Mathematical Model for Optimum Depth of Thermal Insulating Layer

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Abstract

In order to ensure the comfort and health of buildings and resolve contradictory between improvement of thermal insulating function of buildings and reduce of thermosteresis and amount of basic building materials (namely save resourses), heat conduction equation was calculated. This mathematical physics equation which was calculated via finite difference method was attributed to inverse problem of gray distributed parameter systems. The economical model of optimum depth of thermal insulating layer was established. Finally, the optimum materials and depth was determined according to the two models.

Keywords: Heat conduction equation, Inverse problems of mathematical physics, Finite difference method, Economic optimization

1. Background and analysis of the model

Most of settlement housings in the city of China were flattop buildings, the highest surface temperature of which could reach 75°C in summer and the lowest surface temperature could reach -40°C in winter. Many measures were carried out to keep suitable temperature inside buildings. On one hand, heating equipment and conditioner were setting up in buildings; on another hand, the thermal insulation ability of buildings was improving, so that the energy consumption would be decreased. The use of appropriate thermal insulation materials with appropriate depth was very important to ensure the comfort and health of buildings guarantee normal production and living, decrease energy consumption, reduce the weight of the roof system, reduce amount of basic building materials, which would save resources and reduce building costs.

2. Hypothesis of the model

1) A given thermal insulation material that transfer heat steadily.

2) The rooftop surfaced was considered flat, no significant changes of gradient.

3) The temperature distribution of rooftop flat surface was considered uniform, namely there was no temperature change within any flat surface. Only heat conduction at vertical direction was considered to affect temperature distribution.

4) The optimum depth of thermal insulation layer only related to type of thermal insulation materials, present value factor of life cycle, heating degree days, thermal resistance of non- thermal insulation layer and type of fuel for heating.

3. Establishment and calculation of the model

3.1 Model 1: inversion model of thermal insulation layer depth of flattop buildings

The structure of flat rooftop was showed in Figure 1.

1) Flattop buildings heat conduction equation

From model hypothesis, the heat conduction equation was:

$$\lambda(z)\frac{\partial^2 u}{\partial z^2} - \rho(z)c(z)\frac{\partial u}{\partial t} = 0 \quad 0 \leq z \leq l \quad t > 0 \tag{1}$$

λ: Coefficient of heat conductivity of materials; ρ: Dry density of material; c: Specific heat of material; u: Temperature of any site from flattop structure; z: Spatial coordinate which is perpendicular with top flat; l: Depth of flat top; t: Time coordinate.

2) Initial conditions

The initial condition, which was the temperature distribution regularity of various layer of flat top at beginning of calculation, was given for No. 1 equation firstly. The time is 5:00 am. It is supposed that temperature distribution within flat top was uniform, so:

$$u|_{t=0} = \phi(z) \quad 0 \leq z \leq l \tag{2}$$

$\phi(z)$: Function of temperature change of various layers within flat top at beginning of calculation; the meanings of other symbols were same with above.

3) Boundary conditions

The outside surface of rooftop was calculated according to mixed boundary conditions. Namely, heat discharge of outside surface of rooftop is proportional with rooftop surface temperature and atmospheric temperature, so:

$$-\lambda_0 \frac{\partial u}{\partial z}|_{z=0} = h_0(u|_{z=0} - \theta_\alpha) \tag{3}$$

λ_0: Coefficient of heat conductivity of materials; h_0: Coefficient of heat exchange between rooftop and medium around; α: Thermal radiation absorption coefficient of rooftop outside surface; θ_α: Atmospheric temperature of outside rooftop; the meanings of other symbols were same with above.

Further consideration of sun radiation effect:

$$\alpha I(t) - \lambda_0 \frac{\partial u}{\partial z}|_{z=0} = h_0(u|_{z=0} - \theta_\alpha) \tag{4}$$

$I(t)$: Sun radiation intensity on surface of rooftop ; the meanings of other symbols were same with above.

The second boundary condition of inside surface of rooftop, namely

$$\frac{\partial u}{\partial z}|_{z=l} = \phi(t) \tag{5}$$

4) Additional conditions

As it is about inverse problem of geometrical boundary of heat conduction equation, there were additional conditions in process of inversion calculation of thermal insulation layer depth.

$$u|_{z=l} = G(t) \quad t > 0 \tag{6}$$

$G(t)$: Function of temperature changes in various levels of inside rooftop at the moment of t.

5) Numerical methods

Finally, simulation of inside surface temperature of flat rooftop was made by finite difference method. Specific difference discrete format deduced from equation (1) $\sim$ (6) was as following:

$$\begin{cases} u_{i,j} = \frac{\tau}{h_i^2 \rho_i c_i} \lambda_{i+1} u_{i+1,j} + (1 - \frac{\tau}{h_i^2 \rho_i c_i} \lambda_{i+1} - \frac{\tau}{h_i^2 \rho_i c_i} \lambda_i) u_{i,j} + \frac{\tau}{h_i^2 \rho_i c_i} \lambda_i u_{i-1,j} \\ u_{i,0} = \phi(ih) \\ u_{0,j} = \varphi(j\tau) \\ u_{1,j} = \frac{h_1 \alpha}{\lambda_0} I(j\tau) - \frac{h_1 h_0 - \lambda_0}{\lambda_0} u_{0,j} + \frac{h_1 h_0}{\lambda_0} \theta_\alpha \\ u_{n+1,j} - u_{n,j} = \phi(j\tau) \end{cases} \tag{7}$$

u_{ij}: Temperature of the i layer of flat rooftop structure at the moment of j; λ_i: Coefficient of heat conductivity of the i layer of materials; h_i: Coefficient of heat exchange between the i layer of materials and around medium; ρ_i: Dry density of the i layer of materials; τ: Time coordinate; the meanings of other symbols were same with above.

Here, i=0, 1, 2, 3, 4, 5, 6, 7, 8; j=0, 1, 2, 3...26, 27; n=7; τ=5.

3.2 Economical model of thermal insulation layer of flat rooftop

Economical model of optimum depth of thermal insulation layer was established on the base of considering of thermal insulation materials type, present value factor of life cycle (PWF), heating degree days (HDD), thermal resistance of non-thermal insulation layer (R_{wt}) and type of fuel for heating.

1) Calculation of costs within life cycle of thermal insulation layer

The costs within life cycle of thermal insulation layer were composed of investment costs and fuel costs. The cost of thermal insulation materials per unit area was:

$$C_{in} = C_i \times d \tag{8}$$

C_{in}: Price of thermal insulation materials per unit area (Yuan/m^2); C_i: Price of thermal insulation materials per unit volume (Yuan/m^3); d: depth of thermal insulation materials.

Heating fuel costs of outside wall per unit area every year were:

$$C_h = \frac{86400 HDD \times C_f}{[(R_{wt} + d/\lambda_m)H\eta]} \qquad (9)$$

C_n: Costs of heating fuel per unit area of rooftop ($Yuan/m^2 \cdot a$).; C_f: Price of heating fuel ($Yuan/kg$); HDD: Heating degree days (°·d); λ_m: Coefficient of heat conductivity of thermal insulation materials; R_{wt}: Sum of thermal resistance of area inside and outside room and thermal resistance of other materials except thermal insulation layer; H: calorific value of fuel (kJ/kg); η: Efficiency of heating system.

The total costs of fuel used for heating in one unit of wall within the lifespan of thermal insulation layer were calculated by cycle value methods. As following:

$$C_t = PWF \cdot C_h \qquad (10)$$

C_t: The total costs of fuel used for heating in one unit of wall (Yuan/m^2); PWF: Factor of life cycle.

The way of PWF calculation was as following:

When $g > I$:

$$I^* = \pounds I - g)/(1 + g) \qquad (11)$$

When $g < I$:

$$I^* = (g - I)/(1 + I) \qquad (12)$$

So:

$$PWF = [1 - (1 + I^*)^{-N}]/I^* \qquad (13)$$

When $g = I$:

$$PWF = (1 + I)^{-1} \qquad (14)$$

I: Bank interest rate (%); g: The inflation rate (%); $I*$: The modified bank interest rate compared with the inflation rate (%); N: Years of life cycle of thermal insulation layer.

The annual heating costs of building were composed of investment costs of thermal insulation layer and fuel costs. So the total costs for every one unit of thermal insulation layer within life cycle were:

$$C = \frac{86.4 \times PWF \times C_f \times HDD}{(R_{wt} + d/\lambda_m)H\eta} + C_i \times d \qquad (15)$$

Here, C: Total costs for every one unit of thermal insulation layer within life cycle ($Yuan/m^2$). the meanings of other symbols were same with above.

2) The equation for optimum depth of thermal insulation layer

Because $C = \frac{86.4 \times PWF \times C_f \times HDD}{(R_{wt} + d/\lambda_m)H\eta} + C_i \times d$, So the optimum depth of thermal insulation layer was descided as following:

$$\frac{\partial C}{\partial d} = \frac{86.4 \times PWF \times C_f \times HDD}{H\eta h(R_{wt} + d/\lambda_m)^2} - C_i = 0 \qquad (16)$$

So the calculation equation of optimum depth of thermal insulation layer was:

$$d_{op} = \sqrt{\frac{86.4 \times C_f \times HDD \times m \times PWF}{H\eta C_i}} - m \times R_{wt} \qquad (17)$$

4. Test and application of the model

4.1 Test of inversion model of thermal insulation layer depth of flattop buildings

$u_{8,j}$ was calculated by inversion model of thermal insulation layer depth. The equation set (7) was programmed by Matlab software. The detailed procedures were as following:

(1) The thermal insulation layer depth was estimated as [a, b] according to previous building experience.

(2) The temperature of inside surface was calculated by entering the two endpoint a and b. Then two curves of room temperature versus time were drawn up.

(3) Compare the above curves with the curves produced by actual measurement of temperatures at different time point in a day. If they were matched exactly, this range was the best for thermal insulation layer depth. If one endpoint did not satisfy the conditions, new endpoint could be got by split the above range via dichotomy. Then the new endpoint and another suitable endpoint form a new range $[a_1, b_1]$. Begin from ? and repeated.

Figure 2 showed the follow chart.

According to previous construct experience, the thermal insulation layer depth was estimated to be about 150 mm~400 mm in this model, namely, a = 150, b = 400. By inversion model, forward modeling and check of additional conditions, curves produced by program matched with actual curves when the thermal insulation layer depth was in the range of 180 mm~300 mm (Figure 3).

From Figure 3, we can see that the depth of perlite thermal insulation layer was 180 mm~300 mm in summer. Similarly, when the depth of perlite thermal insulation layer was 200 mm~300 mm in winter, curves produced by program matched with actual curves (Figure 4).

From Figure 4, we can see that the depth of perlite thermal insulation layer was 200 mm~300 mm in winter. From above two cases, we can conclude that the optimum depth of thermal insulation layer was 200 mm~300 mm.

4.2 Test of economical model of thermal insulation layer of flat rooftop

The latest four materials (Polyethylene foam, Styrofoam, Rigid Polyurethane foam and PVC rigid foam) were selected for determination of optimum materials as thermal insulation layer. The parameters of the four materials were listed in Table1.

Figure 5 showed relationship between depths of various thermal insulation layer materials and total costs of per unit area of thermal insulation layer within life cycle.

The depths of various materials with lowest total costs were showed in Figure 6.

The detailed data were listed in Table 2.

Analysis of PWF, HDD and R_{wt} were demonstrated in Figure 7~9.

From the figures, we can see that the depth of thermal insulation layer of Rigid Polyurethane foam and PVC rigid foam changed too much when Rwt, PWF and HDD changed. So they are not applicable. The suitable depth sof Polyethylene foam and Styrofoam acquired by inversion model were (54, 65) and (43, 55), respectively. The optimum depths of Polyethylene foam and Styrofoam acquired by economical model were 69.3649 mm, 52.8727 mm, respectively. However, 69.3649 mm was not in the range (54, 65) in compare with 52.8727 mm was in the range (43, 55). Therefore, the optimum material was Styrofoam and its optimum depth was 52.8727 mm.

5. Evaluation of the model

5.1 Advantages of the model

1) The models established in present study are easy to solve.

2) The knowledge used in this study is primary and the methods to solve problems are easy to understand.

3) Data processing and collecting correspond with national standard.

4) It was very scientific and applicable to consider both heat flow and economic in determination of the optimum thermal insulation layer materials.

5.2 Disadvantages of the model

1) Heat Conduction during various lays of rooftop was regarded as linear and unchanged in inversion model. However, this linear relationship did not exist in objective word, because the non-linear relationship such as, mutual restrict, saturation and limitation, could not be evitable.

2) The non-uniform distribution of various layers density of rooftop, thermal bridge existed in heat conduction during various layers and temperature fluctuation inside and outside the buildings will constrain the optimum depth determination.

3) From Figure 7~9, we knew that the depth of thermal insulation layer correspond with PWF, HDD and R_{wt}. However, the changes of PWF, HDD and R_{wt} were not be considered. This may cause deviation. So the model remained to be improved.

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Appendix

Matlab source code:

1. The program to determine suitable thermal insulation layer depth in summer

```
r=input(' thermal insulation layer depth:');
a=zeros(9,28);b=zeros(1,27);
p=[600 1800 250 1800 2400 1800 370];%density
c=[1.47 0.84 0.84 0.84 0.84 0.84 0.84];%specific heat
e=[0.0233 0.17 0.93 0.047 0.93 1.53 0.93 0.075 0.0233];%coefficient of heat conductivity
h=[1 2 r 2 20 1.5 0.1];%*0.001; %depth of various layers
i=[122.4 162.3 203.7 324.2 392.6 467.8 504.0 624.6 630.0 640.6 691.5 691.51 717.8 732.9 723.4 713.4 708.0 682.2 663.9 569.4 418.4 381.1 320.0 294.9 231.3 125.7 121.0 119.2];%intensity of sun radiation
e0=0.0233; %coefficient of heat conductivity of air
f=0.74; %absorption coefficient of rooftop
h0=39.808;%coefficient of heat exchange of rooftop
for n=1:9
if n==1
a(n,1)=22.4;
elseif n==2
a(n,1)=-4.4*sum(n-1)/(2*sum(7))+22.4;
elseif n==9
a(n,1)=18;
else
sum=cumsum(h);
a(n,1)=-4.4*(sum(n-1)+sum(n-2))/(2*sum(7))+22.4;
end
end
a(1,2:28)=[23.5 23 23 23.6 23.8 25 24 24.5 25 26 27 30 32 35 35.5 36 37 38 38 39 38 37.5 33 30 25 22 20];%selected
```

temperature outdoor

for j=2:28

a(2,j)=(h(1)*0.01*i(j)*4.18*1000/3600*f/e0+(h(1)*0.01*23-e0)/e0*a(1,j)+h(1)*0.01*23*25/e0)*0.085-0.15*(0.5*j-12) 2;

end

for j=2:28

if r¡29&r¿23

if j¡19

for m=3:8

a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(30*j)+0.0041*(r-30) 2+0.02;

b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;

end

else

for m=3:8

a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(30*j)+0.0041*(r-30) 2-0.29;

b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;

end

end

else

if j¡19

for m=3:8

a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(r*j)+0.0041*(r-25) 2+0.029;

b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;

end

else

for m=3:8

a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(r*j)+0.0041*(r-25) 2-0.29;

b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;

end

end

end

end

for j=2:28

a(9,j)=a(8,j)+b(j-1);

end

a

x=linspace(5,18.5,28);

y=a(9,1:28);

s=subplot(1,1,1);

z=linspace(5,19,15);

plot(x,y,'o-');

set(s,'XTick',z);

When the depth of thermal insulation layer r=25, the program running results were showed in Table 1.

Table 1

22.4	23.5	23	23	23.6	23.8	25
22.08571	25.634	28.16716	33.48173	37.54067	41.43612	44.78818
22.22946	22.21447	22.73912	23.12285	22.92386	22.68626	23.01987
21.07829	21.06364	21.58846	21.97225	21.77306	21.53506	21.86805
19.92713	19.91222	20.43678	20.82031	20.62087	20.38261	20.71534
18.98915	18.97448	19.49928	19.88305	19.68386	19.44584	19.77881
18.07248	18.05852	18.58405	18.96817	18.76916	18.53124	18.86428
18.00426	18.02616	17.7854	17.8268	17.46803	17.17206	17.49121
18	18.51511	19.44195	19.81883	19.58272	19.33907	19.68304

2. Determine suitable thermal insulation layer depth in winter

r=input(' thermal insulation layer depth:');

a=zeros(9,28);b=zeros(1,27);

p=[600 1800 250 1800 2400 1800 370];%density

c=[1.47 0.84 0.84 0.84 0.84 0.84 0.84];%specific heat

e=[0.0233 0.17 0.93 0.09 0.93 1.53 0.93 0.075 0.0233];%coefficient of heat conductivity

h=[1 2 r 2 20 1.5 0.1];%*0.001; %depth of various layers

i=[122.4 162.3 203.7 324.2 392.6 467.8 504.0 624.6 630.0 640.6 691.5 691.51 717.8 732.9 723.4 713.4 708.0 682.2 663.9 569.4 418.4 381.1 320.0 294.9 231.3 125.7 121.0 119.2];%intensity of sun radiation

e0=0.0233; %coefficient of heat conductivity of air

f=0.74; %absorption coefficient of rooftop

h0=39.808;%coefficient of heat exchange of rooftop

for n=1:9

if n==1

a(n,1)=-17;

elseif n==2

a(n,1)=33*sum(n-1)/(2*sum(7))-17;

elseif n==9

a(n,1)=16;

else

sum=cumsum(h);

a(n,1)=33*(sum(n-1)+sum(n-2))/(2*sum(7))-17;

end

end

a(1,2:28)=[-17.45 -17.3 -17.4 -17 -15.38 -14.5 -14 -14 -13.5 -13.5 -13 -13 -11.5 -9 -7 -6 -4.5 -4.5 -5 -6 -5 -7 -9 -9.5 -10 -11 -13]; %Suitable outdoor temperature

for j=2:28

a(2,j)=(h(1)*0.01*i(j)/60*4.18*1000/3600*f/e0+(h(1)*0.01*23-e0)/e0*a(1,j)+h(1)*0.01*23*25/e0)*0.085-0.15*(0.5*j-12)

```
2-30;
end
for j=2:28
if r¡29&r¿23
if j¡19
for m=3:8
a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(30*j)+0.0041*(r-30) 2+0.02;
b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;
end
else
for m=3:8
a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(30*j)+0.0041*(r-30) 2-0.29;
b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;
end
end
else
if j¡19
for m=3:8
a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(r*j)+0.0041*(r-25) 2+0.029;
b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;
end
else
for m=3:8
a(m,j)=0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)*a(m+1,j-1)+(1-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m)-0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1))*a(m,j-1)+0.5*3.6/(h(m-1) 2*p(m-1)*c(m-1))*e(m-1)*a(m-1,j-1)+0.45*sin(r*j)+0.0041*(r-25) 2-0.29;
b(j-1)=-1*(1+j) (-3.5)*(0.5*j-10) 2+2.221;
end
end
end
end
for j=2:28
a(9,j)=a(8,j)+b(j-1);
end
a
x=linspace(5,18.5,28);
y=a(9,1:28);
s=subplot(1,1,1);
```

```
z=linspace(5,19,15);
plot(x,y,'o-');
set(s,'XTick',z);
3. Optimization of the model for calculation of thermal insulation layer depth
HDD=4471;%HDD
Cf=0.150;%price of fuel
H=21000;%calorific value
h=0.75;%coefficient of heat supply efficiency
Ci=[210 300 230 320];%price of materials
m=[0.047 0.042 0.33 0.48];%coefficient of heat conductivity
Rwt=0.4119;%sum of heat resistance except thermal insulation layer
g=0.05;%rate of inflation
I=0.0414;%bank rate
Iw=(g-I)/(1+I);%modified bank rate compared with inflation rate PWF=(1-(1+Iw) (-10))/Iw;% PWF
C=zeros(4,16);%total costs of a unit area thermal insulation layer within life cycle
dop=zeros(1,4);%the optimum depth
C1=zeros(1,4);%the optimum costs of a unit area thermal insulation layer
d=linspace(0,150,16);
for a=1:4
for b=1:16
C(a,b)=86.4*PWF*Cf*HDD/((Rwt+d(b)*0.001/m(a))*H*h)+Ci(a)*d(b)*0.001;
end
dop(a)=((86.400*Cf*HDD*m(a)*PWF/(H*h*Ci(a))) (1/2)-m(a)*Rwt)*1000;
end
C
dop
for b=1:4
C1(b)=86.4*PWF*Cf*HDD/((Rwt+dop(b)*0.001/m(b))*H*h)+Ci(b)*dop(b)*0.001;
end
C1
x=linspace(0,150,16);
y=C(1,:);
z=linspace(30,100,15);
s =subplot(1,1,1);
plot(x,y,'*-g','lineWidth',2);
%legend('costs of Polyethylene foam',1);
hold on;
y=C(2,:);
plot(x,y,'square-m','lineWidth',2);
%legend('costs of Styrofoam',1);
hold on;
y=C(3,:);
```

```
plot(x,y,' -r','lineWidth',2);
%legend('costs of Rigid Polyurethane foam',1);
hold on;
y=C(4,:);
plot(x,y,'pentagram-b','lineWidth',2);
%legend('costs of PVC rigid foam',1);
xlabel('depth of thermal insulation layer(mm)');
ylabel('Costs(Yuan/m 2)');
set(s,'XTick',x);
set(s,'YTick',z);
grid;
```

The running results were showed in Table 2.

4. Optimization of models to get optimum depth and lowest costs

```
HDD=4471;%HDD
Cf=0.150;%price of fuel
H=21000;%calorific value
h=0.75;%coefficient of heat supply efficiency
Ci=[210 300 230 320];%price of materials
m=[0.047 0.042 0.33 0.48];%coefficient of heat conductivity
Rwt=0.4119;%sums of heat resistance except thermal insulation layer
g=0.05;%rate of inflation
I=0.0414;%bank rate
Iw=(g-I)/(1+I);%modified bank rate compared with inflation rate
PWF=(1-(1+Iw) (-10))/Iw;%PWF
C=zeros(4,16);% total costs of a unit area thermal insulation layer within life cycle
dop=zeros(1,4);%the optimum depth
C1=zeros(1,4);% the optimum costs of a unit area thermal insulation layer
d=linspace(0,150,16);
for a=1:4
for b=1:16
C(a,b)=86.4*PWF*Cf*HDD/((Rwt+d(b)*0.001/m(a))*H*h)+Ci(a)*d(b)*0.001;
end
dop(a)=((86.400*Cf*HDD*m(a)*PWF/(H*h*Ci(a))) (1/2)-m(a)*Rwt)*1000;
end
dop
C
for b=1:4
C1(b)=86.4*PWF*Cf*HDD/((Rwt+dop(b)*0.001/m(b))*H*h)+Ci(b)*dop(b)*0.001;
end
C1
x=linspace(0,150,16);
y=C(1,:);
```

```
z=linspace(30,100,15);
s=subplot(1,1,1);
plot(x,y,'*-g','lineWidth',2);
%legend('costs of Polyethylene foam',1);
hold on;
y=C(2,:);
plot(x,y,'square-m','lineWidth',2);
%legend('costs of Styrofoam',1);
hold on;
y=C(3,:);
plot(x,y,' -r','lineWidth',2);
%legend('costs of Rigid Polyurethane foam',1);
hold on;
y=C(4,:);
plot(x,y,'pentagram-b','lineWidth',2);
%legend('costs of PVC rigid foam',1);
xlabel('depth of thermal insulation layer(mm)');
ylabel('total costs(Yuan/m 2)');
set(s,'XTick',[dop(4),dop(2),dop(1),dop(3)]);
set(s,'YTick',[C1(1),C1(2),C1(3),C1(4)]);
grid;
```

Table 1. The parameters of four latest materials

Material of thermal insulation layer	Coefficient of heat conductivity $(W/(m \cdot K))$	Price ($Yuan/m^3$)
Polyethylene foam	0.047	210
Styrofoam	0.042	300
Rigid Polyurethane foam	0.033	230
PVC rigid foam	0.048	320

Table 2. The corresponding depth of thermal insulation layer with the lowest costs

Materials of thermal insulation layer	Lowest costs (Yuan/m2)	The corresponding depth of thermal insulation layer (mm)
Polyethylene foam	33.1987	69.3649
Styrofoam	36.9136	52.8727
Rigid Polyurethane foam	72.0733	88.7176
PVC rigid foam	83.7359	31.9814

Table 3. Intensity of sun radiation at different moments

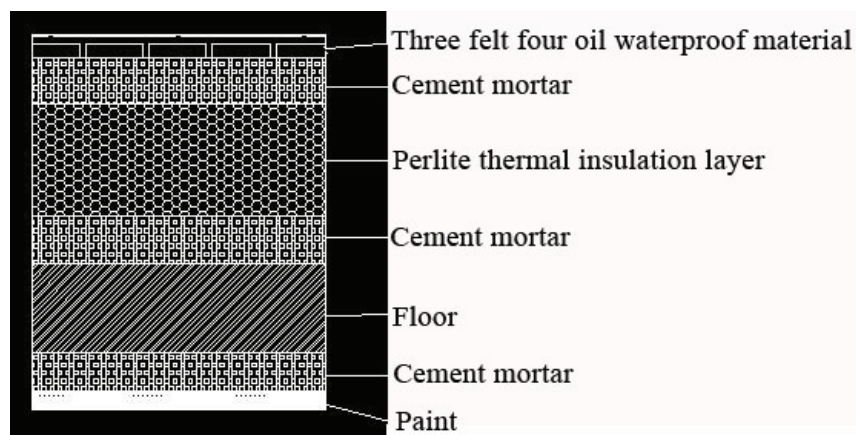| Time	Sun radiation kcal/(m^2·h)	Time	Sun radiation kcal/(m^2·h)	Time	Sun radiation kcal/(m^2·h)
5:00	122.4	10:00	691.5	15:00	418.4
5:30	162.3	10:30	691.51	15:30	381.1
6:00	203.7	11:00	717.8	16:00	320.0
6:30	324.2	11:30	732.9	16:30	294.9
7:00	392.6	12:00	723.4	17:00	231.3
7:30	467.8	12:30	713.4	17:30	125.7
8:00	504.0	13:00	708.0	18:00	121.0
8:30	624.6	13:30	682.2	18:30	119.2
9:00	630.0	14:00	663.9		
9:30	640.6	14:30	569.4		

Figure 1. Structure of flat rooftop
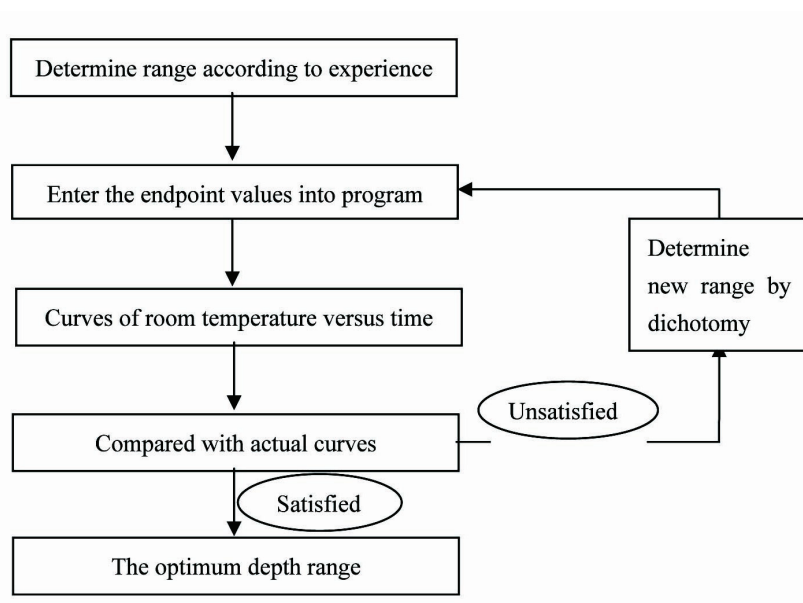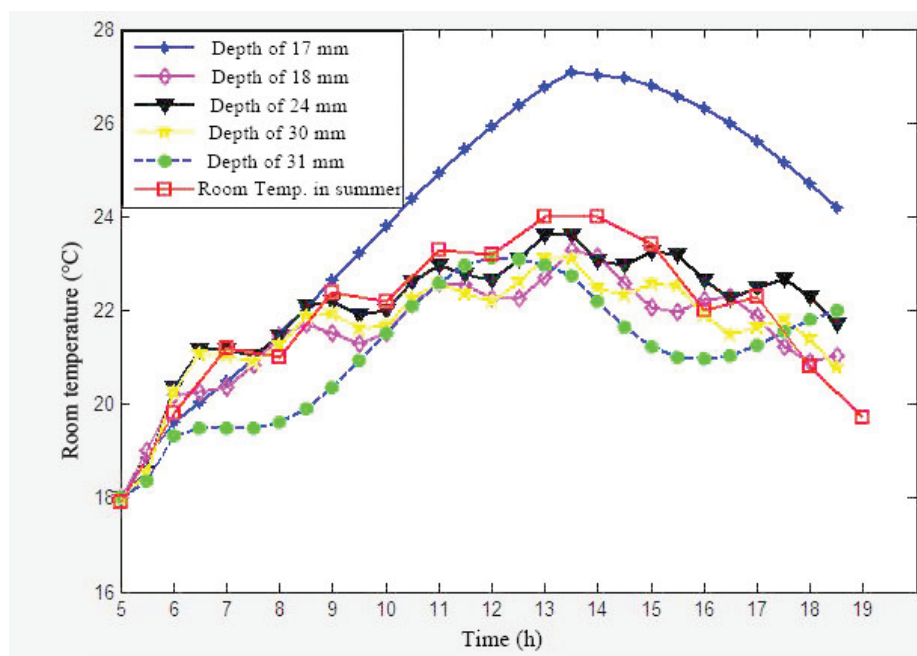
Figure 2. Follow chart of solver

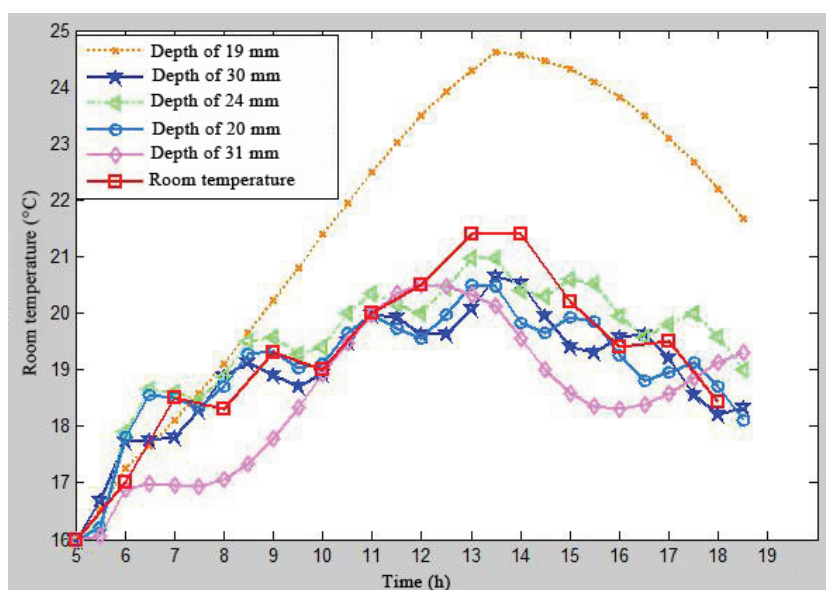Figure 3. Comparison of temperatures calculated by the model with annual temperatures in summer

Figure 4. Comparison of temperatures calculated by the model with annual temperatures in winter
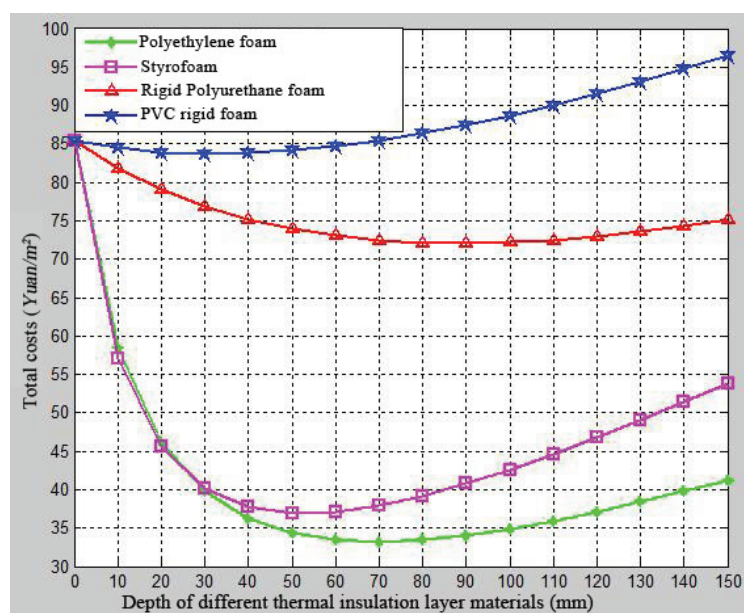

Figure 5. Relationship between depth of different thermal insulation layer materials and total costs
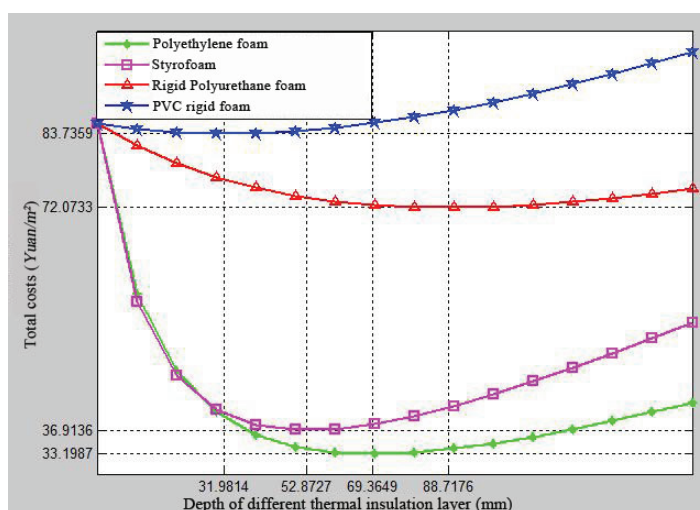
Figure 6. The corresponding depth of thermal insulation layer with the lowest costs
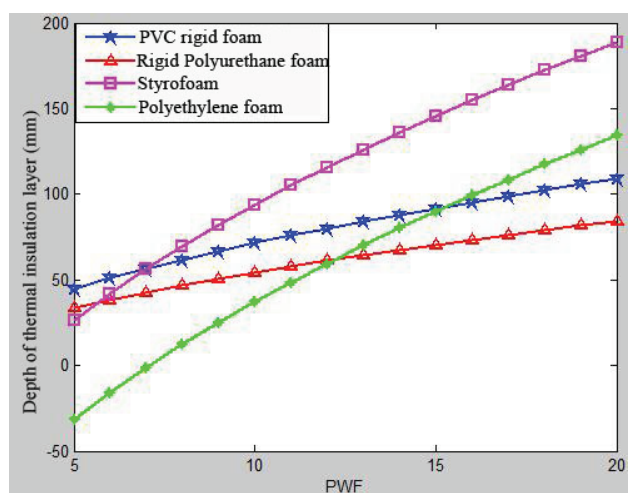

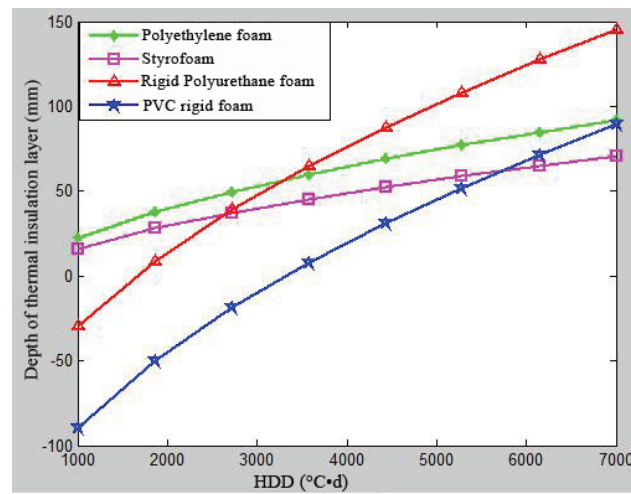Figure 7. Relationship between depth of thermal insulation layer and PWF

Figure 8. Relationship between depth of thermal insulation layer and HDD
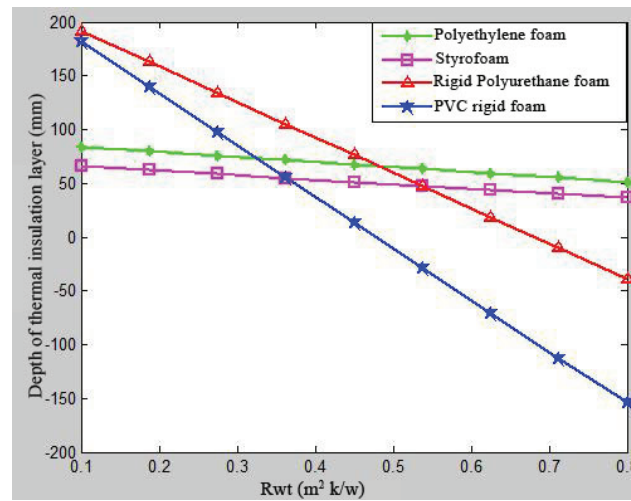

Figure 9. Relationship between depth of thermal insulation layer and R$_{wt}$