

Intraday Volatility Analysis on S&P 500 Stock Index Future

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Abstract

This paper analysed intraday volatility by S&P 500 stock index future product and basic on the high frequency trading strategy. The processes of the model are the GARCH series which including GARCH (1, 1), EGARCH and IGARCH, moreover run such models again by GARCH-In-Mean process. The result presented that EGARCH model is the preferred one of intraday volatility estimation in S&P500 stock index future product. And IGARCH Model is the better one in in-the-sample estimation. Otherwise the IGARCH model is the preferred for estimation in out-of sample and EGARCH model is the better one. GARCH (1, 1) model haven't good performance in the testing. Overall the result will engaged in microstructure market analysis and volatility arbitrage in high frequency trading strategy.

Keywords: Intraday Volatility, High frequency trading, GARCH model, Index future

I. Introduction

Volatility model is one of main approach using in asset return estimation. In high frequency trading field, volatility modelling applied in forecast intraday return varying, the time range almost between 1 minters to 240 minters so that the assets return estimation can be more accurate and efficiency. Current research diversify the volatility model as linear and non-linear, and the most popular and wield use is that generalized autoregressive conditional heteroskedasticity (GARCH, Bollerslev(1986)) model. Pagan and Schwert is the earlier one of apply GARCH model in estimate financial asset return volatility, they estimated stock return volatility by GARCH and E-GARCH model (Nelson (1991)). Franses and Van Dijk (1996) proposed to estimate stock return volatility of Germany, Holland, Spain, and Italy and Sweden stock exchange by non-linear GARCH models, the research result proposed that non-linear GARCH model can significantly improve the linear GARCH model efficiency in volatility forecasting. Anderson and Bollerslev (1998) used them research to confirm that GARCH series is more accurate volatility estimation model. Other hand, Admati and Pfleiderer (1988) have researched price and volume by intraday patterns, it is an earlier intraday price volatility forecast theory. Engle, Ito and Lin (1990) researched intraday volatility in foreign currency market. Bessembinder and Seguin (1993) researched the intraday price, volume and market depth by high frequency data. It is the first empirical high frequency research which focuses on future market. However, current high frequency research almost focuses on equity and foreign currency market but stock index future markets, just few papers had researched about it.

This paper I'm going to study intraday volatility analysis in S&P 500 index future products by GARCH model series, there are GARCH (1, 1), E-GARCH and I-GARCH models. By the S&P 500 historical data, and going to estimate ultra frequency ,each 1 minter, 5 minters, 10 minters and 15 minters return volatilities varies for statistic volatility trading opportunities.

II. Data and Methodology

This paper study S&P 500 stock index future in which is the main financial future contract in the world. The intraday data of S&P 500 index future is obtained from TickData Company, the time period from 1st September 2008 to 30th September 2009. The price records are including 1 minutes, 5 minutes, 10 minutes, 30 minutes and 60 minutes sets. The logarithm returns of different times calculate as $R_t = \log(p_t/p_{t-1})$, where p_t is the future price of the period t and p_{t-1} is the future price of immediately preceding period.

The estimation of will calculate the rate of return firstly. Then test whether the rate of return exist the unit root or not by Augmented Dickey-Fuller (ADF, 1979) and Phillips and Perron (PP, 1988) models, and different the data until it is stationary if it has unit root phenomenon. In volatility forecast, first to test whether the rate of return is or not conditional variance by ARCH-LM model and then forecast the volatility in different time by GARCH-type models.

In case of financial price and return volatility has significantly heteroskedasticity feature. Generalized autoregressive conditional heteroskedasticity model is more usable in return volatility estimation field. According to (Engle (1982)), the GARCH (p, q) model can write as:

$$\alpha_t = \varepsilon_t \sigma_t \quad (1)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \alpha_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

Where σ_t^2 is current time conditional variance, α is intercept, σ_{t-1}^2 is the conditional variance on formal time, β is parameter, means that the influence of old news to market volatility, γ is parameter means that the influence of new positive to market volatility, α_{t-1}^2 is sum of error square on last time and ε_t is white noise.

Normally, the price and return volatility is estimated by GARCH (1, 1) model which is

$$\alpha_t = \varepsilon_t \sigma_t \quad (2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \alpha_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Volatility always has persistent changes, in this case Nelson (1990) designed Integrated GARCH (Note 1) model (IGARCH) process. A GARCH (p,q) process is stationary with a finite variance if

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$$

A GARCH (p,q) process is called IGARCH process if

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j = 1$$

In finance field, the leverage effect predicts that an asset's return may become more volatile when the price decreases. Exponential GARCH (Note 2) EGARCH) model process is designed to model the leverage effect by Nelson (1991). The model is

$$\log(\sigma_t) = \alpha_0 + \sum_{i=1}^q \alpha_i g(\varepsilon_{t-i}) + \sum_{j=1}^p \beta_j \log(\sigma_{t-j}) \quad (3)$$

Where

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma(|\varepsilon_t| - E(|\varepsilon_t|))$$

In some estimations, it make sense to use the conditional standard deviation as one of regression variables, when the dependent variables is a return then we might expect that higher conditional variability causes higher returns, because the market demands a higher risk premium for higher risk. Model where the conditional standard deviation is a regression variable called GARCH-in-mean (GARCH-M) which can write as:

$$r_t = X_t' \gamma + \sigma_t + \varepsilon_t \quad (4)$$

Where σ_t is the GARCH process with conditional standard deviation process σ_t . σ_t and components of X_t are the predictor variables and γ and the components of γ are the regression coefficients. Here σ_t must be estimated.

In the process of estimation, firstly set the model for calculate square of return by as follows equation:

$$\sigma_{t+1}^2 = a + b h_{t+1}^2 + u_t \quad (5)$$

Where σ_{t+1}^2 is the true volatility and h_{t+1}^2 is the volatility in forecast.

According to Pagan and Schwert (1990), function (5) will write as log form as:

$$\log(\sigma_{t+1}^2) = a + b \log(h_{t+1}^2) + u_t \quad (6)$$

Second set the root mean square model for calculate the different between estimation and true value of volatilities, the RMS equation is

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T (h_t^2 - \sigma_t^2)^2} \quad (7)$$

Where T is the number of observations.

III. Empirical Result

1. Descriptive Statistic

From 6 frequency descriptive statistical calculation, we knew that 1 minute unite data has the biggest value of mean standard deviation than others. And the maximum and minimum values of ultra data are biggest and lowest than others. 1 min data has biggest standard deviation which means that each 1 min value has significantly different with the mean and also the data is much more discrete. Ultra and 1 min unites data has left side slopes and others has right slopes and all of the data unites has the feature of leptokurtic distribution.

Insert Table 1 Here

2. Unit Root Test

For test the whether the data is stable, we normally test it by ADF and PP models, the hypothesis are:

H_0 : Unit root is existing

H_1 : Unit root is not existing

Table 2 shows all frequencies ejected H_0 , and then there have not unite root effect, the data series are stable.

Insert Table 2 Here

3. Heteroskedasticity Test

Before estimate the volatility by GARCH-type model, it is necessary to test whether the conditional heteroskedasticity is existing in variance. The hypotheses of the testing are:

H_0 : Non-ARCH Efficiency

H_1 : ARCH Efficiency

Table 1 show all frequencies ejected H_0 . All data series has heteroskedasticity efficiency. Then the GARCH-type testing is available.

Insert Table 3 Here

4. Realized Volatility Statistics

From figure 1 to 6 are the realised volatilities of ultra to 60 minutes.

Insert Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, Figure 6 Here

From the above figures we can easily find that the volatility clustering features in each frequency team. Otherwise in case of the number of observations are too much huge so that the clustering phenomenon maybe not analyse clearly.

5. Forecast Results

The following table presents the results from the forecast models which estimated by both in the sample and out of sample data. For each frequency of the data and each of forecast model the best one ranked estimation are signed as symbol of *.

Table 4 presents the results for the in the sample as the measure of realized volatility. From the results we know that GARCH (1, 1) model has 3 times ranked in the best model which has including GARCH –M type. EGARCH (EGARCH-M) has 4 times ranked in best estimated model and as well as 4 times ranked record for IGARCH (IGARCH) model. General speaking IGARCH model is the preferred model for forecast the intraday volatility according to the value of root mean squared error. And the EGARCH model can be the alternative model. GARCH (1, 1) model has better performance on the ultra and 10 minutes volatility forecast, however in another frequency volatility forecast has low performance. Which means that GARCH (1, 1) model could not be the better model for using in really market. EGARCH model had great performance on both Ultra and 1 minutes frequency volatility estimation, especially in 60 minutes forecast it has lowest root of mean squared error, the model is more accurate in the estimation of 60 frequency. Otherwise in the Ultra-frequency estimation, EGARCH got highest value of R square. IGARCH model has better good performance because it was the much

more accurate model in case of it has three times ranked in lowest root of mean square error. Even though it didn't get higher R square value, however in real market the accurate estimation approach is very important.

Table 5 presents the result of out of same data. From the result we know that EGARCH model is 5 times ranked in best model, and IGARCH model has 5 times great ranking as well. Compare with EGARCH and IGARCH models in out sample estimation, IGARCH model is 4 times ranked in the lowest root of mean square error, which means that IGARCH model is more accurate than other two type of GARCH model. Especially in 60 minutes frequency volatility forecast, IGARCH model had great performance. Otherwise, IGARCH model had great performance on ultra frequency forecast, which mean that IGARCH model is able to capture the ultra market volatility. GARCH (1, 1) model has lowest performance on out of sample forecast.

Insert Table 4, Table 5 Here.

IV. Conclusion

This paper tired to analysis intraday volatility by GARCH series models, the result is that EGARCH model is the best one of intraday volatility estimation in evidence of S&P500 stock index future market. And IGARCH Model is the better one in in-the-sample estimation. Otherwise the IGARCH model is the preferred for estimation in out-of sample and EGARCH model is the better one. GARCH (1, 1) model haven't good performance in the testing. Otherwise, High frequency trading is a most popular trading strategy in current financial market. It has already got huge success from Foreign exchange, future and equity markets. It normally focuses on tick data and intraday volatility and characterized by high number of trades and a lower average gain per trade. Moreover, the crucial factor of high frequency trading successful is that accurate forecast the volatility of asset.

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Notes

Note 1. For more about IGARCH model see Nelson (1990).

Note 2. For more about EGARCH model see Nelson (1991).

Table 1. The result of descriptive statistic

Time Frequency	Ultra	1 min	5 mins	10 mins	30 mins	60 mins
Observations	1048574	1048526	1048347	1048257	1047933	1047722
Mean	5.34e-09	-3.15E-06	5.45E-09	5.04E-09	5.54E-09	5.52E-09
Median	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	0.0011	1.66E-05	8.04E-06	6.11E-06	4.21E-06	2.91E-06
Minimum	-0.001	-0.0331	-8.99E-06	-6.45E-06	-3.87E-06	-3.16E-06
Std Dev	4.64e-06	0.0032	8.65E-07	7.23E-07	4.99E-07	4.27E-07
Skewness	0.0111	0.0640	-0.0115	-0.0442	-0.0563	-0.0882
Kurtosis	6.3097	1.2311	4.3789	4.2345	4.0843	3.6729
Jarque-Bera	4.7861	3.3048	83085.66	66906.07	51892.99	21128.79
(Probability)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 2. Unit root testing by ADF and PP models

Time Unite	Model	Testing value	P-Value
Ultra	ADF	-274.6107***	0.0001
	PP	-3.4302***	0.0001
1 minute	ADF	-19.9243***	0.0001
	PP	-3.9721***	0.0001
5 minutes	ADF	-50.3413***	0.0001
	PP	-383.4098***	0.0001
10 minutes	ADF	-41.7706***	0.0001
	PP	327.0710***	0.0001
30 minutes	ADF	-29.5261***	0.0000
	PP	-241.8626***	0.0001
60 minutes	ADF	-25.8545***	0.0001
	PP	-175.0171***	0.0001

***Significant at the 1% level

Table 3. The result of GARCH testing

Frequency		ARCH Test		
Ultra	F-statistic	130168.3***	P-value	0.0000
	OBS*R-squared	115794.0		0.0000
1 minute	F-statistic	276411.0***	P-value	0.00002
	OBS*R-squared	202529.0		0.00002
5 minutes	F-statistic	9327757.0***	P-value	0.0000
	OBS*R-squared	942426.9		0.0000
10 minutes	F-statistic	13005197.0***	P-value	0.0000
	OBS*R-squared	970066.1		0.0000
30 minutes	F-statistic	25987237.8***	P-value	0.0000
	OBS*R-squared	1007312.33		0.0000
60 minutes	F-statistic	33669215.0***	P-value	0.0000
	OBS*R-squared	1016101.92		0.0000

***significant at 1% level

Table 4. Forecast result (In –the –sample)

	R^2	Likelihood	RMSE
Frequency-Ultra			
GARCH(1,1)	0.00003	9077507	0.000046
GARCH(1,1)-M	0.00001	9077511	0.000037*
EGARCH	0.00033*	9077869	0.000031
EGARCH-M	0.00021	9099176*	0.000052
IGARCH	0.00019	9098547	0.000041
IGARCH-M	0.00019	9098178	0.000039
Frequency-1 min			
GARCH(1,1)	0.000018	9079565	0.000061
GARCH(1,1)-M	0.000012	9078569	0.000049*
EGARCH	0.000025*	9077795	0.000052
EGARCH-M	0.000019	9099864*	0.000050
IGARCH	0.000022	9098979	0.000053
IGARCH-M	0.000021	9098855	0.000052
Frequency-5 min			
GARCH(1,1)	0.00013	13536332	0.000865
GARCH(1,1)-M	0.00010*	13536221	0.000843
EGARCH	0.00016	13537656	0.000801
EGARCH-M	0.00014	13537459	0.000795
IGARCH	0.00018	13537884*	0.006448
IGARCH-M	0.00016	13537256	0.006232*
Frequency-10 min			
GARCH(1,1)	0.00085*	13728807*	0.000732
GARCH(1,1)-M	0.00082	13728791	0.000707
EGARCH	0.00086	13720854	0.000665
EGARCH-M	0.00081	13720800	0.000633
IGARCH	0.00067	13726475	0.000591
IGARCH-M	0.00065	13725598	0.000557*
Frequency-30 min			
GARCH(1,1)	0.00122	14174404	0.0499
GARCH(1,1)-M	0.00116	14173589	0.0463
EGARCH	0.00130*	14118965	0.0236
EGARCH-M	0.00114	14117852	0.0208
IGARCH	0.00108	14116657	0.0115
IGARCH-M	0.00104	14109638	0.0107*
Frequency-60 min			
GARCH(1,1)	0.00877	14372799	0.7133
GARCH(1,1)-M	0.00852	14371082	0.7047
EGARCH	0.00925	14379514	0.5011
EGARCH-M	0.00891	14378458	0.4788*
IGARCH	0.01015	14589647	0.6026
IGARCH-M	0.01007	14586571	0.6953

Table 5. Forecast result (Out –of –sample)

	R^2	Likelihood	RMSE
Frequency-Ultra			
GARCH(1,1)	0.00005	9056981	0.000033
GARCH(1,1)-M	0.00002	9053987	0.000021*
EGARCH	0.00067*	9069327*	0.000044
EGARCH-M	0.00060	9067409	0.000081
IGARCH	0.00023	9066698	0.000047
IGARCH-M	0.00018	9060578	0.000065
Frequency-1 min			
GARCH(1,1)	0.000021	9004953	0.000057
GARCH(1,1)-M	0.000018	9003369	0.000050*
EGARCH	0.000026	9020357	0.000055
EGARCH-M	0.000022	9013998	0.000048
IGARCH	0.000028*	9087895*	0.000059
IGARCH-M	0.000021	9078924	0.000052
Frequency-5 min			
GARCH(1,1)	0.00011	13426119	0.000994
GARCH(1,1)-M	0.00007	13410078	0.000872
EGARCH	0.00019*	13512761*	0.000773
EGARCH-M	0.00015	13510098	0.000769
IGARCH	0.00017	13509298	0.006301
IGARCH-M	0.00013	13509200	0.006298*
Frequency-10 min			
GARCH(1,1)	0.00109	13366072	0.001251
GARCH(1,1)-M	0.00095	13320481	0.001108
EGARCH	0.00113*	13777456*	0.000923
EGARCH-M	0.00101	13740994	0.000874
IGARCH	0.000933	13728920	0.000687
IGARCH-M	0.000884	13719987	0.000634*
Frequency-30 min			
GARCH(1,1)	0.00148	14048689	0.0801
GARCH(1,1)-M	0.00123	14068971	0.0699
EGARCH	0.00151	14035879	0.0558
EGARCH-M	0.00114	14048495	0.0639
IGARCH	0.00167*	14096548	0.0209*
IGARCH-M	0.00132	14109638*	0.0183
Frequency-60 min			
GARCH(1,1)	0.00904	14259769	0.8842
GARCH(1,1)-M	0.00889	14223694	0.8181
EGARCH	0.00934	14310015	0.7915
EGARCH-M	0.00922	14300358	0.7558
IGARCH	0.01007*	14589647*	0.4789
IGARCH-M	0.00992	14586571	0.4242*

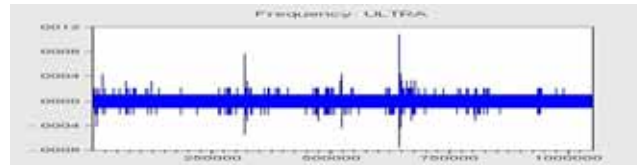


Figure 1. Realized volatility of S&P 500 index future by ultra-Frequency

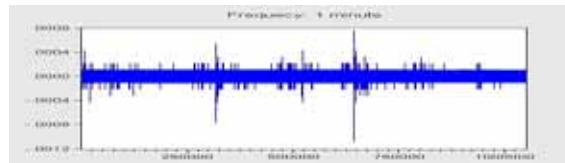


Figure 2. Realized volatility of S&P 500 index future by 1 minute Frequency

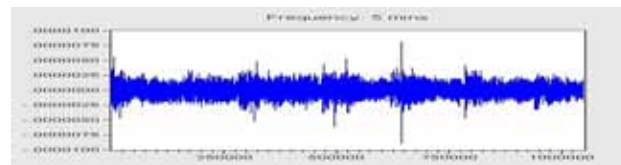


Figure 3. Realized volatility of S&P 500 index future by 5 minute Frequency

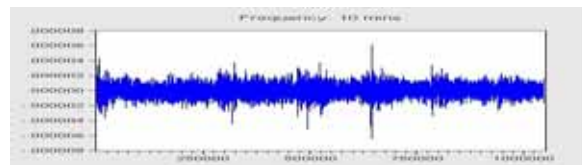


Figure 4. Realized volatility of S&P 500 index future by 10 minute Frequency

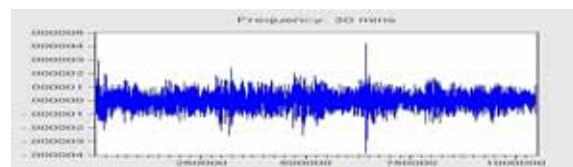


Figure 5. Realized volatility of S&P 500 index future by 30 minute Frequency

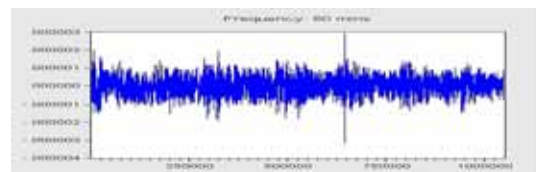


Figure 6. Realized volatility of S&P 500 index future by 60 minute Frequency