Regional Growth in the Euro Mediterranean Countries: Effects of Increasing Returns and Spatial Externalities

Amina NACEUR SBOUI (corresponding author)
Faculty of economic sciences, Tunis El Manar University, Tunisia
Tel: 216-97-439-064   E-mail: naceuramina@yahoo.fr

Mohamed Amine HAMMAS
Faculty of Economic Sciences of Sousse, Tunisia
Tel: 216-22-416-443   E-mail: hammasamine@yahoo.fr

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Abstract
Most of the recent contributions analysing the regional disparities in growth have stressed on the convergence equation which derived from the traditional neoclassic model of Solow (1956). However, this type of approach presents the main drawback that is the lack of consideration for increasing returns to scale, which are at the origin of endogenous growth and new economy geography models. In that purpose, we find more appropriate to use Fingleton’s (2001) model. We develop a spatial model to endogenously detect the presence of spatial externalities and to estimate their effect on regional growth. We extend this specification to a panel of 26 euro Mediterranean countries over the period 1995-2004. Results of estimation with recent tools of spatial econometrics permit to detect the presence of increasing returns to scale. In addition, we conclude that the external effects are geographically burned and they are associated to a substantive phenomenon meaning that the growth in a country depends on the growth in neighbouring countries via pecuniary and technological externalities. These results are related to the predictions of endogenous growth theory and new economy geography model.

Keywords: Regional disparities, Endogenous growth, New economy geography, Increasing returns to scale, Spatial econometrics

1. Introduction
There has been a remarkable surge of interest in ‘geographical economics’ or ‘the new economic geography’, prompted by the publication of the book by Fujita, Venables and Krugman(1999). This new wave of theory put economic geography centre stage within mainstream economics, since it established the notion that increasing returns could coexist within a theoretical framework with explicit microeconomic foundations. Regional science and regional economics, which had tended to be somewhat marginalized, has now become a focus of attention. However the development of formal models has been at a cost, for although the idea of externalities is central to the new economic geography theory, and in related urban economic theory (Abdel Rahman and Fujita, 1990, Rivera Batiz 1988), in the purest form of these models the only externalities present are pecuniary externalities, representing market interdependence. The idea that technological externalities are also relevant is somehow squeezed out, being too difficult to accommodate within formal models. Or, the endogenous growth models (Marshall, 1980; Saxenian, 1994; Jaffe, 1989, Feldman, 1994) show that this type of externalities is likely favour the agglomeration phenomenon. Thus, the endogenous growth theory underlines their role in terms of growth.

The integration between the endogenous growth theory and the geography economic distinguish between the two types of agglomeration admitting a particular importance to the agglomeration of activities of innovation (Englmann and Wals, 1995; Martin and Ottaviano, 1999). In side, of pecuniary externalities (related to market interdependence) influencing the production, are introduced the technology externalities (related to hors market interdependence) influencing particularly the innovation. As in the case of endogenous growth models with R&D, the output of research in a firm is favoured by the importance of activity in other firms in the economy. The activities of innovation are thus incited to locate in the region where the number of innovate firms is higher.

The industrial agglomeration results then from the imbrications of these phenomenon: “an economic agglomeration is created as well by the technological externalities as by pecuniary externalities” (Fujita and Thisse, 1997).

The empirical analysis of these forces of agglomeration can be treated by recent tools of spatial econometrics. Without controlling also for externalities, in the form of spillovers between regions, the spatial models are invariably poorly
specified and fail the diagnostic tests that are the accepted professional standards of the spatial econometrics community. Various approaches have been adopted in attempting to introduce externalities into spatial econometric models, with two main strands appearing in the literature. One treats the externalities in a somewhat ad hoc manner as random shocks, which impact within a specific region and simultaneously spill over into other, frequently neighbouring, regions. In these models there is no attempt to explicitly model the sources of these external effects. The second strand attempts to model the causes of the externalities.

However, in this paper we are interested in investigating the link between the spatial externalities and their impact on the economic growth of the Euro Mediterranean countries. We will base on the theoretical growth model with increasing returns to scale of Verdoorn, to which we introduce the spatial dependence. We propose to apply the appropriate spatial econometrics tools to test the presence of these externalities and to estimate their magnitude on the regional growth.

On this base, the paper is organized as follow: In Section 2 we describe specification of Verdoorn’s law with different types of spatial dependence and we discuss the type of externalities related to the resulting specification. Section 3 presents the major characteristics of a model growth that includes externalities across regions caused by technological diffusion. It is shown the similarity between his empirical specification and the spatial lag model. In section four, we present data and the weight matrices used. Results of estimation based on a panel of 26 euro Mediterranean countries over the period 1995-2004 are reported in section 5. Finally, section 6 concludes.

2. Theoretical model of Verdoorn’s law and spatial dependence

Empirical implementation of the Verdoorn’s law is initially proposed by Kaldor (1957) and successively extended by Kaldor (1970) and Dixit and Thirlwall (1975). They suppose that the motif of growth results from a linear relation between the growth of productivity and output.

Then, in its simplest form, the empirical specification for the dynamic Verdoorn’s Law can be written as:

\[ p = b_0 + b_1 q + \varepsilon \]  

Where \( p \) is the level of growth of productivity, \( q \) is the growth of output and \( \varepsilon \) is an error term. The usual interpretation of parameters in (1) is that a value of \( b_1 \) around 0.5 implies the existence of increasing returns to scale. (Note 1)

The introduction of spatial dependence in the Verdoorn’s Law permits to consider the geographic externalities. Different types of models can be used to treat the spatial dependence in observations. Applied for the Verdoorn’s Law, they give different interpretations of coefficients associated to geographic externalities. We study the case of two models which are more used: the lag spatial model and the spatial error model.

2.1 The spatial Lag model:

In the spatial lag model we allow the growth of productivity in country i to depend on a weighted average of growth rates of its neighbours, in addition to the explanatory variables of Verdoorn’s Law. The model in equation [1] becomes. (Note 2):

\[ p = b_0 + b_1 q + \rho W p + \varepsilon \]  

Where \( \rho \) is a parameter indicating the extent of the spatial interaction between observations with non-zero entries in \( W \), the spatial weights matrix. Note that this implies that the growth rate of technology in each country depends not only on the values of the explanatory variables in that country, but also on the values of the explanatory variables in other countries, subject to distance decay. This can be seen by expressing the model in reduced form:

\[ p = (I - \rho W)^{-1} ( b_0 + b_1 q + \varepsilon ) \]  

This expression indicates that, a marginal increase in output in country i has a direct effect on the growth rate in that country, and an indirect effect on the growth rate of its neighbours. In addition, the original direct and indirect effects result in induced effects in the neighbours of the neighbours of country i, and in turn in the neighbours of those neighbours, and so on throughout the whole system, including some feedback effects on country i itself. The total effect of a marginal increase in output is therefore equal to the sum of the direct, indirect and induced effects, and its magnitude differs across countries.

2.2 The spatial error model

In the spatial error model the spatial dependence is restricted to the error term. Intuitively, we can think of the spatial dependence working through omitted variables with a spatial dimension (climate, social norms, exogenous shocks), so that the errors from different countries are spatially correlated. Equation [1] becomes:

\[ p = b_0 + b_1 q + \varepsilon \]  

\[ \varepsilon = \lambda W \varepsilon + \mu \]
Where, $\lambda$ is a parameter indicating the extent of the spatial correlation between the errors.

Note that since $\varepsilon = (I - \lambda W)^{-1}\mu$, the model can be rewritten as follows:

$$p = b_0 + b_1 q + (I - \lambda W)^{-1}\mu$$

[5]

Estimation of this model using OLS or GLS result unbiased but inefficient estimates. It should therefore be estimated using maximum likelihood or general method of moments (Anselin, 1988). (Note 3).

3. Substantive spatial externalities in the Verdoorn’s law

In its initial form the Verdoorn’s law seems to be more simple and doesn’t permit to characterize the endogenously of progress of technology. Moreover, this specification permits to cutch only the line between the growth of productivity and the growth of output whereas other factors can influence the growth of output especially in a regional level. Then, we carry the specification of Fingleton (2000, 2001) in which the progress of technology depends on the geographic spillovers, the level of initial technology and the level of human capital in regions.

Supposing that technology change is proportional to the accumulation of capital (in form of capital growth) and the capital growth is equal to the productivity, we have the following relation:

$$\lambda = \dot{X} + \phi p + \sigma W p$$

[6]

Where, $\Phi$ and $\sigma$ are coefficients and $W$ is the weight matrix which catches the effects of spatial externalities between the regions. $\lambda$ is proportional to the productivity growth intra-regional and also extra-regional. Thus, we explicitly consider the role of pace by allowing the model to take into account the spillover effect included in the third term on the right-hand side of equation [6]. The growth of productivity in a region depends on the growth in the neighbouring regions by the existence of effect of spillovers via the technology progress. The term $\dot{X}$ depends on the socio-economic conditions of each region such as the initial level of technology and the level of human capital of each region.

The initial level of technology is introduced by the mean of a technology gap between each region and the region leader to cutch the possible effect of diffusion of innovation from a region with high level of technology to a region with law level of technology. This is based on the following hypotheses: first, the differences in level of technology imply differences in productivity (Barro and Sala-i-Martin, 2004). Second, the regions more rich technologically know the growth via the innovation whereas the regions accusing a technological backward proceed by an imitation and adapt technologies of region leader (Baumol, 1986). In consequence, the technology diffusion in backward regions can induce growth faster in these regions and the impact of the region leader is more important as the technology gap is high (Abramovitz, 1986 ; Gomulka, 1987 ; Targetti and Foti, 1997).

Using the approach of Nelson and Phelps (1966), the human capital is supposed to be an increasing function of the level of education, in the measure where an important human capital stock is supposed favour the adoption of foreign technologies successfully and permits to determine the capacity of a nation to develop new ideas.

The introduction of these elements in the specification of Verdoorn permits not only the introduction of concepts of endogenous growth theory, but also those of geographic economy. Indeed, the increasing returns to scale and the effects of spillovers are the fundamental hypotheses of two theories (Englmann and Walz, 1995; Walz, 1996; Martin and Ottaviano, 1999; Baldwin and Forslid, 2000; Rezgui, 2004). Thus, this new specification seems pertinent to study the regional growth following these theories.

After some algebra, we propose a different specification for the Verdoorn’s Law, that is:

$$p = \sigma W_1 p + b_0 + b_1 q + b_2 G + b_3 H + \varepsilon$$

[7]

Where, for yearly data, $p$ denotes annual growth rates, a bold character represents a vector $[N*(T-1) x 1$ with the information for each region and time period ($t=2, ..., T$). $W_1 p$ is the spatial lag for the growth rates of labour productivity.

The other variables are defined as following:

* $G$ a vector $[N*(T-1) x 1$ corresponds to the technology gap (approximated by the differential in productivity) in every year between each region of the sample and the region leader.

* $H$ a vector $[N*(T-1) x 1$ corresponds to the level of human capital. It was measured by the population with secondary education and more.

* Finally, $W_1$ is a $[N*(T-1)]x[N*(T-1)]$ matrix with the following general expression:
\[ W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W & 0 & 0 \\ 0 & 0 & W & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & W \end{bmatrix} \]

\[ W_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W & 0 & 0 \\ 0 & 0 & W & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & W \end{bmatrix} \]

\( 0 \) being a \((N \times N)\) matrix of zeros and \( W \) a \((N \times N)\) spatial matrix of weights.

We note that in the literature of spatial econometrics, the specification [7] corresponds to the spatial autoregressive model, in which the coefficient \( \varpi \) reflects the presence of effects of spatial externalities, i.e. the growth in contiguous regions (defined by the weight matrix \( w \)) affects the growth of productivity (via the technology progress) of the considered region.

4. Data and spatial weights matrix

For yearly data, we use the variable “growth of labour productivity on the period \( t \) and \( t-1, t = 2, \ldots, 10 \)” as an endogenous variable. Labour productivity is obtained by dividing “GDP at constant price 2000” by “number of population in employment”. Data on these two variables and the level of education are obtained from the database: “World Indicator Data; W.I.D” version 2006.

Annual data on technology gap (\( Q \)) are approximated by the difference between value of labour productivity in each country and the value in the Leader country (the country with highest value of labour productivity).

We will expose results of empirical validation on a specific sample of Euro Mediterranean countries which is composed from:

1) First, the fifteen countries of the European Union: Germany, France, Italy, Luxembourg, Netherlands, Greece, Spain, Portugal, Ireland, Denmark, Sweden, Finland, Austria, Belgium and United Kingdom.

2) Second, the eleven south Mediterranean countries: Algeria, Tunisia, Cyprus, Egypt, Israel, Lebanon, Jordan, Morocco, Turkey, Syria and Malta.

NB: for the reason of the absence of data availability, the country « Autonomy Palestinian Territory » will be excluded from the analysis.

The period of our analysis is 1995-2004. This period is particularly interesting for the analysis of the impact of spatial externalities on economic convergence, since it corresponds to crescent economic integration between the Euro Mediterranean countries, marked by acceleration of liberalisation of exchanges and by a widening of market. It corresponds also, after the reform of structural founds, to the existence of important regional political aiming to harmonise the potentialities of territories development.

4.1 Definition of weight matrix \( W \):

The concept of weight matrix constitutes a fundamental element in Spatial Econometrics because it permits to model the interactions between observations. Generally, two principal conceptions are reserved to the determination of the elements of the weight matrix, respectively founded on the principle of contiguity and on the principle of distance.

4.1.1 The matrix of contiguity

The matrix of contiguity reposes on the sharing of a common frontier between spatial unities. Formally, a contiguity matrix represents each localisation of spatial system in line and in column. The “spatial weights” (elements of weight matrix) \( w_{ij} \) of matrix of contiguity \( W \) are then defined by the following expression:

\[ w_{ij} = \begin{cases} 1 & \text{if regions } i \text{ and } j \text{ are contiguous for order } 1. \\ 0 & \text{else.} \end{cases} \] \[ [10] \]

Moreover, a same contiguity matrix can represent different arrangements of spatial units: this is the problem of topological invariance (Cliff et Ord, 1981, p. 21). Then, others weight matrices appear useful.

4.1.2 The matrix of distance

The matrix of distance reposes on the idea that two spatial units know high (respectively low) interaction that the distance between them is low (respectively high). Cliff and Ord (1973, 1981) are the first ones that used this type of specification, by combining a function of the reverse of the distance that separates two localisations and the relative length of their common frontier. However, recently the most current specifications in the empirical studies use expressions more simple for the spatial weights.

In our work, we carry a simple matrix of distance based on the reverse of the distance that separate spatial units. In this case the elements of this matrix \( W_{ij} \) are defined as following:
\[ w_{ij} = \frac{1}{d_{ij}}, \quad \text{where} \quad d_{ij} \text{ is the distance that separate the centroids of countries } i \text{ and } j. \]  

However, the matrices of weights are standardised in lines for facilitating interpretation of spatial parameters after estimation. Thus, each line \( i \) of matrix of weight \( W \) is divided by the sum of elements \( w_{ij} \) that compose it and the resulting spatial weights are:

\[ w_{ij}^* = \frac{w_{ij}}{\sum_j w_{ij}} \]

The standardisation of the matrix of weights permits to compare the spatial parameters issued from different models.

5. Results of estimations

On the base of panel data in the present section, we will proceed stepwise. Firstly, the model without spatial dependence is considered. Then, we will move forward to considering the issues deriving from spatial autocorrelation.

5.1 Estimation of a-spatial model

We start with estimation of a-spatial model (model [2.2] without the endogenous lag variable). We must note that in case of the estimation of a model with the method of panel data, the model can be with individual fixed effects (Within estimator) or with individual random effects (Estimator with GLS). Results of estimation of the equation of convergence under the two specifications of panel are presented in the table 1:

- For the two models, we observe that the coefficient of Verdoorn (coefficient of output) is positive and statistically significant which corroborates the presence of the increasing returns to scale. This coefficient is more significant for the random effects model.

- The coefficient of human capital is also positive, indicating that the level of education facilitates the absorption of foreign technology and the creation of new innovations. An increase of the level of education in one country with one unit raises the labour productivity growth by 31.7% in the case of model with fixed effects and by 45.6% in the case of model with random effects.

- The coefficient of technology gap is not significant in the two specifications.

- We not that the test of fisher is significant at 5% level, indicating the presence of individual effects. To choice between the two types of specification for individual effects (fixed effects or random effects), we use the test of Hausman (1978). The decision rule of this test is the following: if the realisation of the statistic is higher than \( \chi^2(K) \) (Note 4) at \( \alpha \)% level we reject the hypothesis null and we privilege the adoption of individual fixed effects and the Within estimator is unbiased. On the contrary, if the statistic of Hausman is smaller than \( \chi^2(K) \) at \( \alpha \)% level, we accept the hypothesis null and we privilege the adoption of random individual effects and the use of GLS estimator. In our case and for a level of tolerance of 5%, we remark that the value of the test of Hausman is low and not significant, indicating the absence of the correlation between the regional specific effects and the explicative variables of model (the hypothesis \( H_0: \text{corr}(\mu_i, X_{it}) = 0 \) is not rejected). Thus, the estimators of the model with random effects are convergent (Mundlak, 1978). (Note 6).

5.2 Tests of spatial dependence

We use Anselin (1988) and Anselin et al. (1996) tests to detect the presence of spatial dependence. In order to identify the form of the spatial dependence (spatial error model or spatial lag), the Lagrange Multiplier tests (resp. \( LM_{ERR} \) and \( LM_{LAG} \)) and their robust version are performed. The decision rule is subject to Anselin and Florax (1995): if \( LM_{LAG} \) (resp. \( LM_{ERR} \)) is more significant than \( LM_{ERR} \) (resp. \( LM_{LAG} \)) and \( R-LM_{LAG} \) (resp. \( R-LM_{ERR} \)) is significant whereas \( R-LM_{ERR} \) (resp. \( R-LM_{LAG} \)) is not, then the most appropriate model is the spatial lag model (resp. the spatial error model).

Results of these tests and the associated p-value are presented in the table 2 using the matrix of contiguity (first column) and the matrix of distance (second column).

Adopting the decision rule of Anselin and Florax (1995) we can conclude that the spatial model with endogenous variable is the more appropriate specification for the analysis of the effects of spatial externalities on the regional growth in the euro Mediterranean countries. These externalities are associated to a substantive phenomenon implying that the growth of labour productivity in one country affects the growth in neighbouring countries.

In the next paragraph we will present results of estimation of the spatial model with endogenous lag variable. This specification is conformable to the one in Fingleton (2001) (equation [2.2]).

5.3 Estimation of the empirical specification

Table 3 resumes the results of estimation of equation [2.2] in the case of model with random effects, by the Maximum Likelihood method with the matrix of distance (first column) and the matrix of distance (second column).
• We note that in this specification as in the case of a-spatial model, the coefficient of Verdoorn and the coefficient of human capital are also positives and statistically significant for the matrix of distance as well as for the contiguity matrix.

• In this model, we note that the coefficient of technology gap is negative and significant at 5% level meaning the absence of a technology catch-up effect in the euro Mediterranean countries. There are many countries possessing law levels of productivity can’t catch up the Leader country (Germany) which possesses the high level of technology.

• The coefficient of spatial dependence is high and statistically significant ($\hat{\alpha} = 0.831$, with the matrix of distance and $\hat{\alpha} = 0.673$ with the matrix of contiguity). This indicates that when the growth of labour productivity in the neighbours of a country i increase with one unit, the growth of labour productivity in this country increase with 83.1% when we use the matrix of distance and increase with 67.3% when we use the matrix of contiguity. This result confirms that the spatial externalities in the euro Mediterranean countries are associated to a substantive phenomenon implying that the growth in a country affects the growth in neighbours by the effect of spatial externalities, for example the increase in the level of human capital in a country influences on the growth of the labour productivity in this country, but also influences on the growth of labour productivity in the neighbouring countries. Moreover, with the relation of the two weight matrices used in our empirical validation which based on the contiguity and distance effect, we note that the spatial dependence decrease with the distance separating countries of our sample. This shows that the technology diffusion is geographically burned which influences the growth inequality between the north and the south of the euro Mediterranean countries. Thus, the economic integration doesn’t permit to eliminate barriers of the absorptive capacity of new technology from the north to the south.

6. Conclusions

In this paper, our aim is to analysis the effects of spatial externalities on the regional growth on the base of the common hypothesis of new economy geography theory and the endogenous growth theory “increasing returns to scale”. The integration of these two theories suppose that the increasing returns of scale results from the pecuniary externalities (representing market interdependence) and the technology externalities (representing hors market interdependence), and these externalities are geographically burned. To analyse this new approach we have started by the modification the equation of Verdoor’n Law to obtain an empirical specification incorporating the increasing returns to scale and also the effect geographic spillovers via the progress of technology. Thus, we suggested that the recent methods of spatial econometrics constitute powerful tools to endogenously detect the effect of spatial externalities on the regional growth.

In our empirical validation we are based on a panel of an integrated space of the euro Mediterranean countries observed on the period 1995-2004. Results of estimation of our empirical specification indicate that increasing returns are significant in determining the level of growth at the regional level. In addition, because of the significant presence of spatial dependence, the growth of labour productivity in one country could contribute to their neighbours via spatial externalities. Also, two other explanatory variables (education and technological gap) have proven to be significant, and their extent is important: the coefficient of education is positive confirming that the level of education facilitates the absorptive capacity of foreign technology and thus favours the growth of productivity. The coefficient of the technological gap is negative indicating that there are some countries can’t catch up the leader country (Germany) in term of labour productivity.

These results confirm the predictions of new economic geography theory and the endogenous growth theory: first external effects (pecuniary and technological externalities) constitute as a source of increasing returns to scale implying that the increase of factor of production leads to a more proportional increase of production. Second these external effects are geographically burned leading to the concentrations of these factors in space and favour the inequality of regional growth.

This note is not in favour of a positive impact of politics of regional growth destiny to reduce inequality between the euro Mediterranean countries. The technology diffusion from the north to the south constitutes a source of the reduction of these inequalities which in their tour necessitate the investment in the productive resources of the south.

References


Notes

Note 1. For a review of the different approaches to the estimation of the Verdoorn’s Law see Leon-Ledesma (2000).

Note 2. For simplicity we drop the subscript i in all subsequent equations.

Note 3. By rearranging equation (5) it can be shown that the spatial error model is equivalent to an extended version of the spatial lag model that includes both a spatially lagged dependent variable and the set of spatially lagged independent variables (excluding the constant term). This equivalence only holds if a number of non-linear constraints are satisfied. The resulting model is known as the ‘spatial Durbin’ or ‘common factor’ model (Anselin, 2001).

Note 4. k is the number of freedom or also the number of exogenous variables.

Note 5. This value is obtained from the table of $\chi^2$

Note 6. Details on the model with random effects and spatial dependence are presented in the appendix.

Appendix: Random effects model and spatial dependence

The spatial autocorrelation can be incorporated in the random effects model in which the coefficients of regressions are supposed fixed (Anselin, 1988a ; Case, 1991 ; Baltagi et Li, 2002). In this model we have:

$$y_{it} = x_{it}' \beta + \epsilon_{it} \quad i=1,\ldots,N ; t=1,\ldots,T$$  \[1\]

Where $y_{it}$ is a vector $NT \times 1$, i denotes the region and t the period, $x_{it}$ is a vector $k \times 1$ of observations relative to k explicative variables and $\beta$ is a vector $k \times 1$ of parameters.
The term of error is supposed to incorporate the unobservable effects due to space (Hsiao, 1986; Baltagi, 1995):

\[ \varepsilon_{it} = \mu_i + \phi_{it} \quad i=1,\ldots,N; t=1,\ldots,T \]  

[2]

\( \mu_i \) is a vector \( N \times 1 \) of regional specific effects. The vector \( NT \times 1 \) of error terms \( \phi_{it} \) with mean null and variance \( \sigma_{\varepsilon}^2 \), is supposed normal, \( \phi_{it} \sim N(0, \sigma_{\varepsilon}^2) \). The \( \phi_{it} \) are supposed independents of effects \( \mu_i \) and explicative variables of model.

1. The autoregressive spatial model:

A first formulation of spatial dependence is the specification of autoregressive spatial model (cf. Florax et Folmer, 1992). If we consider an impalement of \( T \) observations relative to each region, the random effects model with spatial dependence can be written as:

\[ Y = \rho W'Y + X\beta + Z\mu + \phi \]  

[3]

Where \( Z = \left(I_N \otimes I_T\right) \) is a \( NT \times N \) matrix of regional indicative variables, \( I_T \) is a vector \( T \times 1 \) of 1 and \( I_N \) denotes \( N \times N \) identity matrix. The sign \( \otimes \) is the kronecker product. \( W' = (W_N \otimes I_T) \) where \( W_N \) is the \( N \times N \) spatial weight matrix, standardized. \( \rho \) is the spatial autoregressive coefficient.

The matrix of variances in the case of spatial autoregressive model presents the following structure:

\[ \Omega(\rho, \theta^2, \sigma^2_{\varepsilon}) = \sigma^2_{\varepsilon} M^{-1}(\rho) \left[ Q + \theta^2 \cdot B \right] M^{-1}(\rho) \]  

[4]

Where \( M(\rho) = (I_N - \rho W) \), \( B = (I_N \otimes T^{-1} I_T) \) and \( Q = (I_{NT} - B) \) are respectively the \( NT \times NT \) operators between and within and \( \theta^2 = \frac{\sigma^2_{\varepsilon}}{\sigma^2_{\mu} + \sigma^2_{\varepsilon}} \).

The determinant and the reverse of \( \Omega \) are respectively:

\[ |\Omega(\cdot)| = \sigma^2_{\varepsilon} |M'| |\theta^2|^{-T} \]  

[5]

and \( \Omega^{-1}(\cdot) = \sigma^2_{\varepsilon} M_i'[Q - \theta^2 \cdot B] M_i \)  

[6]

2. Spatial model with autocorrelation in errors:

A second formulation of spatial dependence supposes that the errors \( \phi_{it} \) are spatially autocorrelated (cf. Florax et Folmer, 1992; Baltagi et Li, 1999):

\[ \phi = \lambda W\phi + v \]  

[7]

Where \( \lambda \) is the coefficient of spatial autocorrelation. It catches the effects of spatial variables omitted in the model. The vector \( NT \times 1 \) of errors \( v_{it} \) is supposed normal, \( v_{it} \sim N(0, \sigma^2_{\varepsilon}) \). \( v_{it} \) are also supposed independents of the effects \( \mu_i \) and the explicative variables of model.

In the case of spatial model with autocorrelation in errors, the matrix of variances possesses the following structure:

\[ \Omega(\lambda, \theta^2, \sigma^2_{\varepsilon}) = \sigma^2_{\varepsilon} \left[ Q + \theta^2 \cdot B + (M'(\lambda)M(\lambda))^{-1} - I_{NT} \right] \]  

[8]

Where, \( M(\lambda) = (I_N - \lambda W) \), \( \theta^2 = \frac{\sigma^2_{\varepsilon}}{\sigma^2_{\mu} + \sigma^2_{\varepsilon}} \).

The determinant and the reverse of the matrix \( \Omega(\cdot) \) can be written respectively (cf. Anselin, 1988 : 153-154):

\[ |\Omega(\cdot)| = \sigma^2_{\varepsilon} |A_N|^{-2(\theta^2-1)} \left\{ A_N A_N \right\}^{-1} + \left( \theta^2 - 1 \right) |J_N| \]  

[9]

and \( \Omega^{-1}(\cdot) = \sigma^2_{\varepsilon} \left\{ A_N A_N \otimes (I_T - J_T) + \left[ A_N A_N \right]^{-1} \right\}^{-1} \otimes J_T \)  

[10]

where, \( A_N = I_N - \lambda W \)
3. Estimation by the maximum likelihood method:


The expression of log likelihood with spatial dependence is the following:

\[ L(\alpha, \beta, \sigma^2) = c_0 - \frac{NT}{2} \ln(\det((1/2u')\Omega^{-1}(\alpha, \beta, \sigma^2)u)) \]  

\[ c_0 = -\frac{NT}{2} \ln(2\pi)/2. \]

We deduce the autoregressive spatial model for \( \alpha = \rho, j = 1, \) \( \sigma^2 = \sigma^2_x \) and \( u = M(\rho)Y - X\beta - Z\mu. \) And the spatial model with autocorrelation in errors for \( \alpha = \lambda, j = 2, \) \( \sigma^2 = \sigma^2_v \) and \( u = M(\lambda)(Y - X\beta - Z\mu). \)

The estimation of models with spatial dependence with the likelihood method necessitates a non linear optimisation and implies numeric calculations as much harder and longer when the number of observations is important. In particular, one of difficulties in application of the ML resides in calculation of determinant of the matrix of Jacobean. An alternative proposed by Ord (1975) reposes on proper values of the weight matrix. Then,

\[ |W_T - \alpha W_S| = |W_N - \alpha W_S| = \prod_i (1 - \omega_i) \]  

Where the \( \omega_i \) design the proper values of the matrix \( W_N \) and \( \alpha = \{\rho, \lambda\}. \) The identity [12] implies that the values of coefficients of spatial dependence must satisfy the condition: \( \omega_{\min}^{-1} \leq \alpha \leq \omega_{\max}^{-1} \) where \( \omega_{\max} = 1 \) in the case of weight matrices normalised. The advantage of this procedure is that we can determine the proper values of these matrices before the optimisation (since \( W_N \) is supposed known). This reduces considerably the numeric calculation of likelihood of model, at least in the case of small samples.

Table 1. Estimation of a-spatial model

<table>
<thead>
<tr>
<th></th>
<th>Fixed effects model</th>
<th>Random effects model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{b}_0 )</td>
<td>-1.245 (0.021)</td>
<td>0.125 (0.001)</td>
</tr>
<tr>
<td>( \hat{b}_1 )</td>
<td>0.734 (0.021)</td>
<td>0.817 (0.001)</td>
</tr>
<tr>
<td>( \hat{b}_2 )</td>
<td>-0.081 (0.120)</td>
<td>-0.061 (0.095)</td>
</tr>
<tr>
<td>( \hat{b}_3 )</td>
<td>0.317 (0.012)</td>
<td>0.456 (0.001)</td>
</tr>
</tbody>
</table>

| Test de Fisher | 16.241 (0.000) |
| Test de Hausman | 0.024 (0.094) |

**Note:** N=26, T=10. The values of critical probabilities are in parentheses. Level of significance = 5%.
Table 2. Tests of spatial dependence in a-spatial model

<table>
<thead>
<tr>
<th>tests</th>
<th>Matrix of distance</th>
<th>Matrix of contiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LMERR</strong></td>
<td>899.251 (0.000)</td>
<td>752.426 (0.000)</td>
</tr>
<tr>
<td><strong>LMLAG</strong></td>
<td>1125.524 (0.000)</td>
<td>1005.214 (0.000)</td>
</tr>
<tr>
<td><strong>RLMERR</strong></td>
<td>3.011 (0.999)</td>
<td>1.256 (1.000)</td>
</tr>
<tr>
<td><strong>RLMLAG</strong></td>
<td>1111.452 (0.000)</td>
<td>914.53 (0.000)</td>
</tr>
</tbody>
</table>

**Note:** The values of critical probabilities are in parentheses. Level of significance = 5%.

Table 3. Estimation of the empirical specification: equation [2.2]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{b}_0$</td>
<td>0.214 (0.009)</td>
<td>-0.025 (0.008)</td>
</tr>
<tr>
<td>$\hat{b}_1$</td>
<td>0.871 (0.001)</td>
<td>0.868 (0.001)</td>
</tr>
<tr>
<td>$\hat{b}_2$</td>
<td>-0.031 (0.003)</td>
<td>-0.015 (0.005)</td>
</tr>
<tr>
<td>$\hat{b}_3$</td>
<td>0.598 (0.001)</td>
<td>0.631 (0.001)</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.951 (0.000)</td>
<td>0.899 (0.000)</td>
</tr>
</tbody>
</table>

**Note:** $N = 26, T = 10$; Estimation by ML method. The values of critical probabilities are in parentheses. Level of significance = 5%.