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An Analytic Soluion of Fingering Phoenomenon Arising in Fluid Flow through Porous Media by Using Techniques of Calculus of Variation and Similarity Theory

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Abstract

The present paper represents an analytical solution of fingering phonomenon arising in double phase flow through homogeneous media under certain initial & boundary condition using techniques of calculus of variation and similarity theory. The numerical and graphical representation of solution has been given the graph of saturatin $F(\eta)$ of injected liquid, is increasing after $\eta = 0.5$ for $t > 0$, which indicates that when injected liquid entries into native liquid at common-interface, then suddenly the native liquid enters into injected liquid due to difference in wettability. Hence initial saturation will decrease and then after $\eta > 0.5$ the saturation uniformly increases parabolically which is physically consistent with the available theory.

Keywords: Fingering phenomenon, Double phase flow, Similarity theory, Capillary force, Calculus of Variation, Rayleigh-Ritz method

Nomenclature

V_W	Seepage velocity of Water
V_0	Seepage velocity of Oil
ρ_W	Density of Water
ρ_0	Density of Oil
α	The inclinati of the bed,
g	Acceleratin due to gravity,
P_c	Capillary pressure
P_0	Pressure of Oil
P_W	Pressure of Water
P	Mean Pressure
S_W	Saturation of Water
S_0	Saturation of Oil
$\eta(x,t)$	The interface curve
K	Intrinsic permeability

1. Introduction

It is a very well-known physical fact that when a fluid, contained in a porous media, is displaced by another of lesser viscosity, instead of regular displacement of whole front, perturbations (fingers) occur which shoot through the porous medium at relatively great speed. This phenomenon of occurrence of instabilities is called fingering.

Immiscible flow of heavy oil in a porous formation by high temperature pressurized water has been numerically studied. The physical region is a square domain in the horizontal plane with low and high pressure points at the opposite corners along one of the diagonals. Water, the invading fluid, when introduced at high pressure displaces the *in situ* oil towards the low-pressure production zone. The extent of displacement of oil by water through the porous medium in a given amount of time and the appearance of preferential flow paths (fingers) is the subject of the present investigation.

The resistance to water-oil movement arises from the viscous forces in the fluid phases and the capillary force at their interface. Based on their relative magnitudes, various forms of displacement mechanisms can be realized. As the viscosity ratio of heavy oil to water is large, viscous forces in the oil phase become dominant and constitute the major factor for controlling the flow distortions in the porous formation. A mathematical model that can treat the individual fluid pressures, capillary effects and heat transfer has been employed in the present work. A fully implicit, two-dimensional numerical model has been used to compute the pressure and temperature fields. The domain decomposition technique has been adopted in the numerical solution since the problem is computationally intensive. Naturally occurring oil-rich reservoirs to which the present study is applicable are inhomogeneous and layered. A qualitative study has been carried out to explore the effect of permeability variations on the flow patterns. Numerical calculations show that non-isothermal effects as well as layering, promote the formation of viscous fingers and consequently the sweep efficiency of the high-pressure waterfront.

In the statistical treatment of fingering (Scheidegger, A.E., 1961) only average cross sectional area occupied by the fingers, is taken into account, the size and shape of the individual fingers are disregarded. Scheideger and Johnson (1961) introduce the idea of discussing the statistical behaviour of instabilities in homogeneous porous media and considered the phenomenon without the effecct of capillary pressure. Verma (1964) has examined the behaviour of fingering phenomenon in a displacement process through heterogeneous porous medium from statistical point of view. It has been shown that fingers may be stabilized in homogeneous media statistical view point by (Scheidegger, A.E., 1960) many authors, for example, Chouke (1959), Jecquard (1940), Verma (1924), have investigated this phenomenon with different aspects.

1.1 Formulation of Problem

Let water be injected with constant velocity into a dipping oil saturated porous medium of homogeneous physical characteristic. The displacement of oil by water gives rise to a well-developed finger flow as shown in Figure 1(a), 1(b).

< Figure 1(a) & Figure 1(b) >

From Darcy's law the seepage velocity of water V_w and oil V_0 can be written as

$$V_w = \frac{K_w}{\delta_w} K \left(\frac{\partial P_w}{\partial X} + \rho_w g \sin\alpha \right) \tag{1}$$

$$V_0 = -\frac{K_0}{\delta_0} K \left(\frac{\partial P_0}{\partial X} + \rho_0 g \sin\alpha \right) \tag{2}$$

where ρ_w and ρ_0 are constant densities of water and oil respectively, α is the inclination of the bed, g is acceleration due to gravity.

The equation of continuity of the phases is given by

$$P \frac{\delta S_w}{\partial_t} + \frac{\partial V_w}{\partial X} = 0 \tag{3}$$

$$P \frac{\delta S_0}{\partial_t} + \frac{\partial V_0}{\partial X} = 0 \tag{4}$$

From the definition of phase saturation we have

$$S_w + S_0 = 1 \tag{5}$$

The capillary pressure, which is defined as the pressure discontinuity of the following phases across the common interface is written as

$$P_c = P_0 - P_w \tag{6}$$

Or

$$\frac{\partial P_w}{\partial X} = \frac{\partial P_0}{\partial X} - \frac{\partial P_c}{\partial X} \tag{7}$$

The equation of motion for saturation can be obtained by substitution of the values of V_w and V_0 from equations (1) and (2) in equation (3) and (4) respectively. Thus we have

$$P \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial X} \left[\left(\frac{K_w}{\delta_w} k \right) \left(\frac{\partial P_w}{\partial X} + \rho_w g \sin\alpha \right) \right] \tag{8}$$

$$P \frac{\partial S_0}{\partial t} = \frac{\partial}{\partial X} \left[\left(\frac{K_0}{\delta_0} k \right) \left(\frac{\partial P_0}{\partial X} + \rho_0 g \sin\alpha \right) \right] \tag{9}$$

Substituting the value of $\frac{\partial P_w}{\partial X}$ from Eq. (7), Eq. (8) reduces to

$$P \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial X} \left[\left(\frac{K_w}{\delta_0} k \right) \left(\frac{\partial P_0}{\partial X} - \frac{\partial P_c}{\partial X} + \rho_w g \sin\alpha \right) \right] \tag{10}$$

Now considering Eq. (9) and (10)

$$P\frac{\partial}{\partial t}(S_w + S_0) = \frac{\partial}{\partial X}\left[\frac{\partial P_0}{\partial X}k\left(\frac{K_0}{\delta_0} + \frac{K_w}{\delta_w}\right) - \frac{K_w}{\delta_w}K\frac{\partial P_0}{\partial X} + g\sin\alpha K\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right)\right]$$

Using (5) we have

$$\frac{\partial}{\partial X}\left[\frac{\partial P_0}{\partial X}K\left(\frac{K_0}{\delta_0} + \frac{K_w}{\delta_w}\right) - K\frac{K_w}{\delta_w}K\frac{\delta P_0}{\delta X} + Kg\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right)\right] = 0 \tag{11}$$

Integrating both sides w.r.t. "x" we have

$$K\frac{\partial P_0}{\partial X}\left(\frac{K_0}{\delta_0} + \frac{K_w}{\delta_w}\right) - K\frac{K_w}{\delta_w}\frac{\partial P_0}{\partial X} + Kg\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right) = cons\tan t$$

or,

$$\left(\frac{K_w}{\delta_w} + \frac{K_0}{\delta_0}\right)\frac{\partial P_0}{\partial X}K - \frac{K_w}{\delta_w}K + gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right) = -q \tag{12}$$

where q is the constant of integration
or,

$$\frac{\partial P_0}{\partial X} = \frac{\left[-q + \frac{K_w}{\delta_w}\frac{\partial P_0}{\partial X} - gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_W\rho_W}{\delta_W}\right)\right]}{\left(\frac{K_W K}{\delta_W} + \frac{K_0 K}{\delta_0}\right)} \tag{13}$$

Substituting the value of $\frac{\partial P_0}{\partial X}$ from equation (13), equation (10) becomes (Appendix A).

$$P\frac{\partial S_W}{\partial t} + \frac{\partial}{\partial X}\left[\frac{\frac{K_0 K}{\delta_0}\left\{\frac{\partial P_0}{\partial X} + g\sin\alpha(\rho_0 - \rho_w)\right\}}{\left(1 + \frac{K_0\delta_W}{K_w\delta_0}\right)} + \frac{q}{\left(1 + \frac{K_0\delta_W}{K_w\delta_0}\right)}\right] = 0 \tag{14}$$

The value of the pressure of oil (P_0) can be written as

$$P_0 = (P_0 + P_W + P_0 - P_W)/2 = \frac{P_0 + P_W}{2} + \frac{P_0 - P_W}{2} = \overline{P} + \frac{1}{2}(P_0 - P_W) = \overline{P} + \frac{1}{2}P_c \tag{15}$$

Where $\overline{P}$ is the mean pressure. Since $\overline{P}$, the mean pressure is constant hence we have

$$\frac{\partial P_0}{\partial X} = \frac{1}{2}\frac{\partial P_c}{\partial X} \tag{16}$$

Substituting the value of $\frac{\partial P_0}{\partial X}$ in equation (12) we get (Appendix B)

$$q = \frac{\partial P_c}{\partial X}\left(\frac{1}{2}\frac{K_W K}{\delta_W} - \frac{1}{2}\frac{K_0 K}{\delta_0}\right) - g\sin\alpha\left(\frac{K_0 K\rho_0}{\delta_0} + \frac{K_W K\rho_W}{\delta_W}\right) \tag{17}$$

using the above value of "q" equation (14) reduces to (Appendix C)

$$P\frac{\partial S_W}{\partial t} + \frac{\partial}{\partial X}\left[\frac{K_w K}{\delta_w}\left\{\frac{1}{2}\frac{\partial P_0}{\partial X} - \rho_w g\sin\alpha\right\}\right] = 0 \tag{18}$$

1.2 A Special case study

For definitions of the mathematical analysis, we assume a standard form for the relationship between capillary pressure, permeability of water & permeability of oil with phase saturation as

$$K_w = S_w$$
$$K_0 = 1 - S_w \tag{19}$$
$$P_0 = B\left(S_w^{-\frac{1}{2}} - C\right)$$

where B and C are constant.

Using the above values equation (18) reduces as follow :

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial X}\left[\frac{S_w K}{\delta_W}\left\{\frac{-B}{4}S_w^{-\frac{3}{2}}\frac{\partial S_w}{\partial X} + \rho_w g\sin\alpha\right\}\right] = 0 \tag{20}$$

This is the equation of motion for saturation, with the boundary condition

$$\text{when } x = 0 \text{ then } S_w(0,t) = S_{w0}, \tag{21}$$

Since an exact solution of equation (20) is difficult to obtain due to non-linear forms there in hence we have obtain the approximate solution of the above problem by Rayleigh-Ritz method :

1.3 Solution procedure

Since capillary pressure is very small in porous media, hence assuming that the capillary pressure P_c as zero or B as zero, equation (20) reduces to

$$P\frac{\partial S_w}{\partial t} + \left[\frac{\rho_w g \sin\alpha}{\delta_w}\frac{\partial S_w}{\partial X}K\right] = 0$$

or,

$$\frac{\partial S_w}{\partial t} + \delta\frac{\partial S_w}{\partial X} = 0 \tag{22}$$

where

$$\delta = \rho_w g \sin\alpha\left(\frac{K}{P\delta_w}\right) \tag{23}$$

Using Birkhof's technique of one parameter group transformation, defined as

$$\begin{aligned} T_1 : X &= a^q x \\ T &= a^u t \\ \overline{S_w} &= a^v S_w \end{aligned} \tag{24}$$

where parameter $a \neq 0$, and q, u, v are real numbers to be determined.

Hence we have

$$\frac{\partial S_w}{\partial t} = a^{-v}\frac{\partial \overline{S_w}}{\partial T}\frac{\partial T}{\partial t} = a^{u-v}\frac{\partial \overline{S_w}}{\partial T}$$

$$\frac{\partial S_w}{\partial X} = a^{-v}\frac{\partial \overline{S_w}}{\partial X}\frac{\partial X}{\partial x} = a^{q-v}\frac{\partial \overline{S_w}}{\partial X}$$

Equation (21) using above value becomes

$$a^{u-v}\frac{\partial \overline{S_w}}{\partial T} + \delta a^{q-v}\frac{\partial \overline{S_w}}{\partial X} = 0 \tag{25}$$

equation (24) is absolute conformed invaraint under T_1 provided

$$q - v = u - v$$

or,

$$\frac{q}{u} - \frac{v}{u} = 1 - \frac{v}{u}$$

or,

$$\frac{q}{u} = 1 \tag{26}$$

and choosing an arbitrary constant "A" as follows

$$\frac{v}{u} = A \tag{27}$$

Thus the invariants of group T_1 is given by

$$\eta = \frac{x}{l} \tag{28}$$

and

$$F(\eta) = \frac{S_w(x,t)}{t^A} \tag{29}$$

or,

$$S_w(x,t) = t^A F(\eta)$$

$$S_w = t^A F$$

$$\frac{\partial S_w}{\partial t} = At^{A-1}F + t^A F'\left(-\frac{x}{t^2}\right),$$

where dash represent differentiations w.r.t. 'η'

$$\frac{\partial S_w}{\partial t} = t^{A-1}(AF - \eta F')$$

and

$$\frac{\partial S_w}{\partial x} = t^{A-1}F'(\eta)$$

Substituting the above values in equation (21) we have

$$t^{A-1}[AF(\eta) - \eta F'(\eta) + \delta F'(\eta)]0$$

Since $t^{A-1} \neq 0$, therefore

$$AF(\eta) - \eta F'(\eta) + \delta F'(\eta) = 0 \tag{30}$$

This is an ordinary differential equation of first order.

Case 1.

Let trial solutionis :

$$F(\eta) = a_1 a_2 \eta$$

since

$$\eta \to 0,\ F \to F_0$$

$$F(0.1) = a_1 = F_a$$

or,

$$F = F_0 + a_2\eta$$

hence or,

$$F' = a_2$$

substituting the value of F, F' we have

$$A(F_0 + a_2\eta) - \eta a_2 + \delta a_2 = 0$$

or,

$$(AF_a + \delta a_2) + \eta(Aa_2 - a_2) = 0$$

equating the coefficient of η both sides

$$A = 1$$

$$a_2 = \frac{-AF_0}{\delta} = \frac{-F_0}{\delta}$$

$$F = \left(F_0 - \frac{F_0}{\delta}\eta\right) = F_0\left(1 - \frac{\eta}{\delta}\right) = F_0\left(1 - \frac{x}{\delta.t}\right)$$

Hence from equation (29) that is $S_w = t^2F(\eta)$ we have

$$S_w = t^2F(\eta) = F_0\left(1 - \frac{x}{\delta.t}\right)t^2 = \frac{S_{w0}}{t^A}\left(1 - \frac{x}{\delta t}\right)t^2 = \frac{S_{w0}}{t^{A-1}}\left(t - \frac{x}{\delta}\right)$$

Case 2.

Now we solve the above equation (30) or $AF(\eta) - \eta F'(\eta) + \delta F'(\eta) = 0$ with the boundary condition (23) that is when $x = 0$ then $S_w(0.t) = S_{w0}$ or in other words, when $\eta = 0$, then $F(0, t) = \frac{S_{w0}}{t^A} = F_0$ by Rayleigh-Ritz method as follows :

(1) Writing the given differential equation as the Euler's equation of some variational problem.

(2)Reducing this variational problem to a minimizing problem by assuming an approximate solution in the form

$$\overline{y}(x) = y_0(x) + \sum c, \phi_i(x) \tag{31}$$

Where the trial functions $\phi_i(x)$ satisfy the boundary conditions and $\phi_i(x) = 0$ on the C of its region R.

Let the integral to be extremised be

$$I = \int_a^b f(y, y', x)dx \tag{32}$$

Such that $y(a) = A$, and $y(b) = B$.

Substituting (31) in (32) by replacing y by $\overline{y}$ in I, giving $\overline{I}$ as a function of the unknown's c_i. Then c's become parameters, which are so determiend as to extremise $\overline{I}$. This requires

$$\frac{\partial \overline{I}}{\partial c_i} = 0, \quad i = 1, 2, 3, 4 \ldots \tag{33}$$

Solving these equations, we get the values of c_i, which when substituted in (31) give the desired solution. It's solution is equivalent to extremising the following integral :

$$I = \int_0^{\delta} \phi(\eta, F, F')d\eta \tag{34}$$

The functional of the above problem is as follows :

$$\phi(\eta, F, F') = F^2\left(\frac{A+1}{2}\right) + (\delta - \eta)FF' \tag{35}$$

Since the Euler's equation

$$\frac{\partial \phi}{\partial F} - \frac{d}{dx}\left(\frac{\partial \phi}{\partial F'}\right) = 0$$

gives Eq. (35).

Now assuming the trail function as

$$\overline{F} = F_0\left(1 - \frac{\eta}{\delta}\right)^a \tag{36}$$

Differentiating with respect to η we have

$$\overline{F}' F_0\alpha\left(1 - \frac{\eta}{\delta}\right)^{\alpha-1}\left(\frac{-1}{\delta}\right) \tag{37}$$

Now replacing F by $\overline{F}$, and substituting the values of $\overline{F}$ and $\overline{F}'$ in equation (34) we have

$$\begin{aligned}
\int_0^{\delta} \phi d\eta &= \int_0^{\delta} \left\{(A+1)F_0^2\left(1 - \frac{\eta}{\delta}\right)^{2\alpha}\right. \\
&\quad \left. + \delta\left(1 - \frac{\eta}{\delta}\right)F_0\left(1 - \frac{\eta}{\delta}\right)^{\alpha} F_0\alpha\left(1 - \frac{\eta}{\delta}\right)^{\alpha-1}\left(\frac{-1}{\delta}\right)\right\} d\eta \\
&= \int_0^{\delta} F_0^2\left(1 - \frac{\eta}{\delta}\right)^{2\alpha} [(A+1) - \alpha]\, d\eta \\
&= F_0^2\,[(A+1) - \alpha]\,(-\delta)\left[\frac{\left(1 - \frac{\eta}{\delta}\right)}{\alpha + 1}\right]^{\delta}
\end{aligned}$$

$$I = \frac{F_0^2\delta}{\alpha + 1}(A + 1 - \alpha)$$

or,

$$F_0^2\delta(A + 1 - \alpha) = I(\alpha + 1)$$

or,

$$-F_0^2\delta = \frac{dl}{d\alpha}(\alpha + 1) + I$$

$$I = \frac{F_0^2\delta}{\beta + 1}(A + 1 - \beta) \tag{38}$$

It's stationary value is given by $\frac{dI}{d\beta} = 0$.

Differentiating equation (38) with respect to β we have

$$F_0^2\delta = \frac{dI}{d\beta}(\beta + 1) + I$$

or

$$\frac{dI}{d\beta} = \frac{F^2\delta - I}{\beta + 1} = 0$$

$$\frac{F_0^2\delta}{\beta+1}(A+1-\beta) = F_0^2\delta$$

or,

$$A+1-\beta = \beta+1$$

or,

$$A = \beta \tag{39}$$

Since the trail solutin of the second order functional equation will be of degree at most two hence $\beta = 2$. So the approximate solution is

$$\overline{F} = F_0\left(1-\frac{\eta}{\delta}\right)^2 \tag{40}$$

Hence from equation (29) that is $S_w = t^2F(\eta)$ we have

$$S_w(x,t) = F_0\left(t-\frac{x}{\delta}\right)^2 \tag{41}$$

2. Conclusion

The equation

$$S_w(x,t) = F_0\left(t-\frac{x}{\delta}\right)^2 \tag{42}$$

represents analytical solution of fingering phenomenon arising double phase flow through homogeneous media under initial & boundary condition.

The graphical presentation has been given by figure - 3 that is the graph of $F(\eta)$ verses η. The graph of saturation $F(\eta)$ of injeted liquid is increasing after $\eta = 0.5$ for $t > 0$, which indicates that when injected liquid entries into native liquid at common-interface, then suddenly the native liquid enters into injected liquid due to difference in wettability.

Hence initial saturation will decrease and then after $\eta > 0.5$ the saturation uniformly increases parabolically which is physically consistent with the available theory.

<Table 1>

<Figure 2>

APPENDIX (A)

$$\frac{\partial S_w}{\partial t}\frac{\partial}{\partial X}\left[\frac{K_wK}{\delta_w}\left(\frac{\partial P_0}{\partial X}-\frac{\partial P_0}{\partial X}+\rho g\sin\alpha\right)\right] \tag{A.1}$$

$$\frac{\partial P_0}{\partial X} = \frac{-q+\frac{K_wK}{\delta_w}\frac{\partial P_0}{\partial X}-gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0}+\frac{K_w\rho_w}{\delta_W}\right)}{\left(\frac{K_WK}{\delta_W}+\frac{K_0K}{\delta_0}\right)} \tag{A.2}$$

Substituting the value of (A.2), (A.1) becomes

$$\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial X}\left[\frac{K_wK}{\delta_w}\left\{\frac{-q+\frac{K_w}{\delta_w}\frac{\partial P_c}{\partial X}K-gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0}+\frac{K_w\rho_w}{\delta_W}\right)}{K\frac{K_w}{\delta_w}\left(1+\frac{K_0}{K_w}\frac{\delta_w}{\delta_0}\right)}-\frac{\partial P_c}{\partial X}+\rho_wg\sin\alpha\right\}\right]$$

$$\frac{\partial}{\partial X}\left[\frac{K_wK}{\delta_w}\left\{\frac{-q+\frac{K_w}{\delta_w}\frac{\partial P_c}{\partial X}K-gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0}+\frac{K_W\rho_W}{\delta_W}\right)+\left(\frac{K_WK}{\delta_W}+\frac{K_0K}{\delta_0}\right)\left(\frac{-\partial Pc}{\partial X}+\rho_wg\sin\alpha\right)}{K\frac{K_w}{\delta_w}\left(1+\frac{K_0}{K_w}\frac{\delta_w}{\delta_0}\right)}\right\}\right]$$

$$\frac{\partial}{\partial X}\left[\frac{K_wK}{\delta_w}\left\{\frac{-q+\frac{K_w}{\delta_w}\frac{\partial P_c}{\partial X}K-gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0}+\frac{K_W\rho_W}{\delta_W}\right)+\left(\frac{K_WK}{\delta_W}+\frac{K_0K}{\delta_0}\right)\left(\frac{-\partial Pc}{\partial X}+\rho_wg\sin\alpha\right)}{K\frac{K_w}{\delta_w}\left(1+\frac{K_0}{K_w}\frac{\delta_w}{\delta_0}\right)}\right\}\right]$$

$$= \frac{\partial}{\partial X}\left[\frac{\frac{K_WK}{\delta_W}\left\{-q-\frac{K_0}{\delta_0}\frac{\partial P_c}{\partial X}K+gK\frac{K_0}{\delta_0}(\rho_W-\rho_0)\right\}}{K\frac{K_W}{\delta_W}\left(1+\frac{K_0}{K_W}\frac{\delta_W}{\delta_0}\right)}\right]$$

APPENDIX (B)

$$\left(\frac{K_W K}{\delta_W} + \frac{K_0 K}{\delta_0}\right)\frac{\partial P_0}{\delta_w}\frac{\partial P_c}{\partial X} + gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_W\rho_W}{\delta_W}\right) = -q \quad (B.1)$$

$$\frac{\partial P_0}{\partial X} = \frac{1}{2}\frac{\partial P_c}{\partial X} \quad (B.2)$$

Substituting value of (B.2) in Eq. (B.1) we have

$$\left(\frac{K_w}{\delta_w} + \frac{K_0}{\delta_0}\right)\frac{1}{2}\frac{\partial P_c}{\partial X}K - \frac{K_w}{\delta_w}\frac{\partial P_c}{\partial X}K + gK\sin\alpha\left(\frac{K_0\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right) + q = 0$$

$$\frac{1}{2}K\left(\frac{K_0}{\delta_0} - \frac{K_w}{\delta_w}\right)\frac{\partial P_c}{\partial X} + gK\sin\alpha\left(\frac{K_w\rho_0}{\delta_0} + \frac{K_w\rho_w}{\delta_w}\right) + q = 0$$

APPENDIX (C)

$$P\frac{\partial S_W}{\partial t} + \frac{\partial}{\partial X}\left[\frac{\frac{K_0 K}{\delta_0}\left\{\frac{\partial P_c}{\partial X} + g\sin\alpha(\rho_0 - \rho_w)\right\}}{\left(1 + \frac{K_0\delta_W}{K_w\delta_0}\right)} + \frac{q}{\left(1 + \frac{K_0\delta_W}{K_w\delta_0}\right)}\right] = 0 \quad (C.1)$$

Substituting the value of "q" from Appendix (B).

References

Chouke, R.L. et. al. (1959). The instability of flow immiscible viscous liquid-liquid displacement in premeable media. *Petrol Trans, AIME*, p. 316.

Jecquard, P. and Sequier, P. (1940). Movement debeux fluids on contact unmilien porex. *J. Mech.* 1(4).

Scheidegger, A.E. and Johnson, E.F. (1961). The statistical behaviour of instabilities in displacement process in porous medium. *Canadian Journal of Physic.*

Scheidegger, A.E. (1960). The physics flow through process media. *Univ. Torento Press*, Torento, 217,224, 229, p. 229-231.

Verma, A.P. (1924). Statistical behavior of fingering in displacement process in heterogeneous porous medium with capillary pressure. *Canadian Journal of Physic.* 47(3), p. 319-324.

Table 1.

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$F(\eta)$	0.01	0.0064	0.0036	0.0016	0.0004	0.0	0.0004	0.0016	0.0036

η	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
$F(\eta)$	0.0064	0.01	0.0144	0.0196	0.0256	0.0324	0.04	0.0484	0.0576

η	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5
$F(\eta)$	0.0676	0.0784	0.09	0.1024	0.1156	0.1296	0.1444	0.16

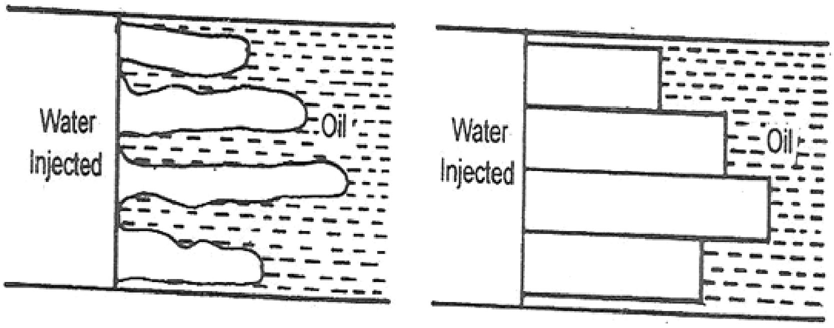

Fig.1(a) Fig.1(b)

Figure 1.

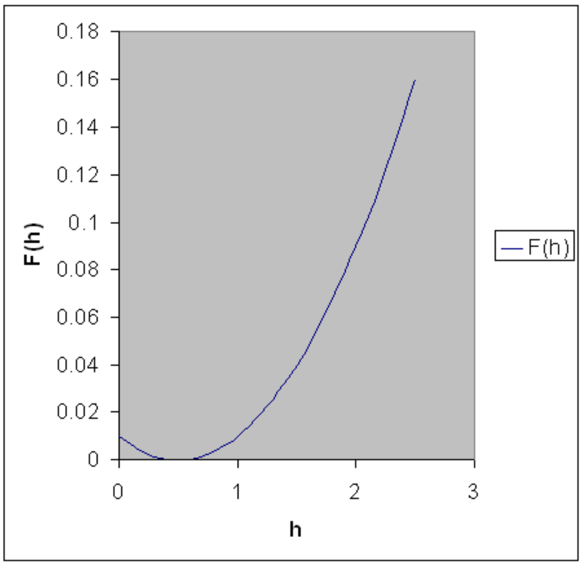

Figure 2.