# Optimization of Launched Parameters of the Railgun with Internal Penalty Function Method 

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#### Abstract

The launched speed of projectile is one of the most important parameters in the design of railgun, in order to explore the influence of the launched speed of projectile by the length of rail and the driving current. Therefore, it is meaningful that gets the optimum speed of projectile. In this paper, main resistances (plasma viscous drag, inertial drag and air drag) was full considered in the motion process of plasma armature, and the optimization model includes the speed of the armature, driving current and the length of rail were build,then it was optimization calculated with internal penalty function method. The optimization result shows that the launched speed of projectile is maximum when the driving current and the length of rail are fixed value. The purpose lies in adjusting the width and the length of the rail and the distance between two rails so as to adjust the launched speed of projectile. The result of research laid a theoretical foundation for designing and manufacture of the railguns.


Keywords: Railgun, Plasma armature, Internal penalty function method, Maximum speed, Optimal calculation

## 1. Introduction

The railgun make use of electromagnetic force to launch projectile, which breakthroughs the limit speed that the projectile is launched by traditional chemical energy. As a new concept weapon, its technology has inestimable application potential not only in the military field, but also in aviation, aerospace, transportation, industrial production, scientific research and other fields. Since the 80s, especially in the recent ten years, with the development of new technology and material, launching launcher, launching weight, projectile speed and high efficiency power source in the railgun have reached a series of achievement (WANG Jing duan,2001). The Su Rense.Livermore Nation Laboratory and the Lowes.Alamos Nation Laboratory, once have cooperated to accelerated a projectile weighed 2.2 g to a supervelocity of $10 \mathrm{~km} / \mathrm{s}$. Fluid Physics Institute of the Chinese Engineering Academy had built the first electromagnetic rail launcher, which can accelerate the projectile weighted 0.34 g to get $16.8 \mathrm{~km} / \mathrm{s}$. While the speed of the conventional projectile can only get $2 \mathrm{~km} / \mathrm{s}$, which is so closed to the limitation of physics that the range is not possible to be farther. The supervelocity of projectile possesses huge kinetic energy which greatly enhance the range and power of the weapon(Liu Wen and Li Min, 2010).

At present the studies about the railgun still continue, involving different fields of researching. About the model problem of the speed of projectile, Parker thinks the speed of the railgun relates to the ablation of chamber wall and increased plasma mass; Ray introduced the resistances related to speed from angle of motion resisting, but the above researches ignored
the air resistance and existed the defects of analysis and calculation of mechanical in their articles.Accurate theoretical analysis and calculation are required in engineering,As a high-tech and high-precision electromagnetic launcher. In this paper, the resistances (plasma viscous drag, inertial drag and air drag) in armature motion was considered, we establish the force and speed model of armature motion and optimize launcher parameters with the penalty function method.

## 2. Constructing Model

The armature pushes the projectile high-speed movement under the effect of electromagnetic force. So it is an important link for selecting the armature to research the railgun. Currently, there are three kinds of armatures was applied to the railgun, including solid armature, plasma armature and composite armature, which were shown in fig.1:
In this paper, the armature that we used is the plasma one.
When big current crossed, the armature was rapidly melted and vaporized in a short time, forming the plasma one, which was affected by electromagnetic force $F_{L}$ in the rail. With the armature current increasing and time continuing, the temperature of plasma is soaringly increasing. This caused local melting and evaporation of inner rail and the materials of a launcher. Moreover, mixed with plasma area and formed viscosity resistance $F_{v}$. Some materials migrated to the armature, the quality of the plasma armature is increased, formed inert resistance $F_{d}$. The plasma armature has affected on the front air resistance $F_{g}$ when the railgun was launched. The several kinds of force were defined as follows(YANG Yu dong and WANG Jian xin, 2008):

1) According to the electromagnetic theory, electromagnetic force $F_{L}$ can be expressed as follows:

$$
\begin{equation*}
F_{L}=\frac{d E}{d L}=\frac{1}{2} L^{\prime} i^{2} \tag{1}
\end{equation*}
$$

Where $F_{L}$ is the driving force, $E$ is the conservative energy of system, mainly showed the magnetic energy of distribution inductance on the rail (Jiang Zhongqiu and Tang Chenglin, 2010). $L^{\prime}$ is Inductance Gradient, reflects inductance of unit length rail. $L$ is the displacement of the armature movement. $i$ is driving current.
2) The viscous resistance $F_{v}$ can be expressed as:

$$
\begin{equation*}
F_{v}=\frac{\lambda v^{2} m_{a}(w+h)}{w h} \tag{2}
\end{equation*}
$$

Where $\lambda$ is viscous factor, relate to armature, processing precision and smoothness of rail. In conditions of higher precision and smoothness, $\lambda=0.0125 ; v$ is armature speed; $m_{a}$ is armature quality; $h$ is width of rail; $w$ is the distance between two rails.
3) The inert resistance $F_{d}$ can be expressed as follows:

$$
\begin{equation*}
F_{d}=v \frac{d m_{a}}{d t} \tag{3}
\end{equation*}
$$

The armature quality $m_{a}$ can be obtained by the solution of (4):

$$
\begin{gather*}
\frac{d m_{a}}{d t}=\alpha i U_{a}  \tag{4}\\
m_{a}=\alpha U_{a} \int_{0}^{t} i d t+m_{0} \tag{5}
\end{gather*}
$$

Where $\alpha$ is ablation coefficient, relates to speed, armature and material of rail. Regard it as a constant. $\alpha \approx 4.7 \times 10^{-8} \mathrm{~kg} / \mathrm{J}$, when use copper material; $m_{0}=0.01 \mathrm{~kg}$ is initial armature quality, $U_{a}$ is arcing voltage. Its size is reasonable key paraments. Arcing voltage was approximately calculated by measuring voltage of two ends of rail when the plasma armature reached smoothly situation. Howewer, we consider the muzzle voltage as a constant according to experimental results of Canberra lab and other laboratory(L EHMANN P RECK B, 2007).
Arcing voltage and armature resistance satisfy the ohm's law:

$$
\begin{equation*}
U_{a}=i R_{a} \tag{6}
\end{equation*}
$$

$R_{a}$ is the resistance armature.
4) The air resistance $F_{g}$ related with armature speed and the cross-sectional area of armature, which was expressed as follows(YANGYu-dong and WANG Jian xin, 2010) :

$$
\begin{gather*}
F_{g} \approx 2.2 \rho_{0} S v^{2}  \tag{7}\\
S=w h \tag{8}
\end{gather*}
$$

where $\rho_{0}$ is air density. the air density is $1.29 \mathrm{~kg} / \mathrm{m}^{3}$ under normal conditions, $S$ is the cross-sectional areas of the inner rail.

The resultanting force of plasma armature can be expressed as follows:

$$
\begin{equation*}
F=\left(m_{p}+m_{a}\right) a=F_{L}-F_{v}-F_{d}-F_{g} \tag{9}
\end{equation*}
$$

where $m_{p}$ is projectile quality; $m_{a}$ is the armature quality; $a$ is the armature acceleration.
The acceleration expressions can be obtained by the solution of (9):

$$
\begin{equation*}
a=\frac{F_{L}-F_{v}-F_{d}-F_{g}}{\left(m_{p}+m_{a}\right)} \tag{10}
\end{equation*}
$$

From Newton's law and kinematics formula, the armature instantaneous speed and displacement respectively can be expressed:

$$
\left\{\begin{array}{l}
v=v_{0}+\int_{0}^{t} a d t  \tag{11}\\
L=\frac{1}{2} \int_{0}^{t} v d t
\end{array}\right.
$$

Where $v_{0}$ is initial armature speed, from (1), (2), (10), (11), we obtain a velocity equation of plasma armature:

$$
\left\{\begin{array}{l}
a=\frac{d^{2} L}{d t^{2}}=\frac{d v}{d t}=\frac{1}{\left(m_{a}+m_{p}\right)}\left(\frac{1}{2} L^{\prime} i^{2}-\frac{\lambda m_{a}(w+h) v^{2}}{w h}-v \frac{d m_{a}}{d t}-1.1 \rho_{0} S v^{2}\right)  \tag{12}\\
m_{a}=\alpha U_{a} \int_{0}^{t} i d t+m_{0}
\end{array}\right.
$$

from (12) we obtain:

$$
\begin{equation*}
\left(m_{a}+m_{p}\right) v^{2}+4 \alpha U_{a} i L v=L^{\prime} i^{2}-\frac{2 \lambda L\left(\alpha U_{a} i 2 L v+m_{0} v^{2}\right)(w+h)}{w h}-2.2 \rho_{0} S v^{2} \tag{13}
\end{equation*}
$$

We have:

$$
\begin{equation*}
\left[m_{a}+m_{p}+\frac{\lambda L(w+h)}{w h} m_{0}+2.2 \rho_{0} S L\right] v^{2}+4 \alpha L U_{a} i\left(1+\frac{\lambda L(w+h)}{w h}\right) v-L L^{\prime} i^{2}=0 \tag{14}
\end{equation*}
$$

The speed of armature can be expressed as follows:

$$
\begin{equation*}
v=\frac{-2 \alpha L U_{a} i\left(1+\frac{L \lambda(w+h)}{w h}\right)+\sqrt{4\left[L \alpha U_{a} i\left(1+\frac{L \lambda(w+h)}{w h}\right)\right]^{2}+L L^{\prime} i^{2}\left(m_{a}+m_{p}+\frac{2 \lambda L m_{0}(w+h)}{w h}+2.2 \rho_{0} L w h\right)}}{\left(m_{a}+m_{p}+\frac{2 \lambda L m_{0}(w+h)}{w h}+2.2 \rho_{0} L w h\right)} \tag{15}
\end{equation*}
$$

## 3. Optimization Design

### 3.1 Model design

The optimization of the railgun was designed by Internal Penalty Function Method, the aim is that we calculate the length of rail, the driving current and the speed of the projectile while the speed of projectile is maximum.

Based on equation (15), the optimization design model of the railgun showed as follows:

$$
\begin{cases}\text { Objective } & \text { function }: \min f(X)=-v  \tag{16}\\ \text { Constraint } & \text { Conditions }: X=X(i, L) \\ v>v_{1} & \end{cases}
$$

Where $X$ is independent variable.
Constraint conditions and the calculated parameters showed as follows:

$$
\left\{\begin{array}{l}
100 K A \leq i \leq 500 K A  \tag{17}\\
1 m \leq L \leq 5 m \\
v_{1}>1500 m / s
\end{array}\right.
$$

$$
\begin{cases}\text { Indctance Gradient } & L^{\prime}=0.5 \mathrm{H} / \mathrm{m}  \tag{18}\\ \text { Width of rail } & h=0.06 \mathrm{~m} \\ \text { Armature quality } & m_{p}=0.005 \mathrm{~kg} \\ \text { Initial speed of armature } & v_{0}=0 \mathrm{~m} / \mathrm{s} \\ \text { The distance between two rails } & w=0.05 \mathrm{~m} \\ \text { Aracing voltage } & U_{a}=200 \mathrm{~V}\end{cases}
$$

### 3.2 Algorithm design

[1] Given initial point $X^{(0)}$;
[2] Select penalty factor $\mu>1$, reduce coefficient $c>0$, convergent precision $\varepsilon>0$, let $k=1$;
[3] The mathematic optimization model was established by Internal Penalty Function Method,Calculating min $\phi\left(X, \mu^{(k)}\right)$, we can get optimal solution $X^{*}\left(\mu^{(k)}\right)$;
[4] Examine $\left\|X^{*}\left(\mu^{(k)-1}\right)-X^{*}\left(\mu^{(k)}\right)\right\|<\varepsilon$; if it is right, $X^{*}\left(\mu^{k}\right)$ is the minimum, stop, or turn [5].
[5] Reduce penalty factor, and calculating $\mu^{(k+1)}=c \mu^{(k)}$, let $\mathrm{k}=\mathrm{k}+1, X^{(k)}=X^{*}\left(\mu^{(k)-1}\right)$, turn [3].

## 4. The results and analysis of optimization

The key parameters driving current and the length of railgun were optimized in different penalty factor and reduce the coefficients and their calculation results were shown table 1:
Tab. 1 shows that the launched speed of projectile is approximately maximum and its value is $v=1630.52 \mathrm{~m} / \mathrm{s}$ when the length of rail $L=1.4868 m$, driving current $I=499999.8646 A$. The projectile speed value above is consistent with the one in(YANG Yu dong and WANG Jian xin,2008), these calculation results are proved that it is ture.The width and the distance between two rails (Asghar keshtkar and Toraj Maleki, 2009)have obviously effect on the speed of the projectile from the model of armature motion. Fig. 2 shows that the launched speed of the projectile is increasing with the distance between two rails and width of rail increasing, then it is decreasing with their changing. When the distance between two rails and width of rail reach 0.0589 m , the speed of projectile is approximately maximum. The main reason is influence of the air resistance. The launched speed of the projectile is increasing with the distance between two rails and width of rail increasing without influence of the air resistance .Fig. 3 shows that the launched speed of the projectile is directely related with the air resistance.
The optimization of the railgun was designed by Internal Penalty Function Method, which includes two variables (driving current and the length of railgun) that all directely affect the launched speed of the projectile. Driving current is a power that it provides acceleration of the projectile, obviously Driving current is the larger, acceleration of projectile is larger, and the launched speed of the projectile as well.The results is consistent with the one in (J.Mankowski, 2007; Asghar Keshtkar, 2009; Wang Zijian, 2009 and Thomas G.Engel, 2006), need to introduce, driving current is the larger,the temperature of the plasma armature and rail is also larger, then the rail is easily burnt out and lead Scattered resistance and Energy consumption all larging, but the Launch efficiency of projectile don't increase,so the above maximum of driving current is 500 KA .
Although the rail is place where it provides acceleration of projectile, it might not be good if the length of railgun is vary. Fig. 4 shows that the launched speed of projectile is increasing with the length of the rail increasing, then it is decreasing with their changing. The main reason is the influence results of the resistances (plasma viscous drag, inertial drag and air drag) on the plasma armature motion process.

## 5. Conclusion

(1) Main resistances (plasma viscous drag, inertial drag and air drag) was full considered in the motion process of plasma armature, and the mathematic model between armature speed, driving current and the length of railgun were established on the optimization.
(2) The optimization of railgun was designed with Internal Penalty Function Method, the launched speed of projectile is approximately maximum and its value is $v=1630.52 \mathrm{~m} / \mathrm{s}$ when the length of rail $L=1.4868 \mathrm{~m}$, driving current $I=499999.8646 A$. The purpose lies in adjusting the width and the length of rail and the distance between two rails so as to adjust the launched speed of projectile.
(3) The optimization results, which compared with the other simulation and experiment, are proved the above mathematic model is ture. It provides theoretical basis for the transition from the experimental to Application stage.

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Table 1. Results of optimization

| penalty factor | ruduce coefficient | drive current(A) | rail length(m) | projectile launch speed(m/s) |
| :---: | :---: | :---: | :---: | :---: |
| 15 | 0.9 | 499997.2865 | 1.6516 | 1630.43 |
| 13 | 0.8 | 499997.2948 | 1.6678 | 1630.49 |
| 10 | 0.6 | 499998.2847 | 1.6712 | 1630.50 |
| 8 | 0.5 | 499998.2949 | 1.6749 | 1630.50 |
| 6 | 0.5 | 499999.8646 | 1.6848 | 1630.52 |
| 5 | 0.4 | 499998.2537 | 1.7015 | 1630.49 |

Solid armature


Figure 1. Commonly used the armature


Figure 2. The influence of projectile speed by the width of the rail


Figure 3. The contrast picture of air resistance


Figure 4. The influence of launched speed of the projectile by the length of the rail

