Chaotic Structures in Brent & WTI Crude Oil Markets: Empirical Evidence

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Abstract

This article provided empirical evidence of existence of chaotic structures in the time series of Brent and WTI crude oil price returns, by means of the Phase Space Reconstruction Technique (PSRT) and fractal integral method. This paper analyzed the time series of oil prices, attained the fractal dimensions, positive Lyapunov exponents and Kolmogorov entropy, and thereby identified the existence of chaos in the markets. Our results can deepen the understanding of crude oil dynamics and the complexity of the analyzed markets.

Keywords: crude oil price, PSRT, correlation dimension, Largest Lyapunov exponent, Kolmogorov entropy

1. Introduction

Various daily financial time series present empirical evidence of the existence of chaotic structures, which are also found in many different financial markets in different economic sectors or economies (Alvarez-Ramirez *et al.* 2008; Alvarez-Ramirez *et al.* 2002; He and Chen 2010; He *et al.* 2007, 2009; He and Zheng 2008). The study of petroleum prices is largely based on the main stream literature of financial markets, whose benchmark assumptions are that returns of stock prices follow a Gaussian normal distribution and that price behavior obeys 'random-walk' hypothesis (RWH), which was first introduced by Bachelier (Bachelier 1900), since then RWH has been adopted as the essence of many asset pricing models. Another important context on this domain is the efficient market hypothesis (EMH) proposed by Fama which states that stock prices already reflect all market information available in evaluating their values (Fama 1970). However, RWH and EMH have been widely criticized in current financial literature (Alvarez-Ramirez *et al.* 2008; Alvarez-Ramirez *et al.* 2002; He and Chen 2010; He *et al.* 2007, 2009; He and Zheng 2008). Many important results in current literature suggest that returns in financial markets have fundamentally different properties that contradict or reject RWH and EMH. These ubiquitous properties identified are: fat tails (Gopikrishnan *et al.* 2001), long-term correlation (Alvarez-Ramirez *et al.* 2008), volatility clustering (oh *et al.* 2008), fractals/multifractals (He and Chen 2010; He *et al.* 2007, 2009; He and Zheng 2008), chaos (Adrangi *et al.* 2001), etc. Therefore, interesting problems arouse such as: are crude oil markets chaotic? Can price behaviors be predictable, and on what time scale?

To answer these questions, one must investigate the markets by means of time series analyses, which can be roughly categorized into two norms: canonical statistics method and nonlinear dynamics analysis. The former norm is widely applied if the time series is random, while the other would be selected if the time series is deterministic chaotic. Crude oil prices do not follow so-called random walks (Ng and Pirrong 1996; Tabak and Cajueiro 2007), but exhibit fractal phenomena (Alvarez-Ramirez *et al.* 2008; Alvarez-Ramirez *et al.* 2002; He and Chen 2010; He *et al.* 2007, 2009; Panas and Ninni 2000; Tabak and Cajueiro 2007) and chaotic features (Panas and Ninni 2000) which is derived from both innate randomness driven by chaotic dynamics and exogenous factors such as market noise, bounded rationality of market participators (Voituriez 2001), heterogeneous belief or expectations, etc. Thereby, the latter norm is applied to study the crude price dynamics to obtain some critical parameters such as fractal dimension, Largest Lyapunov exponents (LLEs for short), Kolmogorov entropy.

applied to the time series of monthly and daily crude oil price within the same observation period, obtained the critical characteristic parameters to describe the strange attractors; thereby empirical evidence is found to support the existence of chaotic structures in the markets, and the qualitative scale-free similarity among the four price systems at different time scales, which implies that there exist fractals phenomena in these systems. Due to the chaotic features, short-term predictability of price behaviors is guaranteed whilst it is impossible to predict longer

term price behaviors.

We contribute to the current literature that we integrate chaotic and fractal analyses, and provide many pieces of inter-related and supportive empirical evidence to show the nonlinear properties in international crude oil markets. Therefore, our results can deepen the understanding of crude oil dynamics and the complexity of the analyzed markets.

2. Methodology

PRST

Due to its complexity and our limited knowledge of its intrinsic underlying dynamics, it is impossible for researchers to set up a perfect analytical dynamic functions (or models) for crude oil market. But, on the other hand, the characteristic quantities, such as LLEs, fractal dimensions, Kolmogorov entropies, can be relatively easier to be estimated to detect the chaotic structures in actual price systems (Rosenstein *et al.* 1993). In current literature, one of the most applicable and promising ways to obtain these quantities is PRST, whose procedure can be described as follows (Rosenstein *et al.* 1994):

Firstly, as for the actual price time series $p = \{p_1, p_2, L, p_n\}$ ($n \to \infty$), let us choose an optimal time delay τ and imbedded dimension m. Then define $\{P_i | P_i \in R^m\}$ as follows:

$$P_i = \left\{ p_{i\tau}, p_{i+1\ \tau}, L, p_{i+(m-1)\ \tau} \right\}, i = 1, L, n - (m-1) - \tau$$

Thus, an *m* dimensional phase space is obtained. Then the fractal integral is given by

$$C_m(r) = \frac{1}{N(N-1)} \sum H\left(r - \|P_i - P_j\|\right),$$

where $H(x)$ stands for a Heaviside function which is defined as:

$$H(x) = \begin{cases} 1, x > 0 \\ 0, x \le 0 \end{cases}.$$

In equation (2.2), $N = n - m + 1$ and $P_i (i = 1, \cdots, N)$ is a point (vector) in the phase space. $\|P_i - P_j\|$, or so-called critical distance, is the Euclidean distance between any two different points in the space. Any two vectors whose distances are less than the critical measure *r* are correlated ones. The accumulated distance distribution function $C_m(r)$ means that the probability of the distances which are less than *r* in the space.

It is critically important to choose the proper numerical values of the parameters, e.g., *r* cannot be too large to correlate all points in the space, i.e. $C_m(r) = 1$, nor too small to be unable to detect fractal dimensions. Ying and David argued that although the researchers can change imbedded dimension *m* and time delay τ simultaneously, it is critical to keep $m\tau$ constant to obtain numerical result for optimal time delay τ (Ying and David 1998). To choose an optimal time delay, the auto-correlation function introduced in ref.(Theiler 1990) is applied in this article, which can be defined as:

$$R_{xx}(\tau) = \frac{1}{N} \sum_{i=1}^{N-\tau} (p_i - \overline{P})(p_{i+\tau} - \overline{P}),$$

where *N* stands for the number of the observation points, which in this article is the length of the price (returns) time series; x_i is the *i*th sample in the series. $\overline{P}$ means the arithmetic mean of the series. If R_{xx} decays to $\left(1 - \frac{1}{e}\right)$ of the original numerical values as τ increasing, the optimal value for τ is thereby estimated.

Fractal dimension

The fractal dimensions is calculated by means of fractal integer:

$$D_2 = \lim_{r \to 0} \frac{\ln C_m(r)}{\ln r}.$$

Largest Lyapunov Exponent

A positive largest Lyapunov Exponent is one of the most important features in a chaotic system. The method introduced in ref. (Adrangi *et al.* 2001) is applied to obtain those exponents.

Kolmogorov Entropy

Kolmogorov entropy K_2 is practical evidence to indentify the chaotic dynamics. Usually $0 < K_2 < \infty$, the larger K_2 gets, the more chaotic the system is. K_2 is defined as:

$$K_2 = \frac{1}{t} \ln \frac{C_m(r)}{C_{m+1}(r)}.$$

3. Empirical analysis and discussion

In this article, the most representative crude oil markets are analysed, that is, daily and monthly Brent and WTI prices, dating from July 1, 1996 to August 31, 2005. All our data are taken from the sources: http://stockchart.com and www.Economagic.com/em–cgi/data.exe/var/west-texas-crude-long.   The daily logarithmic returns are obtained from the original series (see Fig. 1).

When choosing the parameters τ and m, Takens proved that τ and m can be chosen independently for an infinite, noise-free time series (Rosenstein *et al.* 1993). As the critical value of m remains unknown, m is altered from 2 to 54 with the step of 1 to probe the critical value, while the numerical results of $\ln r$ ($\ln r \neq 0$) is selected within [-10, 10] with the step of 0.05. According to equation (2.4), the optimal time lag ($\tau = 1$) is chosen (see Fig. 2).

The fractal dimensions

By means of fractal dimensions, the freedom degrees can be estimated in dynamic system of crude oil prices (returns), i.e., how many variables are needed in representing the nonlinear dynamic system? Furthermore, if the dimensions are non-integer, the system is characterized by fractals. The relationships of $\ln r$ *vs.* $\ln C_m(r)$ are estimated by equation (2.2). The numerical results are illustrated in Fig. 3. Under different m, it is obvious to find many scale-free areas in Fig.3, which implies that there exist fractal phenomena in the price systems. By estimating the slopes of the scale-free relationships of $\ln r$ *vs.* $\ln C_m(r)$, the numerical results of D_2 can be obtained.

The numerical results in Table 1 and Figure 4 imply that:

The numerical results of D_2 are relatively small, which implies that the dynamic systems have few degrees of freedom, i.e., the price dynamics can be represented by very few variables;

The results of D_2 are numerically similar under different imbedded dimensions;

On different time scales, the relationships of $\ln r$ vs. $\ln C_m(r)$ are similar qualitatively; monthly and daily results are much alike, which implies that the dynamics is scale-invariant.

All D2 are non-integer, which represents emergent fractals phenomena in the systems. The results are consistent with those of our previous researches (He and Chen 2010; He *et al.* 2007; He and Zheng 2008).

The largest Lyapunov exponents

One of the most characteristics of a chaotic system is its sensitivity to different initial conditions, while a positive LLE is evidence of existence of deterministic chaos in the system (Rosenstein *et al.* 1993). A positive LLE also implies that it is impossible to forecast or predict a dynamic system in the long run; because even a trivial noise would be amplified exponentially thus the evolution tracks of price dynamics are significantly different from the original ones.

According to the results (see Fig. 4 and Table 2):

All the evolution tracks of the LLEs converge or stabilize to some certain positive values, featuring Crude oil markets with chaos, i.e., the initial condition is highly sensitive to even a slightest perturbation, which means even apparently negligible market noise must be considered for it may distort rational equilibrium price over time so that it may be a dominant factor and major driving force determining the actual price behaviors, e.g., an irrational but self-reinforcing forecast of future mark-up by some individuals sometimes ends up with psychological panic and rapidly rising prices in crude oil markets.

Due to the extreme sensitiveness to initial conditions, it is impossible to predict long-term price behaviors accurately. But at the same time, the small values of LLEs imply that although a small noise may distort the original evolution track exponentially, shorter-term prediction is possible to offer us better understanding of this domain.

LLEs of monthly Brent and WTI are numerically identical; but LLEs of daily Brent are slightly smaller than those of daily WTI, which imply that market noise has greater impact on the latter than the former; thereby, forecast error and forecasting risk of the latter system are greater than those of the former one accordingly.

Kolmogorov entropies

To further probe into its chaotic feature, the Kolmogorov entropy,one of the measures of chaotic systems, is estimated. By equation (2.2), the numerical results of the entropies are obtained (see Fig. 5).

The results in the Fig. 5 and Table 2 show that

All the entropies, like LLEs in section 3.2, are positive, which is another evidence of chaotic characteristics in the systems, while the differences of the entropies among the four systems are insignificant and trivial.

Kolmogorov entropies can be interpreted as the degrees of distortion of market information in the price system. Therefore, the small but positive values of the entropies means within a short time horizon, the market information can still be applicable to understand the market dynamics.

To analyze the exponential distortion of market information over time quantitatively, multiplicative inverse of the entropies is applied as a plain and plausible estimate. For example, the reciprocal of the Kolmogorov entropy of daily Brent system is $\frac{1}{0.2787}$, that is, the time scale of a rational and effective forecast for that system should be within 36 days, after that any prediction may be distorted exponentially by any slightest noise and thus be ineffective.

4. Conclusions

By means of PSRT, the Brent & WTI price time series are reconstructed to obtain LLEs to find empirical evidence of existence of chaos in the systems; furthermore, by means of Correlation Integral Method, the correlation dimensions and Kolmogorov entropies are estimated, whose numerical results offer more evidence of chaotic structures in the system; in addition, plausible estimations are provided of time scales of effective and rational prediction for the systems. Our results are qualitatively consistent with those of daily returns of Rotterdam and Mediterranean crude oil markets in (Panas and Ninni 2000).

According to the analyses and discussions,

All four price systems reveal chaotic characteristics. It may be rational and applicable to make short term prediction, e.g., less than 36 days for Brent daily prices, but it makes no sense to predict oil prices at longer time scales.

All four price systems are featured with fractal phenomena. The price dynamics is time-scale invariant, in that price behaviors of daily and monthly prices are qualitatively identical.

The price behaviors of Brent and WTI crude oil prices are qualitatively similar; therefore, the knowledge acquired from one market can be the reference to the other.

Since these markets are found to be chaotic with fractal features, the classical statistical and analytical norms may be impotent and ineffective in explaining the underlying market dynamics. The nonlinear dynamics, on the contrary, can obtain more reasonable, more accurate qualitative and quantitative results, and thus offers us better understanding of this domain.

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Table 1. Numerical results for the fractal dimensions

m	Monthly Brent	Monthly WTI	Daily Brent	Daily WTI
2	0.61417	0.43049	0.62258	0.61669
3	0.82497	0.56715	0.82151	0.81621
4	1.02687	0.70755	1.01383	1.01388
5	1.21721	0.84145	1.20387	1.20644
6	1.40244	0.98551	1.38936	1.39360
7	1.59308	1.12892	1.57469	1.57753
8	1.77674	1.26537	1.93233	1.75937
9	1.96104	1.41005	2.11250	1.94283
10	2.13879	1.54947	2.29057	2.11567
11	2.32407	1.68542	2.26364	2.28243

Table 2. the Kolmogorov entropies and LLEs of Brent and WTI

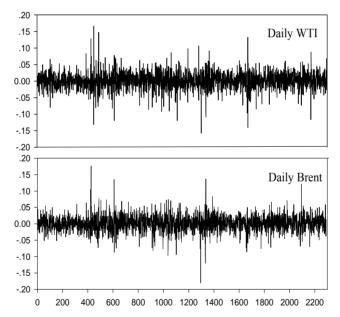|	λ_1	k_2	Eckman-Ruelle condition Satisfied?	Chaotic?
Monthly Brent	0.01139	0.02036	Yes	Yes
Monthly WTI	0.01139	0.01733	Yes	Yes
Daily Brent	0.00036	0.02787	Yes	Yes
Daily WTI	0.00130	0.02519	Yes	Yes

Figure 1. illustrates the logarithmic returns of daily Brent & WTI crude oil time series.
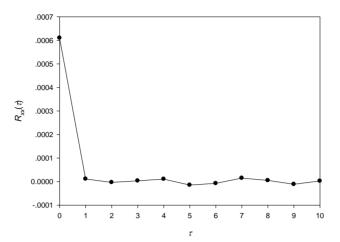
Figure 2. depicts that the numerical results of autocorrelation function R_{xx} decay as the time lag $\tau = 1$ increases by means of equation (2.4). It is obvious that the time lag $\tau = 1$ is optimal in that R_{xx} decays to $\left(1 - \dfrac{1}{e}\right)$ of the original numerical value at this point.
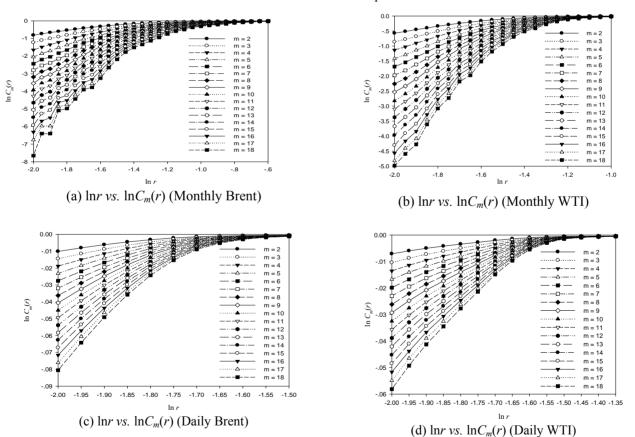

(a) ln*r vs.* ln$C_m(r)$ (Monthly Brent)

(b) ln*r vs.* ln$C_m(r)$ (Monthly WTI)

(c) ln*r vs.* ln$C_m(r)$ (Daily Brent)

(d) ln*r vs.* ln$C_m(r)$ (Daily WTI)

Figure 3. plots the relationships between ln*r* and ln$C_m(r)$ of monthly Brent & WTI and daily ones respectively as the imbedded dimension *m* is increasing. All four relationships display distinct scale-free zones, which imply that there exist fractal phenomena in the systems.
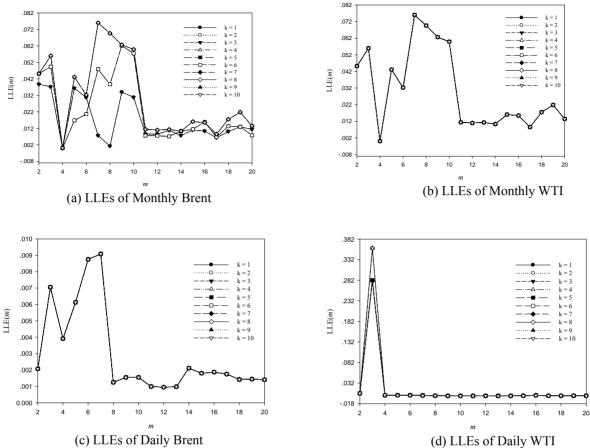
(a) LLEs of Monthly Brent

(b) LLEs of Monthly WTI

(c) LLEs of Daily Brent

(d) LLEs of Daily WTI

Figure 4. illustrates the numerical results for largest Lyapunov exponents for monthly Brent &WTI and daily Brent & WTI respectively. Axis y "*LLE* (*m*)" means the evolution tracks of Largest Lyapunov Exponents (*LLE* for short) as the imbedded dimension *m* is increasing. All four panels display that all the evolution tracks converge to positive LLEs as the imbedded dimension *m* is increasing, which imply that all these systems display deterministic chaotic features.
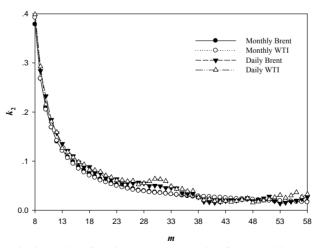

Figure 5. illustrates the numerical results of Kolmogorov entropies for Monthly Brent & WTI and Daily Brent & WTI when the imbedded dimension *m* increases.