,$$

And for incidence:

$$\underset{A}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_1} \underset{A_1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_2} \underset{A_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \xrightarrow{f_3} \underset{A_3}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}_2} \ldots\ldots\ldots \underset{A_m}{\xrightarrow{\lim_{n\to\infty} f_{k_m}}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_3.$$

2.3 Retraction of geometric tape graph

There are many types of retractions of the new tape graph. Consider the geometric tape graph *G (V, E)* where*V(G)* = $\{V^0, V^1\}$ and *E(G)*= $\{e^1\}$.

Now let we consider the retractions as:

(a) Vertices retractions:

$r_1 : (G - V^0) \to G_1$, and $r_2 : (G - V^1) \to G_2$, note that; G_1 is not a graph but lim $r_{1_n} = v^1$ is a graph also lim r_{2_n} is a graph see Fig.(11) .

< Figure 11 >

The retraction of a tape graph becomes a null tape graph, its adjacent and the incidence matrices are given by: *A(G)* $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, *I(G)* = **Φ**.

(b) Edges retractions: If we remove an interior point from the edge vertically, i.e $r_3 : (G - e) \to G_3$, where $e \in e^1$. So G_3 is not a graph see Fig.(12).

< Figure 12 >

Theorem 2.3.1

Retraction for the geometric tape graph induces a new tape graph geometric or simple graph or null graph.

Proof:

Let R_1 *:(G-p)→ (G-p),p* is a point inside a tape, then the retraction is not a tape see Fig. (13).

< Figure 13 >

If p_i is a point in every tape, then R_2 :*(G - p_i) (G - p_i)*.Then R_2 is a new graph of one dimension and (degree of vertex=degree of edges=1) seeFig.(14).

< Figure 14 >

If R_3*; (G –* 1-boundary of every tape). Then R_i *:(G -b_i) (G -b_i)*, such that *R (G -b_i)*=$\overline{G}$, where $\overline{G}$ is a simple graph see Fig.(15).

< Figure 15 >

Lemma 2.3.1

The end of the limit of retractions of the tape graph is a null graph.

Applications:

(1) A ribbon of cassette recorder it is geometric tape graph.

(2) A ribbon of train is a connected tape graph.

(3) Any empty tube is a tape graph in two dimensions, and full tube is a tape graph in three dimensions.

(4) A fork is a tree tape graph.

(5) A Prickly pear is a tree tape graph.

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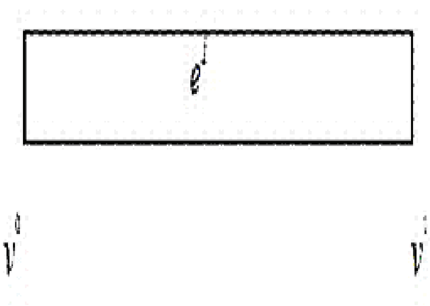

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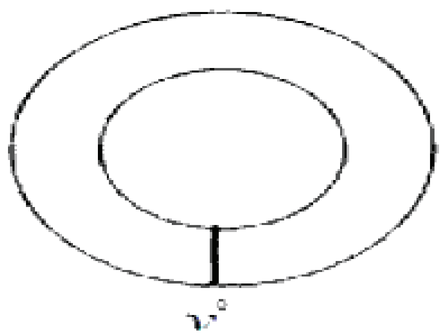

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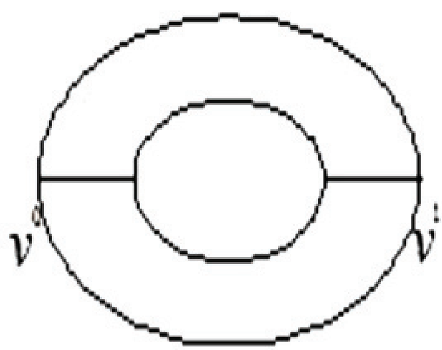

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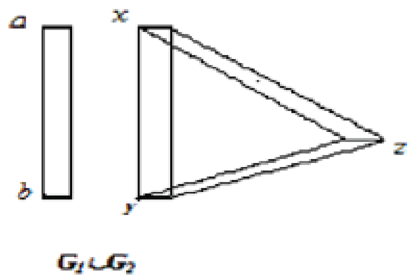

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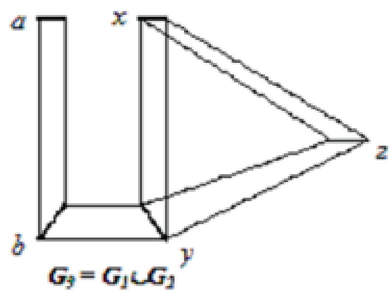

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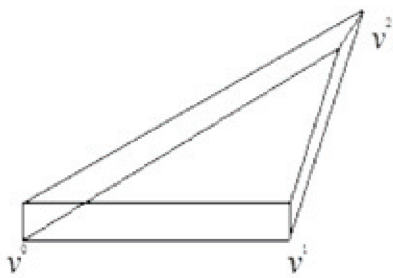

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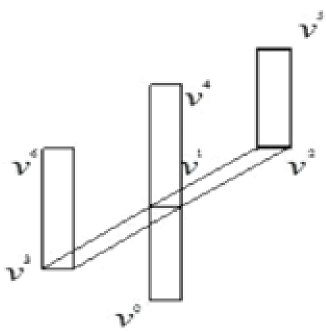

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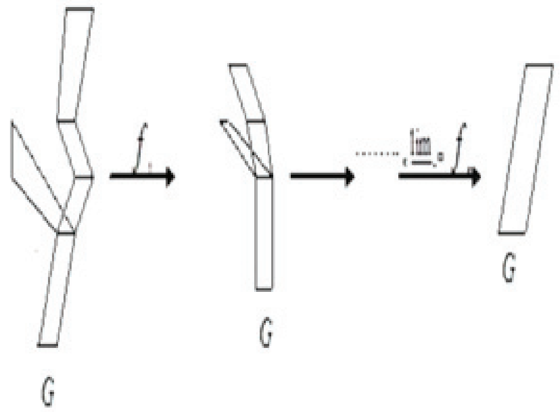

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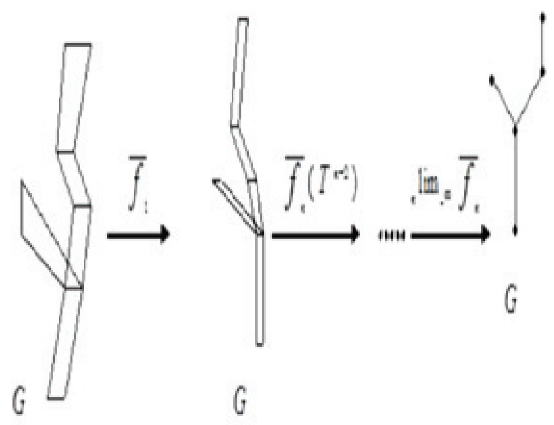

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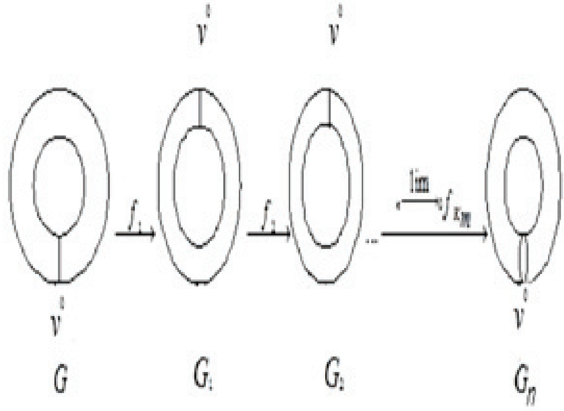

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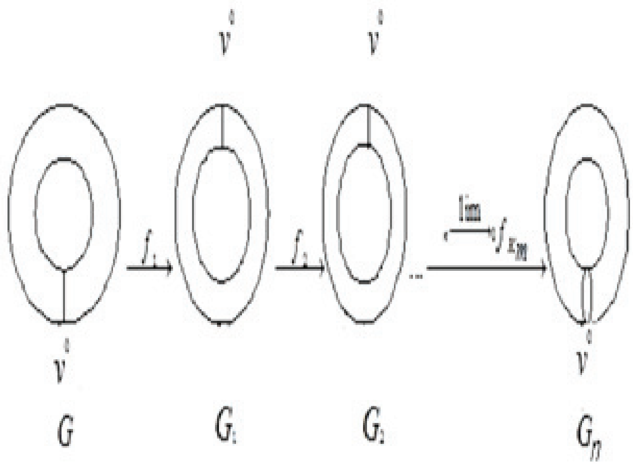

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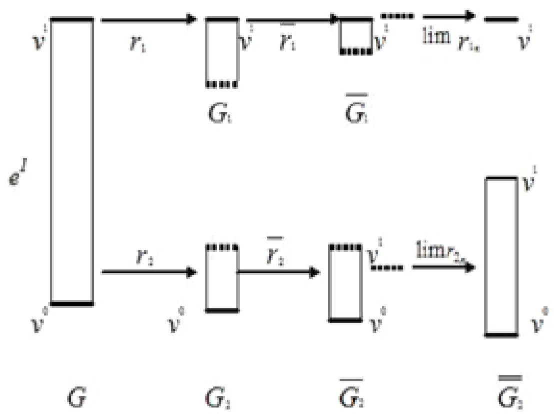

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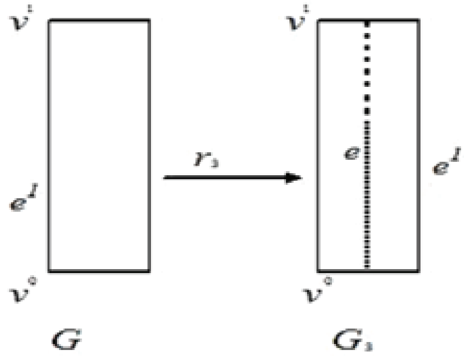

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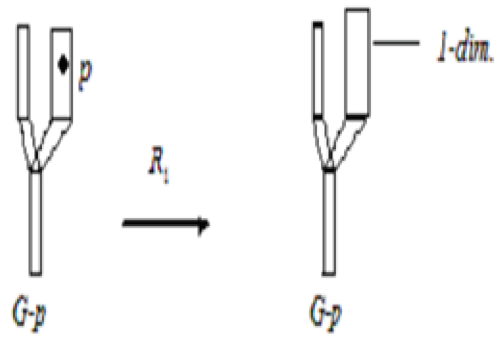

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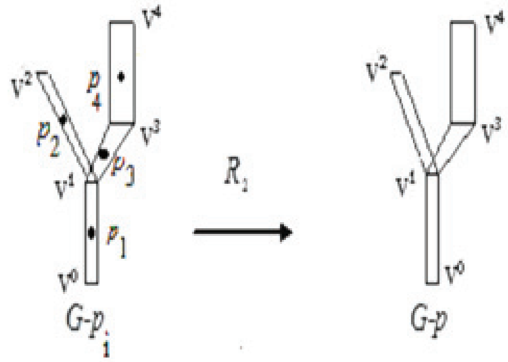

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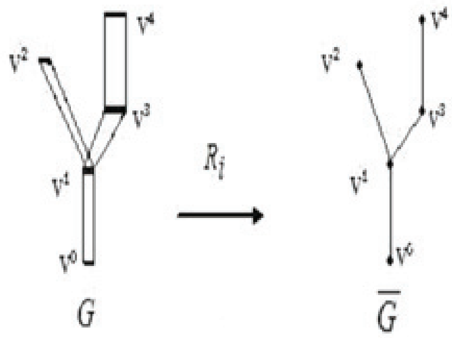

Figure 15.