

Physical Continuous Model of Heating, Evaporation and Ignition of Polydisperse Fuel Droplets

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Abstract

A new modeling of heating and evaporation of fuel droplets and ignition of a fuel vapour/ air mixture in continuous form is suggested. The size distribution of fuel droplets is assumed to be continuous and found from the solution of the kinetic equation for the probability density function (PDF). The semi-transparency of droplets, the difference between the gas temperature and the external temperature are taken into account. The model represents in dimensionless form, and the dynamics of the system is present in terms of the dynamics of a multi-scale, singularly perturbed system (SPS).

Keywords: computational fluid dynamics (CFD), nonlinear ordinary differential equations, polydisperse fuel spray, probability density function

A	pre-exponential rate factor (s^{-1})
a_r	polynomial of the external temperature
b_r	polynomial of the external temperature
B	universal gas constant ($Jkmol^{-1}K^{-1}$)
B_T	Spalding number
B_M	Spalding mass number
b_r	polynomial of the external temperature
C	molar concentration ($kmolm^{-3}$)
c_{pF}	specific heat capacity of fuel vapour ($Jkg^{-1}K^{-1}$)
c_l	specific heat capacity liquid ($Jkg^{-1}K^{-1}$)
E	activation energy ($Jkmol^{-1}$)
h	convection heat transfer coefficient ($Js^{-1}m^{-2}K^{-1}$)
$\bar{k}_g$	average gas thermal conductivity ($Js^{-1}m^{-1}K^{-1}$)
k_{cr}	rate of reaction ($mole(cm^{-3})s^{-1}$)
L	liquid evaporation energy (i.e., latent heat of evaporation, Enthalpy of evaporation) (Jkg^{-1})
m_{d_i}	droplet mass (kg) $i = 1, \dots, w$
n	number of droplets per unit volume (m^{-3})

Nu	Nusselt number
$\bar{Q}_a$	average absorption efficiency factor
Q_{fv}	heat released per unit mass of burnt fuel vapour (Jkg^{-1})
$\bar{Q}_R$	radiative temperature (K)
R	radius of droplet
r	dimensionless radius
T_g	gas temperature (K)
T_d	droplet temperature (K)
T_{ext}	external temperature (K)
t	time (s)
t_{react}	characteristic reaction time (s) defined in equation (17)
V	the volume over all droplet (m^3)
w	number of droplets

Greek symbols:

α_g	volume fraction of gas (dimensionless)
β	dimensionless reduced initial temperature (with respect to the so-called activation temperature E/B)
γ	dimensionless parameter that represents the reciprocal of the final dimensionless adiabatic temperature of the thermally insulated system after the explosion has been completed
ϵ_i	for $i = 1, \dots, 10$ dimensionless parameters defined in equation (17)
θ_g	dimensionless gas temperature
θ_d	dimensionless droplet temperature
μ	molar mass ($kgmol^{-1}$)
ρ_{fv}	density of fuel vapour (kgm^{-3})
ρ_{Ox}	density of Oxygen (kgm^{-3})
ρ_d	density of droplet (kgm^{-3})
σ	Stefan-Boltzmann constant ($W m^{-2}K^{-4}$)
τ	dimensionless time

1. Introduction

Nothing that the combustion of sprays is a complex interaction of many different processes such as droplet heating, evaporation, ignition, combustion, interaction of different droplets etc. (Maas et al., 2006). A calculation of practical combustion systems from first principles i.e., using direct numerical simulations is not possible. Therefore, in analogy to the description of turbulent flames (Faeth, 1987), statistical methods have to be used. Because the spray is characterized by a statistical distribution of droplets sizes, it is natural to describe the spray by a distribution function (William, 1985). In addition to representing the drop size distribution with a frequency plot, it is also informative to use a cumulative distribution representation. This is essentially a plot of the integral of the frequency curve, and it may represent the percentage of the total surface area or volume of a spray contained in drops below

a given size (Lefebvre, 1989). Because the graphical representation of drop size distribution is laborious and not easily related to experimental results, many workers have attempted to replace it with mathematical expression whose parameters can be obtained from a limited number of drop size measurements (Aggrawal, 1998)

In the absence of any fundamental mechanism or model on which to build a theory of drop size distributions, a number of functions have been proposed, based on either probability or purely empirical considerations, that allow the mathematical representation of measured drop size distribution. Those in general use include normal, log-normal, Nukiyama-Tanasawa, Rosin-Rammler, and upper-limit distribution. As basic mechanisms involved in atomization are not clearly understood and no single distribution functions can be represent all drop size data, it is usually necessary to test several distribution function to find the "best" fit to any given set of experimental data (Daniel Christopher, 2002; Durand, 1999).

The aim of this work is to reformulate the model that proposed in (Bykov et al., 2007; Abramzon & Sirignano, 1989; Sirignano, 1999) by using probability density function i.e., in a continuous way as in our previous papers (Nave, 2010, 2011a, 2011b).

2. Discrete Model

According to (Abramzon & Sirignano, 1989; Sirignano, 1999) the basic equations for spray modeling are as follows:

$$\frac{dm_{d_i}}{dt} = -4\pi \frac{\bar{k}_g R_{d_i}}{c_{pF}} \ln(1 + B_M), \quad i = 1, \dots, w, \quad (1)$$

$$m_{d_i} c_l \frac{dT_d}{dt} = 4\pi R_{d_i}^2 h(T_g - T_d) - L \frac{dm_{d_i}}{dt} + 4\pi R_{d_i}^2 \sigma \bar{Q}_a T_{ext}^4, \quad (2)$$

$$\alpha_g \frac{d\rho_{fv}}{dt} = -\alpha_g A \rho_{fv}^a \rho^b e^{-\frac{E}{RT_g}} + \frac{1}{V} \sum_{i=1}^w \frac{dm_{d_i}}{dt}, \quad (3)$$

$$\frac{d\rho_{O_2}}{dt} = -18.5 \rho_{fv}^a \rho^b e^{-\frac{E}{RT_g}}, \quad (4)$$

$$\begin{aligned} c_{mix} \rho_{mix} \frac{dT_g}{dt} = & \alpha_g Q_{fv} A \rho_{fv}^a \rho^b e^{-\frac{E}{RT_g}} - \frac{1}{V} \sum_{i=1}^w c_l m_{d_i} \frac{dT_d}{dt} - \frac{1}{V} \sum_{i=1}^w L \frac{dm_{d_i}}{dt}, \\ & - \frac{1}{V} \sum_{i=1}^w c_{pF} \frac{dm_{d_i}}{dt} (T_g - T_d). \end{aligned} \quad (5)$$

The initial conditions at $t = 0$ are:

$$T_d = 400K, \quad T_g = 880K, \quad \rho_{fv} = \rho_{O_2} = 2.73 \frac{kg}{m^3}, \quad R_{d_i} = 9\mu m, \quad \rho_d = 600 \frac{kg}{m^3}. \quad (6)$$

Equation (1) is the equation for stationary droplet mass, (2) is the equation for stationary droplet temperature, (3) is the equation for conversion of fuel vapour, (4) is the equation for density of oxygen, and (5) energy balanced equation for the gas temperature.

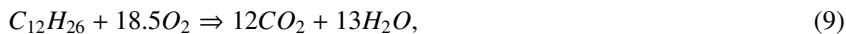
The equation of the fuel vapour density (3) follows directly from the conversation of fuel vapour where the expression of the rate of reaction have the form of (Westbrook & Dryer, 1981);

$$k_{cr} = A [fuel]^a [O_2]^b e^{\left(-\frac{E}{RT_g}\right)}. \quad (7)$$

The values of these coefficients given for $C_{10}H_{22}$ will be used. These are the closest to *n* - dodecane i.e., $C_{12}H_{26}$ (the closest approximation for diesel fuel):

$$\begin{aligned} A &= 3.8 \times 10^{11} \frac{1}{s} \left(\frac{mole}{cm^3} \right)^{1-a-b} = 2.137 \times 10^9 \frac{1}{s} \left(\frac{kmole}{cm^3} \right)^{1-a-b}, \\ E &= 30 \frac{kcal}{mole} = 1.255 \times 10^8 \frac{J}{kmole}, \quad a + b = 1 \end{aligned} \quad (8)$$

The equation for the density of Oxygen (4) derived from the assumption for a single step global reaction for n – dodecane combustion as:



where in our calculations we used the following values for the molar mass of Oxygen and fuel respectively as: $M_{O_2} = 32 \text{ kg/kmole}$ and $M_f = 170 \text{ kg/kmole}$.

The relation between the average gas thermal conductivity and Nu is as follows (Abramzon & Sirignano, 1989):

$$Nu = \frac{2hR_d}{\bar{k}_g} = 2 \frac{\ln(1 + B_T)}{B_T}. \quad (10)$$

The radiative temperature, Q_R , calculated from the $P - 1$ -model and equal to $Q_R = T_{ext}$ for optically thick gases and $Q_R = T_g$ for optically thin gases.

The average absorption efficiency factor as appear in the equation for the droplet temperature, i.e., equation (2) can be calculated from the equation (Sazhin et al., 2004):

$$\bar{Q}_a = a_r R_d^{b_r}, \quad (11)$$

where an explicit expressions for several types of diesel fuel are given in (Sazhin et al., 2004).

3. The Time Evolution of the Probability Density Function

A statistical description of the spray may be given by the distribution function (or density function) $P(R, \vec{x}, \vec{v}, t) dR d\vec{x} d\vec{v}$ which is the probable number of particles in the radius range dR about R located in the spatial range $d\vec{x}$ about $\vec{x}$ with velocity in the range $d\vec{v}$ about $\vec{v}$ at time t .

An equation describing the time rate of change of the distribution function $P(\cdot)$ may be derived phenomenologically by using reasoning analogous to that employed in the kinetic theory of gases. In many combustors, the intensity of burning is comparatively low in the neighborhood of the atomizer, and the main part of the combustion occurs in regions where particle interactions and sources are of no more than secondary importance. In our numerical simulations we focus on the burning process and neglected from the rate of change of the distribution function that caused by collisions with other particles and from the rate of increase of the distribution function with time through particle formation or destruction by processes such as nucleation or liquid break-up. Using these assumptions, according to (William, 1985) the evolution of the size distribution of droplets due to the evaporation process that describe by the kinetic equation for the PDF given by:

$$\frac{\partial P_R}{\partial t} = \frac{\partial}{\partial R} \left(\frac{j_m}{\rho_l} P_R \right). \quad (12)$$

The approximation of this equation as suggested in (Volkov et al., 2004) is:

$$\frac{\partial P_R}{\partial t} = \frac{\partial}{\partial R} \left(\frac{J_m}{\rho_l} P_R \right), \quad (13)$$

where:

$$J_m = \frac{\int_0^\infty R^2 j_m P_R dR}{\int_0^\infty R^2 P_R dR}. \quad (14)$$

Equation (13) coincides with equation (12) in the case when the rate of evaporation does not depend on the droplet radius, as well as the droplets are treated as a monodisperse system. However, the averaging of j_m is performed in such a manner that equation (13) would yield the same balance equation for the mass of the liquid equation i.e., equation (1) as that yielded by equation (12). The integro-differential equation (13) with (14) has a self-similar solution that satisfies the initial distribution $P_R(0, R) = P_{R0}(R)$,

$$P_R = P_{R0}(R + \delta), \quad \delta = \int_0^t \frac{J_m}{\rho_l} dt, \quad (15)$$

and δ is found from the solution of the equation:

$$\frac{d\delta}{dt} = \frac{J_m}{\rho_l}, \quad \delta(0) = 0. \quad (16)$$

4. The Continuous Model in Dimensionless Form

As in our previous work (Nave Ophir et al., 2010, 2011a) the system (1)-(5) can be rewritten in a continuous way by using the following dimensionless parameters:

$$\begin{aligned}
 \beta &= \frac{BT_{g0}}{E}, \quad \tau = \frac{t}{t_{react}}, \quad t_{react} = \frac{e^{1/\beta}}{A\rho_{ffv}^{a-\frac{1}{2}}\rho_{O_20}^{b-\frac{1}{2}}}, \quad r = \frac{R}{R_0}, \quad \tilde{P}_{r0} = \frac{R_0}{n_{d0}}P_{R0}, \\
 \gamma &= \beta \frac{c_{mix}\rho_{mix}T_{g0}}{\sqrt{\rho_{ffv}\rho_{O_20}}Q_{fv}\alpha_g}, \quad R_0 = \frac{\int_0^\infty R^3 P_{R0} dR}{\int_0^\infty P_{R0} dR}, \quad \tilde{\rho}_{O_2} = \frac{\rho_{O_2}}{\rho_{O_20}}, \\
 \tilde{\rho}_{fv} &= \frac{\rho_{fv}}{\rho_{ffv}}, \quad \theta_g = \frac{1}{\beta} \frac{T_g - T_{g0}}{T_{g0}}, \quad \theta_d = \frac{1}{\beta} \frac{T_d - T_{g0}}{T_{g0}}, \quad \theta_g^{ext} = \frac{(T_g^{ext})^4}{T_{g0}^4}, \quad \Upsilon = \frac{\delta}{R_0}, \\
 \epsilon_1 &= \frac{4\pi\rho_d\beta T_{g0}c_l}{3\rho_{ffv}^{\frac{1}{2}}\rho_{O_20}^{\frac{1}{2}}Q_{fv}\alpha_g}, \quad \epsilon_2 = \frac{4\pi\rho_d L}{3\rho_{ffv}^{\frac{1}{2}}\rho_{O_20}^{\frac{1}{2}}Q_{fv}\alpha_g}, \quad \epsilon_3 = \frac{3\bar{k}_g \ln(1+B_M)e^{(\frac{1}{\beta})}}{AR_0^2\rho_d c_{pF}\rho_{ffv}^{a-\frac{1}{2}}\rho_{O_20}^{b-\frac{1}{2}}}, \\
 \epsilon_4 &= \frac{4\pi\rho_d}{3\alpha_g\rho_{ffv}^{\frac{1}{2}}\rho_{O_20}^{\frac{1}{2}}}, \quad \epsilon_5 = \sqrt{\frac{\rho_{O_20}}{\rho_{ffv}}}, \quad \epsilon_6 = \sqrt{\frac{\rho_{ffv}}{\rho_{O_20}}}, \quad \epsilon_7 = \frac{\ln(1+B_T)}{B_T \ln(1+B_M)}, \\
 \epsilon_8 &= \frac{L}{T_{g0}c_l\beta}, \quad \epsilon_9 = \frac{4\sigma T_{g0}^3 R_0^{b_r+1} a_r}{\bar{k}_g}, \quad \epsilon_{10} = \frac{3\bar{k}_g e^{(\frac{1}{\beta})}}{A\rho_{ffv}^{a-\frac{1}{2}}\rho_{O_20}^{b-\frac{1}{2}}R_0^2\rho_d c_l}, \quad \tilde{\epsilon}_3 = \frac{c_{pF}}{c_l}\epsilon_3
 \end{aligned} \tag{17}$$

as follows:

$$\frac{d\Upsilon}{d\tau} = \epsilon_3 \frac{\int_0^\infty r \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^2 \tilde{P}_{r0}(r + \Upsilon) dr}, \tag{18}$$

$$\begin{aligned}
 \frac{d\theta_d}{d\tau} &= \frac{\tilde{\epsilon}_3 \epsilon_7 (\theta_g - \theta_d) \int_0^\infty r \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr} - \frac{\epsilon_8 \frac{d}{d\tau} \int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr} \\
 &\quad + \frac{\epsilon_9 \epsilon_{10} (1 + \beta \theta_g^{ext})^4 \int_0^\infty r^{b_r+2} \tilde{P}_{r0}(r + \Upsilon) dr}{4\beta \int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr},
 \end{aligned} \tag{19}$$

$$\epsilon_6 \frac{d\tilde{\rho}_{fv}}{d\tau} = -\tilde{\rho}_{fv}^a \tilde{\rho}_{O_2}^b e^{\left(\frac{\theta_g}{1+\beta\theta_g}\right)} + \epsilon_4 \frac{\frac{d}{d\tau} \int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^3 \tilde{P}_{r0}(r) dr}, \tag{20}$$

$$\epsilon_5 \frac{d\tilde{\rho}_{O_2}}{d\tau} = -\tilde{\rho}_{fv}^a \tilde{\rho}_{O_2}^b e^{\left(\frac{\theta_g}{1+\beta\theta_g}\right)}, \tag{21}$$

$$\begin{aligned}
 \gamma \frac{d\theta_g}{d\tau} &= \tilde{\rho}_{fv}^a \tilde{\rho}_{O_2}^b e^{\left(\frac{\theta_g}{1+\beta\theta_g}\right)} - \frac{\epsilon_1 \frac{d\theta_d}{d\tau} \int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^3 \tilde{P}_{r0}(r) dr} \\
 &\quad - \frac{\left\{ \epsilon_1 (\theta_g - \theta_d) + \epsilon_2 \right\} \frac{d}{d\tau} \int_0^\infty r^3 \tilde{P}_{r0}(r + \Upsilon) dr}{\int_0^\infty r^3 \tilde{P}_{r0}(r) dr},
 \end{aligned} \tag{22}$$

the dimensionless initial conditions at $\tau = 0$ are:

$$\theta_g = \theta_d = 0, \quad \Upsilon = 0, \quad \tilde{\rho}_{fv} = \tilde{\rho}_{O_2} = 1. \tag{23}$$

5. Analysis and Results

We applying the Frank-Kamenetskii approximation (Frank-Kamenetskii, 1969) to our numerical analysis hence, $\beta \ll 1$ and $\beta\theta_g \ll 1$. This simplification implies that $e^{\left(\frac{\theta_g}{1+\beta\theta_g}\right)} \approx e^{\theta_g}$. In our numerical simulation we compared between the models with and without the impact of the thermal radiation i.e., $\epsilon_9 \neq 0$ and $\epsilon_9 = 0$ respectively in the form of: 1: The continuous model with PDF in the form of Gamma distribution:

$$\tilde{P}_{r0} = \frac{m}{\Gamma\left(\frac{n+1}{m}\right)} \left(\frac{\Gamma\left(\frac{n+4}{m}\right)}{\Gamma\left(\frac{n+1}{m}\right)} \right)^{\left(\frac{n+1}{3}\right)} \cdot r^n \exp \left\{ -r^m \left(\frac{\Gamma\left(\frac{n+4}{m}\right)}{\Gamma\left(\frac{n+1}{m}\right)} \right)^{\left(\frac{m}{3}\right)} \right\}, \quad (24)$$

where $n = 2$, $m = 1$ are parameters and $\Gamma(\cdot)$ is the Gamma function, 2: Nukiyama-Tanasawa distribution [?]:

$$f(2r) = \frac{m(2r)^5}{n^6 \Gamma\left(\frac{6}{m}\right)} e^{-\left(\frac{2r}{n}\right)^m}, \quad (25)$$

3: parcel approximation model for 500 parcels i.e., $w = 300$, 4: monodisperse model for $m, n \rightarrow \infty$ in equation (24). The parameter ρ_{mix} for the parcel model was based on the equation $\rho_{mix} = \sum_i \rho_i$ where ρ_i are the densities of individual components. Also in our numerical simulations we ignored from the droplets dynamics, break-up and coalescence, and the effects of the temperature gradient inside the droplets. The equations presented for a stationary droplets. We plot the solution profiles for the gas temperature $\theta_g - \tau$, droplet temperature $\theta_d - \tau$, density of Oxygen $\rho_{O_2} - \tau$ and fuel vapour density $\rho_{fv} - \tau$. We compared between the different models with and without the impact of the thermal radiation. Let us start with the analysis of the solution profiles of the continuous model with the Gamma distribution. The delay time, i.e., the time from the beginning of the processes until the explosion, for the corresponding model with the impact of thermal radiation is approximately at $\tau \approx 0.0012$. The values of the different variables of the model that corresponds to this thermal explosion point are as follows: $\rho_{O_2}, \rho_{fv} \approx 0.8$ (this variables are decrease very fast in comparing to m), $m \approx 0.25$ (this variable is decrease more slowly in comparing to the other variables), $\theta_g, \theta_d \approx 2.5 - 3$. The delay time for this model without the impact of the thermal radiation is expected to be longer, and our results are confirm these expectation $\tau \approx 0.015$. The values of the different model without the impact of the thermal radiation are as follows: The delay time is $\tau \approx 0.0015$. The values of the different variables of the model without the impact of the thermal radiation are as follows: $\rho_{O_2}, \rho_{fv} \approx 0.6$, $m \approx 0.2$, and $\theta_g, \theta_d \approx 3 - 4$. Simultaneously, we compared the continuous model with the polydisperse approximation model and with the monodisperse model. The delay time for the polydisperse model with the impact of the thermal radiation is $\tau \approx 0.0018$, the delay time of the polydisperse model without the impact of the thermal radiation is $\tau \approx 0.0024$. The delay time of the monodisperse model with the impact of the thermal radiation is $\tau \approx 0.003$, and the delay time of the monodisperse model without the impact of the thermal radiation is $\tau \approx 0.0034$ (see figures 1 – 5). The same analysis can be applied to the continuous model with the Nukiyama-Tanasawa PDF (see figures 6 – 10). In contrast to the Gamma PDF, the solution profile of the mass is different in this case. The decrease of the mass of the system is approximately linear.

6. Conclusions

In this paper we suggested a new physical continuous model that described the heating, evaporation and ignition of fuel droplets by using a probability density function. The results of numerical investigation of the problem have shown that the polydispersity of liquid fuel promotes, and thermal radiation prevents, the ignition of a two-phase combustible mixture. We take into account the evolution of size distribution of droplets by using the kinetic equation for probability density function. The presence of fine droplets in a polydisperse system accelerates the delivery of gaseous fuel and reduce the time ignition. Thermal radiation leads to decrease the delay time for all the different models i.e., the continuous model with the Gamma and Nukiyama-Tanasawa PDF, the discrete polydisperse approximation model and for the monodisperse model. According to our numerical results the continuous models (Gamma and Nukiyama-Tanasawa PDF) have the shorter in the delay time, after these models, the polydisperse approximation model and finally the monodisperse model have the longer in the delay time.

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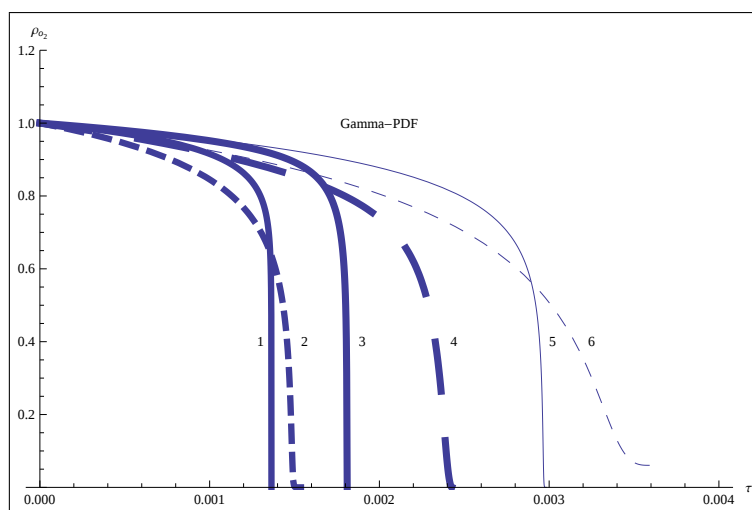


Figure 1. Solution profiles of the Oxygen: $\rho_{O_2} - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

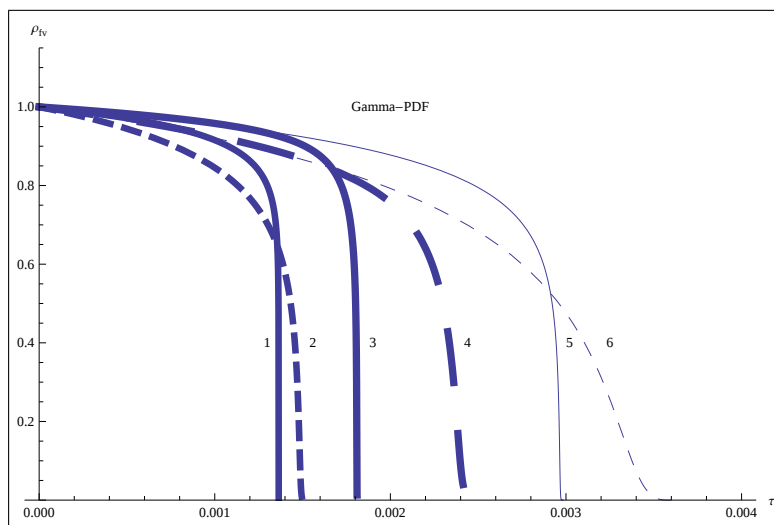


Figure 2. Solution profiles of the fuel vapour: $\rho_{fv} - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

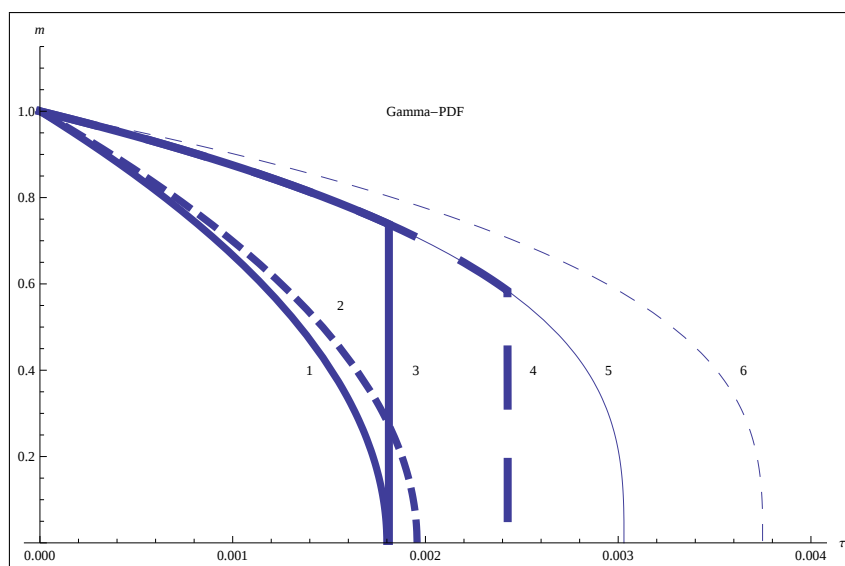


Figure 3. Solution profiles of the mass: $m - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

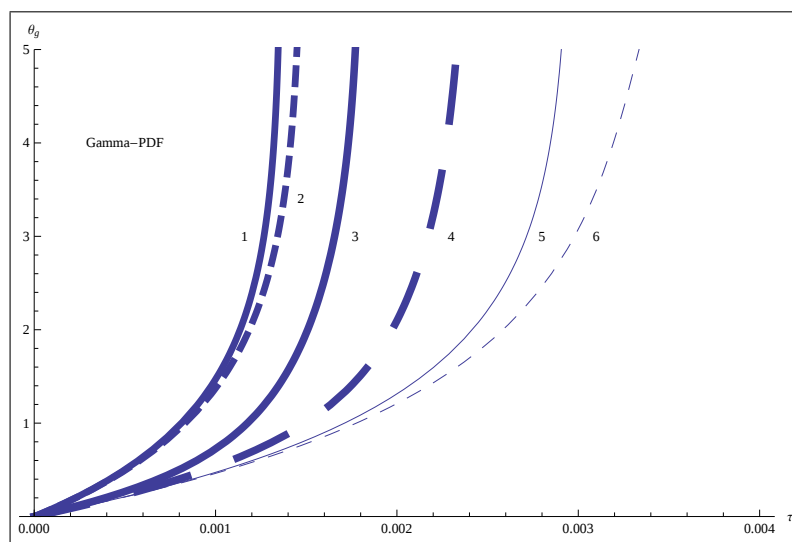


Figure 4. Solution profiles of the gas temperature: $\theta_g - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

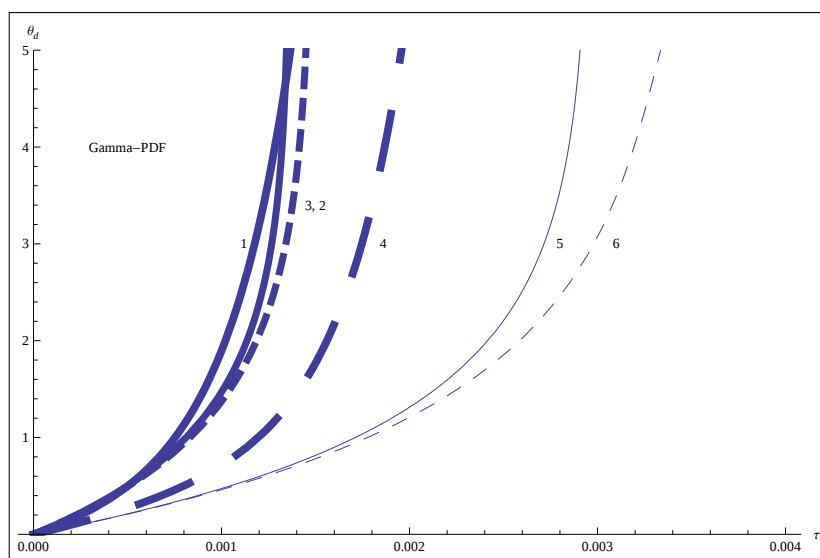


Figure 5. Solution profiles of the droplet temperature: $\theta_d - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

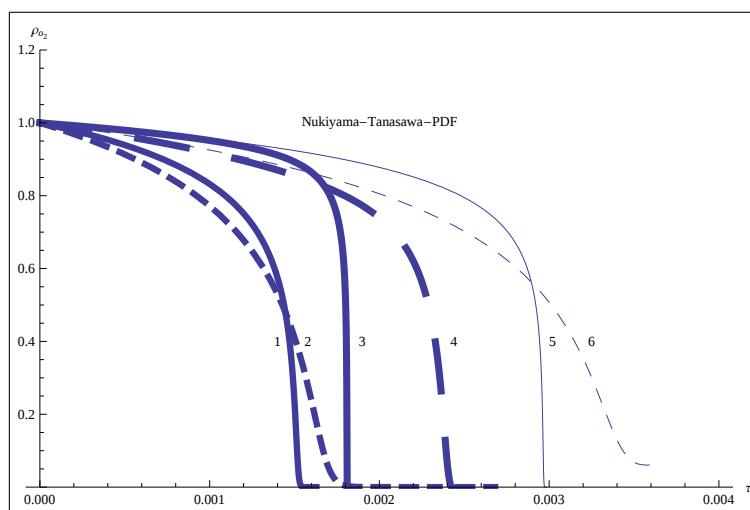


Figure 6. Solution profiles of the Oxygen: $\rho_{O_2} - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

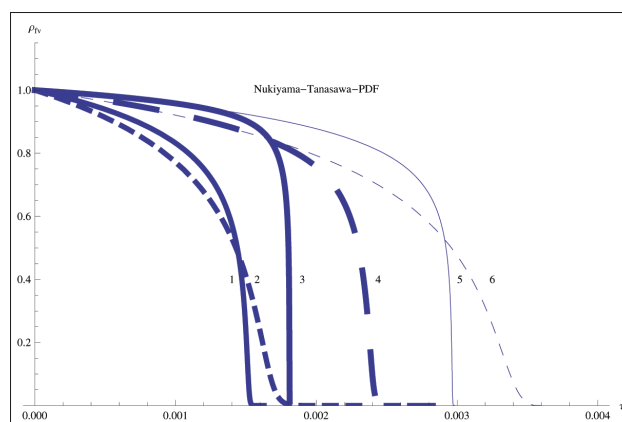


Figure 7. Solution profiles of the fuel vapour: $\rho_{fv} - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

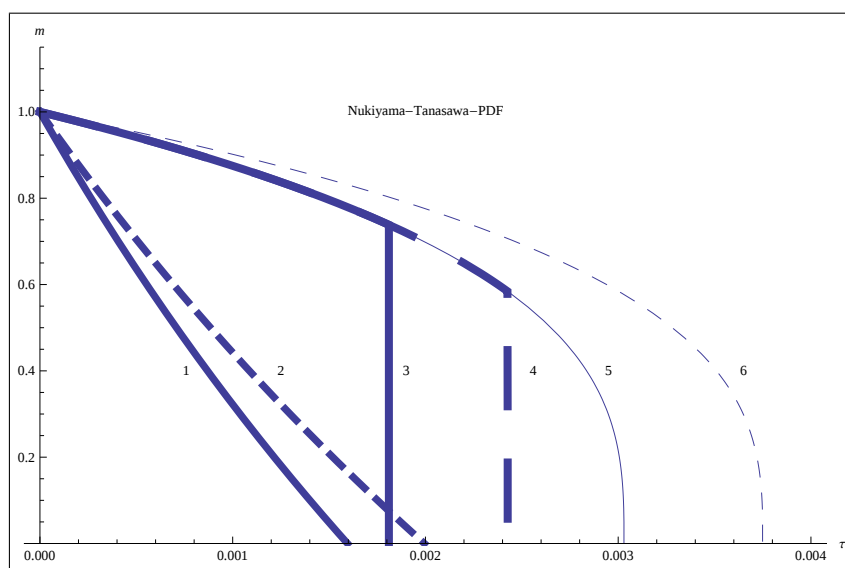


Figure 8. Solution profiles of the mass: $m - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

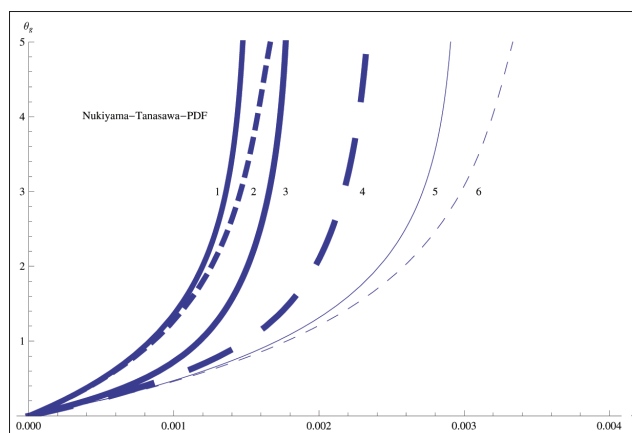


Figure 9. Solution profiles of the gas temperature: $\theta_g - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation

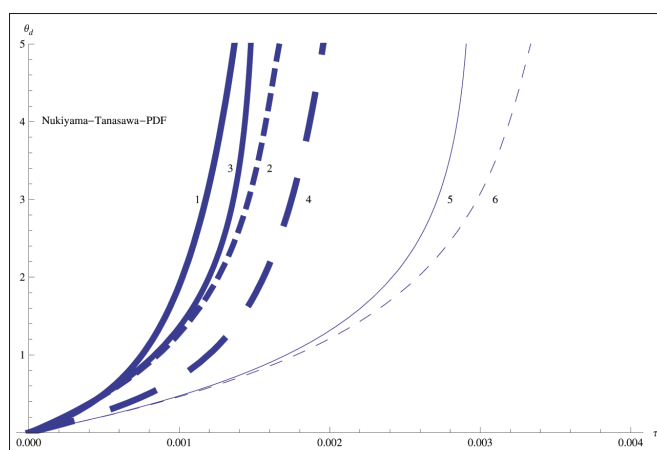


Figure 10. Solution profiles of the droplet temperature: $\theta_d - \tau$; 1: the continuous model with the impact of the thermal radiation, 2: the continuous model without the impact of the thermal radiation, 3: the polydisperse approximation model with the impact of the thermal radiation, 4: the polydisperse approximation model without the impact of the thermal radiation, 5: monodisperse model with the impact of the thermal radiation, 6: monodisperse model without the impact of the thermal radiation