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Studies on the Probabilistic Model for Ship-Bridge Collisions

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Abstract

Shocking ship-bridge collisions indicate that there's large space in the previous bridge anti-collision technology research. There are several advantages in the risk-based anti-collision technology of the bridges. Thus the databases such as SpringerLink, Elsevier ScienceDirect and CNKI, the Chinese database, are included to collect literature for the purpose of examining the probabilistic models. Reviewing the current representative models, this paper argues some limitations in the models, such as the questionable applicability of models, the neglected affects of pier turbulent zones as well as some inaccuracies in the mathematical formulations. Accordingly, the paper revises the current models and also addresses increasing the representativeness of samples with sufficient experiments. This paper explores the topic for its potential applications, and aims to make some contribution to the references on the topic so as to popularize and promote the technology in a real sense.

Keywords: vessel-bridge collision, probabilistic model, limitations, further studies

1. Introduction

Shocking ship-bridge collisions indicate that bridge anti-collision technology research still has very large space in that the practicality and operability of the research results need further proof. Therefore, the author used PDF-Geni and Google as search engines to collect literature, with *bridge anti-collision* as key words. Meanwhile, the databases such as SpringerLink, Elsevier ScienceDirect and CNKI, the Chinese database, were also included. The literature review found that previous risk-based studies on the bridge anti-collision at home and abroad are comparatively deficient. In the case of optimizing the placement of the bridge sensors, the method proposed by Guo (2010) can not only alarm the collision between ships and bridges, but also can note down the data of accidents to evaluate the degree of damage in the bridge. However, studies like Guo's excessively focused on those mathematical models of the collision probability, underestimating the impact of the turbulent zone around piers on the collision probability. Accordingly, Jiang and Wang (2009) recommended using AASHTO model and LARSEN model if the calculation is adjusted to the domestic situation. Based on the previous researches at home and abroad, this paper reviews and comments on the present researches on the probabilistic models for ship-bridge collision, discusses the limitations of the studies, and addresses the corresponding improvements, attempting to make some contributions to the present literature.

2. Literature Review and Commentary

2.1 Literature Review

Bridge anti-collision technology is classified into passive technology and active technology and the domestic researches mostly focus on the passive one, for instance, setting the mechanical anti-collision device to reduce the impact of the collision between the ship and bridge. However, it is acknowledged that it's impossible to block all the collisions unless it depends on the bridge itself (Larsen, 1993; Vrouwenvelder, 1998). Furthermore, the bridge anti-collision device costs too much, for example, the cost of the flexible energy-absorbing anti-collision device for the main pier of *Zhanjiang Bay Bridge* in China remains twenty million RMB, which is an unacceptable cost for the regular bridges. The existing bridge anti-collision devices are restricted to the critical bridges in the dense waterway. In contrast, bridge anti-collision technology based on risk ideas has the advantage of preventing accidents in advance. Thus we should pay enough attention to the risk-based anti-collision research on the bridges, and studies on the probabilistic models for ship-bridge collision catch the author's eye.

The result of applying the AASHTO Method II design procedure is the calculation of an annual frequency of collapse for a given bridge. For critical bridges, the risk acceptance criterion is less than or equal to 0.0001, or once every ten-thousand years. For regular bridges, the acceptable risk is less than or equal to 0.001, or once every thousand years (AASHTO, 1994. *AASHTO LRFD Bridge Design Specification and Commentary*). Collision risk models consider the effects of the vessel traffic, the navigation conditions, the bridge geometry with respect to the waterway, and the bridge element strength with respect to the impact loads (Knott & Pruca, 2000). By reviewing the previous literature, 5 models are currently found to the most representative ones. These models taken from original works are listed as following, whose inaccuracies will be put off until the 3^{rd} section.

2.2.1 AASHTO Model

The 1991 AASHTO Specifications provide three methods (Methods I, II, and III) for designing a bridge while taking into account potential vessel impact. Method II is the only method presented in the 2001 AASHTO LRFD Bridge Design Specification, whose essential data include vessel description, speed and loading conditions, waterway geometry, navigable channel geometry, water depths etc. Under AASHTO Method II, bridges must be assigned an importance classification as a Regular or Critical bridge, based on society/survival demand and security/defense requirements (AASHTO: 2009. *Guide Specification and Commentary for Vessel Collision Design of Highway Bridges*). The equation for the calculation of an annual frequency of collapse for a given bridge is generally formulated as follows:

$$AF = N \cdot PA \cdot PG \cdot PC$$

where,

AF = The annual frequency of bridge element collapse due to vessel collision;

N = The annual number of vessels classified by type, size, and loading which can strike the bridge element;

PA = The probability of vessel aberrancy;

PG = The geometric probability of a collision between an aberrant vessel and a bridge pier or span;

PC = The probability of bridge collapse due to a collision with an aberrant vessel.

To provide an alternative means for calculating the probability of aberrancy, the 2001 AASHTO Specifications allow this probability to be approximated using the equation below:

$$PA = BR \cdot R_B \cdot R_C \cdot R_{XC} \cdot R_D$$

where,

PA = The probability of aberrancy;

BR = The aberrancy base rate;

R_B = The correction factor for bridge location;

R_C = The correction factor for current acting parallel to vessel transit path;

R_{XC} = The correction factor for crosscurrents acting perpendicular to vessel transit path;

R_D = The correction factor for vessel traffic density.

The AASHTO model uses dynamic analysis to determine the force of ships and also provides a simplified way to design a probability model to simulate the ship-bridge collision. The AASHTO model is based on the results of accidents, and the movement of ships is not related to the probability of vessel aberrancy (*PA*), or the geometric probability of a collision (*PG*). The probability calculation is larger than the truth value unless the probability of a collision not between the ship and the vessel is eliminated.

2.1.2 Larsen Model

In 1991, Larsen proposed the collision risk model at IABSE's annual conference (Larsen, 1993), which is expressed in the following form, where the first summation refers to all ship classes considered and the second summation refers to all bridge piers and superstructure spans:

$$F = \sum N_i \cdot P_{C,j} \cdot \sum P_{G,i,k} \cdot P_{F,i,k}$$

where,

F = Expected number of annual collisions to the bridge (bridge piers and/or superstructure);

N_i = Annual number of vessels belonging to a certain class (i) of the vessels passing the bridge;

$P_{c,j}$ = The "causation probability" related to the actual class of vessel (i);

$P_{G,i,k}$ = The "geometrical probability" or "rate of collision candidates" related to the actual class of vessel (i) and to the actual part (pier or span) of the bridge (k);

$P_{F,i,k}$ = The "failure probability" related to the actual class of vessels (i) and to the actual part of the bridge (k).

A probabilistic approach is based on a probabilistic model for the vessel impact force and a spatial stochastic model of the resistance properties of the bridge elements. Larsen model calculates the probability of bridge failure, which means not until $P_{F,i,k}$ being removed can the calculation truly represent the probability of the bridge collision (Jiang & Wang, 2009). As described in AASHTO model, the "causation probability" $P_{c,j}$ does not change with the sailing course. And the accidents are classified into the linear impact, meeting impact and random drifting impact, related to the angle of attack and the different failure modes of the bridge elements (e.g. crushing, rotation, sliding, etc.). Meanwhile, the linear impact can be subdivided into impacts on the axis of channel and those at the turns or bends in the navigation route. When applying the model at a certain river, we should get the gross impact probability by considering the ratio of the three situations in all accidents.

2.1.3 Eurocode Model

In 1997, Eurocode proposed a model to calculate the probability of the ship-bridge collision in volume 1. The model uses the centerline of the channel as X axis and parallel Y axis with the bridge axis, and the pier is located at $X = 0$ and $Y = d$ (Vrouwenvelder, 1998). Ship-bridge collision is considered as a non-homogeneous Poisson process, assuming that the error of Poisson process is $\lambda(x)$ so that the probability of the collision in a referenced period T can be expressed as follows:

$$P_C(T) = nTP_{na}\iint \lambda(x)P_C(x,y)f_s(y)dxdy$$

where,

$P_c(T)$ = The probability of not avoiding at least one collision within the reference period (usually 1 year);

n = The number of ships per time unit (traffic intensity);

T = The reference period (usually 1 year);

P_{na} = The probability that a collision is unavoided in spite of human intervention;

$\lambda(x)$ = The probability of a failure per unit traveling distance, determined with reference to data of previous accidents;

$P_c(x, y)$ = The probability of situations where a collision occurs with a given initial ship position (x, y);

$f_s(y)$ = The distribution of the ship position in the y-direction.

Eurocode and AASHTO Specifications share the similarity in the basic design philosophy. Eurocode 1, Part 2.7 refers in a note to ISO (Draft Proposal DP 10252): 1995. *Accidental Action due to Human Activities*, which specifies the representative value of an accidental action should be chosen in such a way that there is an assessed probability less than p=10^{-4} per year for one structure (Vrouwenvelder, 1998). Although the acceptable risk criterion is determined by each country government, but the acceptable annual frequency of collapse they recommend for the critical bridge is less than or equal to 1×10^{-4}, or once every ten-thousand years (Knott, 1998; AASHTO, 2009. *Guide Specification and Commentary for Vessel Collision Design of Highway Bridges*). Various collision risk models have been developed to achieve design acceptance criteria, while determining the risk acceptance criteria is based on the society's willingness to pay for the risk reduction.

2.1.4 KUNZI Model

Based on the variables describing the accidental course of the ship, a mathematical risk model was formulate by the German researcher Kunz (1998), in which a deviation on the maneuvering path with angle φ and the stopping distance x are chosen. Given the numerous affecting elements, the minimum distance x necessary for avoiding the pier should be a normal random variable. The collision model is outlined here in the following:

$$P(T) = N \cdot \int \frac{d\lambda}{ds} \cdot W_1(s) \cdot W_2(s)ds$$

where,

$P(T)$ = The probability of not avoiding at least one collision within the reference period (usually 1 year);

N = The number of ships per time unit (traffic intensity);

T = The reference period (usually 1 year);

$d\lambda/ds$ = The failure rate per travel unit;

$W_1(s)$ = The probability of collision course;

$W_2(s)$ = The probability not to come to a stop before collision to structure.

where,

$$W_1(s) = F_\varphi(\varphi_1) - F_\varphi(\varphi_2)$$

$$F_\varphi(\varphi) = \frac{1}{\sqrt{2\pi}\sigma_\varphi}\int_{-\infty}^{\varphi} \exp\left\{\frac{(\varphi-\overline{\varphi})^2}{2\sigma_\varphi^2}\right\} d\varphi$$

where, $\bar{\varphi}, \sigma_\varphi$ are mean value and standard deviation of the angle φ between the planned course and the maneuvering course path;

$$W_2(s) = 1 - F_x(s)$$

$$F_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\int_{-\infty}^{x} \exp\left\{\frac{(x-\overline{x})^2}{2\sigma_x^2}\right\} dx$$

where, $\bar{x}, \sigma_x$ are mean value and standard deviation of stopping distance x, referring to the distance between the ship and the pier when the ship detecting the danger of collision and taking urgent measures.

By calculating the probability $W_1(s)$ and $W_2(s)$ for each position along the approaching course of the ship, any probability of collision can be determined. The failure rate is mainly determined by accidents analysis, simulation, or by transferring such value from other technical systems (Galor, 2005). KUNZI model as well as Eurocode model focus on the process of the ship bridge collision. The former calculates the probability of a collision between the ship on a course to a bridge, while the latter does offer the mathematical equation for $P_C(x, y)$ in the given location (x, y). Therefore, the equation $W_1(s) \cdot W_2(s)$ in KUNZI model are recommended to use when calculating $P_c(x, y)$ in Eurocode model, meanwhile the distribution of the ship location in the y-direction $fs(y)$ should be taken into consideration when calculating the probability of collision. As a result, KUNZI model becomes a concrete formulation of the Eurodecode model. However, it is not so convenient to determine the probability of collision in Eurocode model and KUNZI model as to determine in AASUTO model and Larsen model (Jiang & Wang, 2009).

2.1.5 Dai Tongyu Simplified Model

Based on numerous experiments and data analyses, Dai et al. (2003) formulated a simplified model to calculate the probability of a collision, which applies more to the navigational conditions in China. It is hypothesized that the collision frequency F_i of ship class (i) is relevant to the probability of a collision p_i on the course with potential collisions and the value affecting collisions f_i, the model is then defined as follows,

$$F = \sum_i N_i \cdot f_i \cdot p_i$$

where p_i is determined based on the normal distribution of navigation courses. Based on the distribution of navigation courses of the ship passing the bridge, the mean value μ and the standard deviation σ are calculated in the following model:

$$p_i = \int_A^B \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

The probability of a collision to a bridge refers to the summation of the probabilities that passing ships come into collision with the pier and other structures of the bridge. The mathematical equation is formulated in the following:

$$F = \sum_i N_i \cdot f_i \cdot p_i = \sum_i N_i \cdot f_i \int_A^B \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

where,

N_i = The number of ship class (i) per time unit (traffic intensity);

f_i = The value affecting the collisions ship class (i), such as navigation course, current, weather, ship size, speed, direction etc.;

p_i = The probability of a collision on the course with potential collisions;

μ = The mean value of the location that a ship passes the axis of a bridge;

σ = The standard deviation of the location when a ship passing the axis of the bridge.

The feasibility and applicability of the simplified model has been proved by verifying the ship-bridge collision accidents of *Nanking Yangtse River Bridge* in China. Based on the relevant statistics of the waterway of the mentioned bridge, the value affecting the collisions f varies from 0.05 to 0.12. However, as far as the bigger ship sizes are concerned, the affecting value f may be a bit smaller.

2.2 Limitations in Relevant Models

AASHTO model and Larsen model attach attention to extreme situations, and the calculation focuses on the probability of the bridge destruction, while Eurocode model, Kunzi model and Dai Tongyun simplified model pay attention to all the accidents including the situations that the bridge is not destroyed. Excluding the failure probability, AASHTO Model and Larsen Model will be as practical as the other three models. However, these mathematical models have additional drawbacks in that their samples are not representative enough, the influence of the turbulent zones around piers is not considered in the calculation, and there're some mathematical inaccuracies in the equations.

2.2.1 Insufficient Samples and Doubtful Applicability of Models

Admittedly, excessive stress on the affecting factors is not significant because some of the factors do not affect a lot and even can be ignored. However, when the river system is different, the applicability of those models should be doubtful. Let's take Dai Tongyu simplified model as an example. In the case of *Huangshi Yangtse River Bridge* in Hubei with 20 ship-bridge collisions after it came into use, its hydrological conditions around piers is comparably more complex than those around *Nanking Yangtze River Bridge*. Whether Dai Tongyu simplified model can still be applied to this bridge or not obviously needs further consideration and verification. Dai Tongyu simplified model only verified its applicability in the middle and lower *Yangtze River* and was formulated only based on the hydrological conditions around *Nanking Yangtse River Bridge*. The upper reaches is fast-flowing, with a straight and smooth river way and a "V" font river valley, while the middle and lower reaches is mostly slow-flowing, with a winding river way and a "U" font river valley. Obviously, there is a significant difference between the hydrogeological conditions of the upper and lower reaches. In comparing the upper and lower reaches in just one river system, we do find that the net width of navigable channel, angle between the sailing direction and the axis of a bridge, the stopping distance have changed a lot. The applicability of the models is doubtful, let alone applying Dai Tongyu simplified models to a totally different river system such as *Great Canal* and *Yellow River*.

2.2.2 Neglected Impact of Turbulence Zones around Piers

When a current flows by the piers, there will be vortex which gives attraction to the surface layer around the piers. It's called the turbulence zones, whose width depends on the type of the pier as well as the size and shape of the river under the bridge. When the ship enters the turbulent zones, it will be exerted by an attraction which points to the pier. If we still use the present mathematical models to evaluate the risk regardless of the turbulence zones, we will underestimate the probability of the ship-bridge collision. Some domestic researchers simply include the width of the turbulence zones into the calculation (Gong, 2010), which may fall into the wrong idea that "any boat moving into any area of the turbulence zones will have a collision". Nowadays, the peripheral area of a turbulence zones perhaps can not make any difference to the ships with increasing weight and velocity, so counting the whole width of the turbulence zones without careful consideration can shorten the navigation span, which may cause problems to some bridges. To conclude, the relevant researches to date lack the accurate verification on the influence of turbulence zones on the calculation of collision probability.

2.2.3 Some Inaccuracies in the Mathematical Equations

There're some inaccuracies from the viewpoint of mathematics. Taking Dai Tongyu simplified model as an example, an index i should not be included in the formulation of p_i. The index i means different types of ships, but when calculating the value of p_i, the model uses data and courses of all the ships to get the value of μ and σ, which indicates that the value of p_i means no difference to different types of ships. Thus the model should be revised as follows.

$$F = \sum_i N_i \cdot f_i \cdot p = \sum_i N_i \cdot f_i \int_A^B \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Also, the sum in Larsen Model should be more clearly stated. The index i refers to different types of ships and k refers to different parts of a bridge, which should be indicated more directly. The model should be completed as follows.

$$F = \sum_i N_i \cdot P_{C,j} \cdot \sum_k P_{G,i,k} \cdot P_{F,i,k}$$

3. Ideas on Further Studies

AASHTO model is an empirical formula though most of its parameters are statistical. Those statistical parameters mainly focus on the main piers, which leads to the errors in calculating the collision possibility concerning the transition piers and the piers of approach bridges. KUNZI model and Eurocode model focus on the process of the accidents, without taking into account the wind speed, visibility and navigational aids so that the results of the calculation tend to be relatively larger. Accordingly, the paper makes the recommendations as follows.

3.1 Enlarging the Capacity of Samples

Vessel collision accidents to bridge structures are relatively rare and conditions differ from bridge to bridge. Therefore, the estimation of the risk of collision can not be based on vessel/bridge collisions alone. Collision risk models, stimulating potential collision scenarios are necessary (Larsen, 1993), thus simulating the collision accidents to enlarge the capability of samples is recommended hereby. The stimulation of collision consists of the computer-assisted technique as well as the realistic stimulation technique. The computer stimulation technique, such as FEM stimulation approach, can stimulate numerous characteristics such as the collision force, deformation of the structure or collision energy change. As for the realistic technique, we may choose one bridge which is going to be abandoned in each different river system and put them into a second use. Samples need to be representative, so that we can simulate the collisions with different vessel number (in the morning, at noon, in the afternoon) and in different situations (at night with light interference, upstream, downstream, different visibility etc.). To make the statistics more representative, we should relax the drivers or even deliberately distract the drivers. Getting too close to the piers or being fairly difficult to manipulate the ship when coming into the turbulence zones should be counted as most collisions have actually taken place on account of human errors. Of course, the experiment safety is to be guaranteed by some well planned protective measures.

3.2 Applying Revised Models in Bridge Design

Larsen (1993) and Vrouwenvelde (1998) addressed that risk assessment of the bridges should be based on the probabilistic models. Thereby the paper suggests using the newly revised models to calculate the bridge collision. As is pointed out, the impact of the turbulence zones should be included in the calculation of the probabilistic models. The parameter f_T is here used to represent the influence coefficient of the turbulence zones:

$$f_T = k \cdot f(D, \beta, v_1, v_2, h)$$

The parameter k represents the actual correction factor to influence the moving of the ships, D represents the size of the piers, β represents the angle between the moving direction of the river and the axis of the bridge, v_1 represents the velocity of the water flow in front of the piers, v_2 represents the velocity of the wind in front of the bridge and h represents the depth of the river around the piers. With the influence coefficient of the turbulence zones considered and removing the term of the failure probability, new models with better applicability are addressed in the following:

AASHTO model

$$AF = N \cdot PA \cdot PG \cdot f_T$$

Larsen model

$$F = \sum_i N_i \cdot P_{C,j} \cdot \sum_k P_{G,i,k} \cdot f_T$$

Eurocode model

$$P_C(T) = nTP_{na} \cdot f_T \cdot \iint \lambda(x) P_C(x, y) f_s(y) dx dy$$

KUNZI model

$$P(T) = N \cdot f_T \cdot \int \frac{d\lambda}{ds} \cdot W_1(s) \cdot W_2(s) ds$$

Dai Tongyu simplified model

$$F = \sum_i N_i \cdot f_i \cdot p \cdot f_T$$

From the design point of view, the bridge characteristics would be adjusted or the risk reduction requirements would be implemented until the risk acceptance is satisfied (Knott, 1998). The purpose of the risk assessment is to reduce the collision probability and provide theoretical support for the adjustment and perfecting of the bridge design. After the design proposal of a bridge is scheduled, the latest probability model should be used to simulate, analyze and predict all the possible bridge collisions so that the probability of a collision is minimized before putting the design proposal into construction. Likewise, anti-collision devices and better shipping management are also necessary after a bridge is constructed, for instance we can turn to alarming facilities.

4. Conclusive Remarks

Despite its good practicality, AASHTO model presents larger in the calculation results. Eurocode model features focusing on the process of the accidental action, in which a collision occurs when a vessel approaching the bridge becomes aberrant, or the aberrant vessel hits a bridge element, or the stricken bridge element fails. KUNZI model as well as Eurocode model merely focuses on the process of vessel bridge collision. Therefore, Jiang and Wang (2009) proposed to calculate the collision probability in AASHTO model or KUNZI model, on condition that some adjustments should be taken into consideration based on the domestic navigation conditions. On the basis of the previous researches, this paper has reached the following conclusions:

1) Analyzing the representative models, the paper has further discovered the questionable applicability of models, the neglected affects of pier turbulent zones in the models and some mathematical inaccuracies in the probabilistic models.

2) Accordingly, the paper has completed the probabilistic models with mathematical inaccuracies, and further revised the current models with the influence coefficient f_T aiming to improve the practicality of the probabilistic models.

3) This paper has also proposed increasing the representativeness of samples with sufficient experiments, the application of current researches into the design of bridges, and improving the system of shipping management with the aid of alarming facilities.

The paper has attempted to apply the more verified research findings to the anti-collision technology of the bridges so as to popularize and promote the technology in a real sense. Of course, the bridge anti-collision technology based on risk idea has its limitations. No matter how strong the risk idea-based anti-collision capacity is, even if a pier has the least probability to be impacted and the most accurate alarming systems, we do not have enough time to stop a collision when a ship is fairly close to that pier. Therefore, we still cannot delay the research on the anti-collision devices. Furthermore, ship owners have, in principle, the same interest as bridge owners, since the collision will bring damage and losses to both ship owners as well bridge owners (Manen & Frandsen, 1998). Thereby, only by improving the comprehensive anti-collision technology can we fundamentally ensure the safety of the bridge to fulfill their designed life, as well as the ship owner to escape the losses.

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