Optimum Production-Distribution and Transportation Planning in Three-Stage Supply Chains

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Abstract

We develop an integrated mixed-integer linear programming model for production-distribution and transportation planning in three-stage supply chains (SCs). The model has two versions, the fully optimized version (FOV) and the less fully optimized (LFOV). Each of the versions determines the optimal quantity of each product to be produced at each plant in each period, kept in inventory at each plant in each period, transported to each distribution centre (DC) from each plant in each period, subcontracted at each DC, kept in inventory at each DC, and transported from each DC to each retailer in each period. It also determines the optimal amount of extensions needed at each DC in each period. Examples given to test and compare the two versions of the model show that they work very well. In all the examples, the optimal objective-function values produced by the FOV are greater than those produced by the LFOV. The examples also show that using a single model to integrate and optimize all the stages and key components of a SC simultaneous can greatly enhance an organization's operational efficiency and financial performance.

Keywords: Mixed integer linear programming, Model flexibility, Model versions, Produce-or-subcontract decisions, Supply chain performance

1. Introduction

Problems involving supplier selection, plant location, design and location of distribution centers (DCs), warehouse (WH) location and capacity, production planning, transportation and selection of transportation modes, customer-distribution centre assignments, selection and location of retail outlets, inventory management, and decision synchronization are some of the important strategic and operational problems in supply chain management (SCM). The growing importance and popularity of supply chain management in recent times have lead to the growing need for the developments of optimization models or techniques that integrate some or many of these problems simultaneously for best decision outcomes and for better financial and operations performance. Unlike before, industries are now beginning to realize that modeling and solving these problems separately lead to sub-optimal solutions or "less-than-best" decisions outcomes.

Despite these, published works on the developments of models/techniques for solving these problems still lag behind needs. The few most integrated models in these publications integrate about three to five of these problems. Some of these models can be seen in Armtzen et al. (1995); Jolayemi and Olorunniwo (2004); Liang (2008); Rizk et al. (2008); Routry et al. (2009); Tsiakis and Papageorgiou (2007); Tiwari et al. (2010); and You and Grosmann (2009).

Armtzen et al. (1995) develop a mixed integer linear programming (MILP) global SC model for determining product – plant and customer – distribution center assignments, number of SC echelons, and number and locations of DCs. Jolayemi and Olorunniwo (2004) formulate a two-stage SC model that determines the optimal quantities of products to be produced at each plant, transported from each plant to each WH, subcontracted at each WH, and kept in inventory at each WH. The model also determines the optimal amount of extensions needed at each WH. It is one of the few most highly integrated models. However, due to the large numbers of its constraints and binary variables, its size increases rapidly as the numbers of products, plants, and WHs increase.

A fuzzy multi-objective linear programming model for solving integrated production-transportation planning decision problems in supply chains fuzzy environments is developed by Liang (2008). The focus of the research by Rizk et al. (2008) is on flow synchronization between a manufacturing location and multiple destinations.

Multiple products can be shipped from the manufacturing location to different locations via multiple transportation modes.

Routry et al. (2009) develop multi-echelon inventory planning models with lead time and demand uncertainties. Tsiakis and Papageorgiou (2007) consider a model that integrates production, facility location, and distribution alongside with other business issues like import duties, plant utilization and maintenance, and exchange rates.

A hybrid Taguchi-Immune approach is applied by Tiwari et al. (2010) to optimize and integrate supply chain design problem with multiple shipping options, distributed customer demands, and fixed lead times. You and Grosman (2009) present mixed integer linear programming models and computational strategies for the problem of multi-echelon supply chains with inventory under uncertainty.

More examples of these types of integrated models can also be seen in Guo and Tang (2008); Park (2003); Sargut and Komeijn (2007); and Subramanya and Sharma (2008), to mention a few.

Like Jolayemi and Olorunniwo's model (J-O model), most of these other models are developed for designing SCs with less than three stages. This creates a great need for more integrated models for designing SCs with three or more stages. (A stage in a SC can be a set of parallel suppliers or parallel plants or parallel WHs/DCs or parallel retailers).

In our research, we develop an integrated mixed-integer linear programming (IMILP) model for production-distribution and transportation planning in three-stage SCs. The development of the model is based on an intensive modification, extension, and expansions of the J-O model.

The model has two versions. Each version determines the optimal quantity of each product to be produced at each plant in each period, kept in inventory as safety stock at each plant in each period, transported to each DC from each plant in each period, subcontracted at each DC, kept in inventory in each DC in each period, and transported from each DC to each retailer in each period. It also determines the optimal amount of extensions needed at each DC in each period. Thus, the model is developed for planning and managing the operations of SCs with multiple plants, multiple DCs, and multiple retailers. In addition to these, one of the versions of the model automatically determines the optimal level of demand that has to be satisfied for each product in each DC while DC-demands have to be specified in the other version.

In many practical situations, there may be no reasons or problems that warrant specifying the level of requirement for each product in each DC. Specifying the level of demands in each DC may lead to suboptimal decisions. The first version of the model is developed for these situations.

In many other situations, it may be necessary to specify the level of demand for each product in each DC in each period for some strategic, operational/tactical, marketing, or logistic reasons; or for space and resource limitations at the DCs. The second version of the model is developed for addressing these types of problems. Numerical examples will be given to test and to illustrate each version of the model and how each version works.

Additionally, we will use illustrative examples to demonstrate the benefits of integrating some of the stages and key components of SC management and operations into the model, particularly into the version that automatically determines the optimal levels of demands in the DCs (first version). Besides Chandra and Fisher (1994) and Park (2003) who investigated the benefits of integrating and optimizing plant and DC operations or production and distribution planning in two-stage SCs, we have not come across any published literature in which the benefits of integrating some of the stages and key components of SC is investigated. Thus, there has been a general failure on the part of previous authors to do this. This failure has left many potential users of integrated SC models with the impression that these key components are integrated into the models just to make them look sophisticated and complex and nothing more. To disabuse this and encourage the adoptions of integrated SC models in industries, the advantages of integrating these key components into them must be demonstrated.

2. Model Formulation

2.1 Definitions of Symbols

The symbols are grouped into two categories as follows:

2.1.1 Decision Variables

γ_{ipt} : a binary variable which is 1 if product i is produced in plant p in period t and zero otherwise.

Φ_{ipt} : the quantity of product i kept in inventory, as safety stock, in plant p in period t.

π_{it}: a binary variable which is 1 if the quantities of product i produced in all plants in period t meet customers'

demands and zero otherwise.

v_{ipjt}: the quantity of product i from plant p kept in inventory in DC j in period t.

z_{ijct}: the quantity of product i shipped from DC j to retailer c in period t.

q_{ijt}: the quantity of product i subcontracted in DC j in period t.

v_{ipj}: the quantity of product i from plant p kept in inventory, as a safety stock, in DC j in period t.

w_{jt}: the amount of extension in m3 that is needed in DC j in period t.

x_{ipt}: the quantity of product i produced in plant p in period t.

y_{ipjt}: the quantity of product i (in tons) shipped from plant p to DC j in period t.

z_{ijct}: the quantity of product i (in tons) shipped from DC j to retailer c in period t.

2.1.2 Parameters

a_{ript}: the amount of resource r required to produce a unit of product i in plant p in period t.

β_i: conversion factor in m3 per ton of product i.

b_{rpt}: the total amount of resource r available in plant p in period t.

c_{ipt}: the production cost per unit of product i in plant p in period t.

DD_{ijt}: demand for product i from DC j in period t.

e_{jt}: the cost of construction/extension per m3 of DC j in period t.

f_{ipt}: the setup cost of plant p with respect to product i in period t.

g_{ijt}: the cost per unit of subcontracting product i in DC j in period t.

h_{ijt}: the holding cost per unit of product i in DC j in period t.

k_{ipjt}: the cost of transporting a unit of product i from plant p to DC j in period t.

LG: a very large number.

m_{ipt}: the maximum possible capacity of plant p in period t with respect to product i.

s_{ijct}: the selling price of product i to retailer c in period t.

RD_{ict}: demand for product i from retailer c in period t.

θ_{ipt}: the holding cost per unit of product i kept in inventory as safety stock in plant p in period t.

u_{ijct}: cost of transporting product i from DC j to retailer c in period t.

w_{0jt}: the initial capacity of DC j in m3 in period t.

2.2 The Model

Using the decision variables and input parameters defined in section 2.1, we modify and extend some of the components and linear functions of the J-O model and develop new ones (components). In modifying the J-O model, some constraints and variables were completely eliminated from the model. We put all the extended and new components and linear functions together to obtain our three-stage SC model as follows:

$$\text{Maximize } \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{i=1}^{N} s_{ijct} z_{ijct} - \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{N} \theta_{ipt}\Phi_{ipt} - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{N} g_{ijt} q_{ijt} - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{i=1}^{N} h_{ijt} v_{ijpt}$$

$$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{N} (f_{ipt}\gamma_{ipt} + c_{ipt} x_{ipt}) - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{i=1}^{N} k_{ipjt} y_{ipjt} - \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{j=1}^{J}\sum_{i=1}^{N} u_{ijct} z_{ijct} - \sum_{t=1}^{T}\sum_{j=1}^{J} e_{jt} w_{jt}$$

Subject to:

$$\sum_{i=1}^{N} a_{ript} x_{ipt} \le b_{rpt}, \quad r = 1, 2, ..., R;\ p = 1, 2, ..., P;\ t = 1, 2, ..., T. \quad .... (2.2\text{-}1)$$

$$x_{ipt} \le m_{ipt}, \quad i = 1, 2, \ldots N;\ p = 1, 2, \ldots, P;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-2)$$

$$\sum_{j=1}^{J} y_{ipjt} + \Phi_{ipt} - \Phi_{ipt-1} = x_{ipt}, \quad i = 1, 2, \ldots, N;\ p = 1, 2, \ldots, P;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-3)$$

$$LG\,\gamma_{ipt} - x_{ipt} \ge 0, \quad i = 1, 2, \ldots, N;\ p = 1, 2, \ldots, P;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-4)$$

$$\sum_{j=1}^{J} q_{ijt} \le \pi_{it}\left(\sum_{c=1}^{C} RD_{ict} - \sum_{p=1}^{P} m_{ipt}\right), \quad i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T \quad \ldots (2.2-5)$$

$$\sum_{p=1}^{P} y_{ipjt} + \sum_{p=1}^{P} v_{ipjt-1} + q_{ijt} - \sum_{p=1}^{P} v_{ipjt} = DD_{ijt}, \quad i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, J;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-6)$$

(As can be seen in sections 2.3 and 2.4, 2.2-6 is developed for situations where d_{ijt} have to be specified for some strategic or operational reasons)

$$\sum_{p=1}^{P}\sum_{i=1}^{N} \beta_i y_{ipjt} + \sum_{p=1}^{P}\sum_{i=1}^{N} \beta_i v_{ipjt-1} + \sum_{i=1}^{N} \beta_i q_{ijt} - \sum_{m=1}^{t} w_{jm} \le w_{0jt}, \quad j = 1, 2, \ldots, J;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-7)$$

$$\sum_{c=1}^{C} z_{ijct} - \sum_{p=1}^{P} (y_{ipjt} - v_{ipjt} + v_{ipjt-1}) - q_{ijt} = 0, \quad i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, J;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-8)$$

$$\sum_{j=1}^{J} z_{ijct} = d_{ict}, \quad i = 1, 2, \ldots, N;\ c = 1, 2, \ldots, C;\ t = 1, 2, \ldots, T. \quad \ldots (2.2-9)$$

$\pi_{it} = 1$ for $\sum_{j=1}^{J} RD_{ict} > \sum_{p=1}^{P} m_{ipt}$ and 0 otherwise; $q_{ijt} = 0$ for $\sum_{j=1}^{J} RD_{ict} \le \sum_{p=1}^{P} m_{ipt}$; e_{ipt} and $v_{ipjt} = 0$ for t = T;

e_{ipt-1} and $v_{ipjt-1} = 0$ for t = 1; $q_{ijt}, v_{ijpt}, w_{jt}, x_{ipt}, \Phi_{ipt}, y_{ipjt}, z_{ijct} \ge 0$; γ_{ipt} and $\pi_{it} = 0$ or 1 for all I, j, t.

2.3 Model Interpretation

The first term of the objective function is the total revenue from the sales of all products shipped to the retailers from all DCs in all the T periods. The second term is the total inventory holding cost for all products kept in inventory, as safety stock, in all plants over the T periods. The third term is the total cost of products supplied to the DCs by subcontractors. The fourth term is the total inventory holding cost for all products in all DCs over the T periods. The fifth term is the total production cost, including the total setup cost, during the planning horizon. The sixth term is the total cost of transporting all products from all plants to all DCs over the T periods. The seventh term is the total cost of transporting all products from all DCs to all retailers. The last term is the total cost of extension needed in all the J DCs in all the T periods. Thus, the objective function maximizes the total profit obtained after subcontracting the production, transportation, inventory, DC-extension/construction, and setup costs from the total sales revenue.

Constraint (2.2-1) ensures that a manufacturer does not plan beyond the resources that are available at each plant in each period. Constraint (2.2-2) expresses the fact that no plant can produce beyond its maximum capacity.

Constraint (2.2-3) balances the flows of all manufactured products at each plant. The constraint ensures that the quantity of any product i produced in any plant p in period t and the quantity of product i remaining in inventory from the previous period is equal to the quantity shipped from plant p to DC j in period t and the quantity kept in inventory at the plant, as safety stock, in period t. Constraint (2.2-4) ensures that if product i is produced in plant p in any period t, there will be a setup cost charged against him with respect to i.

In many situations, a producer would like to operate his plants to maximum capacity before subcontracting. Constraint (2.2-5) makes this possible. Due to some strategic, operational, and marketing reasons, a SC manager may have to specify the level of requirement for each product in each distribution center in each period.

Constraint (2.2-6) makes this possible. The constraint ensures that such specified levels of demand in each DC in each period are satisfied exactly as specified. It also allows a quantity of product i to be kept in inventory in each DC in each period. The constraint is the reason why we have two versions of the model (see section 2.4). A manufacturer would not like DC space to be a constraint to his operations. Constraint (2.2-7) takes care of this. The constraint enables an extension to be made at any DC at any period whenever necessary.

Constraint (2.2-8) performs three major functions. First, it is the link that connects constraint (2.2-9) to the rest of the model's constraints; second, it ensures that the total quantity of product i shipped to all retailers from DC j in period t and the total quantity kept in inventory in period t is equal to the total quantities of the product that are (1) shipped from all plants to DC j in period t, (2) kept in inventory in DC j in period t-1 and, (3) subcontracted at DC j in period t; and third, it balances the flows of products at each DC. Constraint (2.2-9) enables a retailer's demands to be satisfied. This is achieved by shipping each quantity of product i needed by the retailer to his destination from some or all the DCs.

The bound constraints "e_{ipt-1} and $v_{ipjt-1} = 0$ for t = 1 and for all i, p, and j (i = 1, 2, …., N; p = 1, 2, …..,P; and j = 1, 2, …., J)" ensure that inventory is not carried from one planning horizon to the next planning horizon in any plant and in any DC. Similarly, the bound constraints "e_{ipt} and $v_{ipjt} = 0$ for t = T and for all i, p, and j (i = 1, 2, …., N; p = 1, 2, …..,P; and j = 1, 2, …., J)" ensure that no item is in inventory at any plant and at any DC at the end of the planning horizon.

2.4 The Two Versions of the Model

As noted earlier, constraint (2.2-6) plays a special role in the model. As we mentioned in section 1, it may be sometimes necessary to specify the level of requirement for each product in each DC in each period for some strategic, operational/tactical, marketing, or logistic reasons; or for space and other resource limitations at the DCs. Constraint (2.2-6) has been developed for placing some restrictions on the model or some of its variables to ensure that DC-demand quantities are delivered or satisfied as specified under these situations.

In many other situations, there may be no reasons, or any resource limitations that warrant specifying the level of requirement for each product in each DC. This means that constraint (2.2-6) has to be dropped. Doing this removes some of the restrictions on the model and its variables with respect to DC-demand requirements. The removal allows the model to automatically determine the optimal demand or the level of requirement that have to be satisfied with respect to each product in each DC in each period in order that enough goods may be available in the DCs to satisfy each retailer's specified demands. Thus the removal of constraint (2.2-6) allows for a full optimization of the three-stage SC model while its inclusion makes the model to be less fully optimized. Therefore, it is very obvious that the results produced under the two practical situations or by the two models will be different.

For ease of reference, we will henceforth refer to the fully optimized version as the FOV and the less fully optimized version as the LFOV

3. Numerical Examples and Illustrations

3.1 Statement of Problem

We give some numerical examples to test and illustrate the model. The examples are based on a SC with three plants, two DCs, and three retail centres. The plants and the DCs are operated by a single company. The plants produce three different products and are located in different parts of a country. Any product produced in any plant can be transported to any of the two DCs to satisfy the DC's demand. The DCs are also located in different parts of the country.

Inventory of each product can be kept at the plants and the DCs. Whenever a plant's production capacity for any product cannot satisfy the demand for the product at any DC, the production shortfall is subcontracted and delivered to the DC by the subcontractor. Any quantity of any of the three products can be shipped from any of the two DCs to any of the three retailers to fulfill the retailer's order in any period. There are two products involved.

The problem is to determine the optimal quantity of each product to be produced at each plant in each period, transported to each DC from each plant in each period, transported from each DC to each retailer destination in each period, subcontracted at each DC, and kept in inventory as safety stock at each plant and in each DC. It is also to determine the optimal amount of extensions needed at each DC in each period and the optimal DC-demand quantities (for the case of FOV).

3.2 Examples and Comments

Based on the above general problem statement, we give four numerical examples to test, illustrate, and examine the properties of each of the two versions (the FOV and the LFOV) of the model. In the first example, we fed all relevant data (including the specified value of each DC's demand for each product in each period) directly into the LFOV without making any changes to its structure. We also fed the same data into the FOV without changing its (the FOV's) structure – but this is after excluding the specified DC-demand data. (Recall that FOV does not have DC-demand constraints). The resulting LP problem for FOV has 124 constraints and 142 continuous and 22 binary variables. The number of constraints of the resulting LP problem for LFOV is 136 while the numbers of its continuous and binary variables are also 142 and 22 respectively. Optimal solutions were obtained to each of the two LP problems, using the LINDO solver. The results are presented under example 1 in table 1 (Appendix).

The results show that the optimal objective-function value (the profit) for the FOV is $224,585,648.00 while the optimal value for the LFOV is $219,222,640.00. Thus, the FOV's objective-function value is greater than that of the LFOV by $5,363,008.00. This amounts to a big difference. The difference is due to the fact that the FOV is fully optimized while (due to the presence of constraint (2.2-6)) the LFOV is not optimized fully. These results show that it is more profitable to apply the FOV when there are no critical reasons or problems that warrant the specifications of demand levels for each product at the DCs.

The demand quantities that are automatically determined by the FOV for the DCs in this example are shown in table 2 (Appendix). The corresponding values specified for the DCs for input into the LFOV are also shown in the table. As can be expected, the table shows that the demand quantity determined for each product for each DC in each period by the FOV is different from the corresponding quantity specified for input into the LFOV. For example, the demand-quantity determined for product 1 in period 2 for DC 2 is 88000 while the corresponding quantity specified for input into LFOV is 60000. Thus, if the reason for this specification is due to the limitations of critical resources at DC 2 in that period, the value produced by FOV could lead to serious problems if it (the FOV) is applied instead of the LFOV. On the other hand, specifying demand quantities when there is no need to do so may lead to a big financial loss. In conclusion, the results show that both the FOV and LFOV will work well in their respective applicable situations in any practical application.

The LINDO solver instantaneously provides optimal solution to each problem at a click of the Solve command. To be able to compare the solution time for each problem, we solved each of them in excel, using the Excel 2007 solver (which is not as fast as the LINDO solver). The results are shown under example 1 in table 3 (Appendix). As can be seen in the table, Excel 2007 solver produced optimal solution to each problem within 21 seconds. Thus, the solution times for the two versions are the same. The optimal solutions produced with the Excel 2007 solver is the same as the solution produced by the LINDO solver. This means that the Excel 2007 solver can be a good tool for solving each problem, except that it is not as fast as the LINDO solver.

As a further test of the two versions of the model, we changed the constraint that balances the flow of goods in the DCs (constraint (2.2-8)) to "less-than-or-equal-to" constraint in both the FOV and LFOV. We also changed constraint (2.2-6) (the constraint associated with the specified demand requirements at the DCs) in LFOV in the same way. We make these changes to check whether, with the changes, the two models will be easier to solve.

As expected, the optimal solution to the resulting LP problems (see example 2 in table 1) is the same with the corresponding solution in example 1 for each version. The solutions times for the FOV- and LFOV-related problems with Excel 2007 are 18 and 24 respectively (see table 3 Appendix), which are not too different from their respective solution times in example 1. The optimal solution to each problem is the same as the corresponding solution in example 1, where the constraints are of the equality type. This shows that changing constraints (2.2-6) and (2.2-8) to "less-than-or-equal-to" constraints will not affect the optimal solution to both FOV and LFOV as long as the "equality" in constraint (2.2-9) remain unchanged. Again, as can be seen in table 3, the excel solution to each version of the model in this example is the same with the LINDO solution.

In example 3, we dropped constraint (2.2-5) completely from each version of the model. This removes the condition that a producer should not subcontract until all his plant capacities are fully utilized. The purpose of this is to illustrate how each version can be used for making make-or-subcontract decisions solely on cost considerations and to check if there is any advantage in using each version this way. The input data used in the previous examples for each version was used here. The LP problem obtained for the FOV from this has 118 constraints, 142 continuous variables and 18 binary variables. The one for the LFOV has 130 constraints and the same number of continuous and binary variables as that of FOV. Thus, the numbers of constraints and binary variables for each problem are less than their corresponding numbers in the two previous examples.

The optimal objective-function values produced by the LINDO solver are $224,615,648.00 and $219,252,640.00 for the FOV and LFOV respectively (see example 3 in table 1). These are each greater than their respective corresponding values in example 1 by $30,000.00. This shows that under each of the model's versions, it is much more profitable for a producer to subcontract than to produce whenever it is cheaper to do so and whenever the prevailing condition allows him to subcontract. These results indicate that the two versions of the model are flexible and very good for making make-or-subcontract decisions. Like in the two previous examples, the Excel solution to the model is very much the same with the LINDO solution (see table 3). The excel solution times are also not appreciably different from those in example 1. This again shows that the Excel 2007 solver can be a good solution tool for the two model versions.

After dropping constraint (2.2-5) in example 4 (see table 1), we changed constraint (2.2-8) to "less-than-or-equal-to" constraint in both the FOV and LFOV. Constraint (2.2-6) in the LFOV was also changed to "less-than-or-equal-to" constraint. The LINDO and excel solutions to the resulting LP problem for each version are the same with the corresponding solution obtained for them in example 3 (see examples 3 and 4 of tables 1 and 3). The solution times in example 4 are not appreciably different from those in example 3. The example again shows that changing constraints (2.2-6) and (2.2-8) to "less-than-or-equal-to" constraints will not have any effect on the optimal solution to both FOV and LFOV as long is the "equality" constraints in (2.2-9) remain unchanged.

Many other interesting results of the four numerical examples can be seen in table 1. As can be seen in the table, the total cost of transportation is greater for the FOV than for the LFOV in each of the four examples while, in each example, the total cost of DC extension is greater for the latter than for the former. The number of iterations before obtaining optimal solutions is larger for the FOV than for the LFOV in three of the examples. This shows that the FOV requires more computational time to solve than the LFOV, and this is due to its fully optimized nature.

3.3 The Effects of Integrating the Model's Stages and Key Components on Its Performance

Here in this section, we will use more examples to determine and demonstrate the effects of each of the components of our model – namely: plant- and DC-inventory components, subcontract components, and DC-extension components – on the model's performance. The purpose is to determine the benefits of integrating these components into the model. We will also use examples to demonstrate the effects of having all the three stages of the model integrated together. Since the model's FOV is the only one that is fully integrated, it will be used here to represent our three-stage SC model. The same input data used in the previous examples will be used in our illustrations and the results obtained for the FOV in example 1 of section 3.2 will be used as standards for comparisons.

The first illustration was with the integration of all the stages of the SC networks. We broke the SC model into two separate parts. The first part (FSP) is made up of constraints (2.2-1) to (2.2-7). Recall that these constraints cover plant productions, plant inventories (inventories of finished product), plants-to-DCs shipments of goods, DC operations (i.e. DC inventories and DC extensions), and subcontracting. (Note that since this part is supposed to be a separate model, constraint (2.2-6) has to be included for it to work). The specified DC-demands used for the LFOV under example 1 in section 3.2 is used here. The objective function for this part is made up of the production, plant- and DC-inventory, subcontracting, plant-to-DC shipment, and plant extension costs.

The second separate part (SSP) covers DCs-to-retailers shipments of goods. This takes the form of the classical transportation problem whose constraints are derived from constraints (2.2-8) and (2.2-9) of the original model. Constraint (2.2-9) applies directly but constraint (2.2-8) is restructured to properly represent the DC-supply constraints of the separate transportation model. The specified DC-demands used in the FSP are used as DC supplies in the SSP and the specified retailers demands used in both FOV and LFOV under example 1 in section 3.2 are also used as retailers' demands in it (the SSP).

The objective function for the model maximizes the net profit after the costs of the shipments of goods to retailers are subtracted from the revenue derived from the sales of all goods to retailers. The same values of sale prices and shipping costs used in FOV and LFOV under example 1 of section 3.2 are used.

The two (the FSP and SSP) were solved separately. The optimal objective-function value or total cost for FSP was $48,197,360.00 while the one for SSP was $267,420,000.00. Since the latter represents net profit, we subtract the former from it to obtain a final profit of $219,222,640.0. (See illustration 1 in table 4 (Appendix)). This is less than the net profit produced by the FOV under example 1 of section 3.2 by $5,363,008.00. This shows that using a single model to integrate and optimize all the stages of a SC simultaneously can be very financially rewarding and can greatly enhance operations performance

In the second illustration, we dropped the subcontract component by dropping all the subcontract variables from the FOV. An infeasible solution was obtained after feeding the values of the input parameters into the resulting LP problem and solving it. We were not able to obtain an optimal solution to the problem until all the "equality" constraints in (2.2-8) and (2.2-9) were changed to "less-than-or-equal-to" constraints. The results of the optimal solution are presented as illustration 2 in table 4 (Appendix).

The solution shows that the total quantity of goods delivered to retailers is 597000, which is less than the total quantity of goods delivered in example 1 (under FOV in section 3.2) by 14000. The results also show that the optimal objective-function value is $220,401,312.00, which is less than the value produced by FOV in example 1 of section 3.2 by $4,184,329.00. From these results, it can be concluded that it is necessary to integrate subcontract component into SC models to save producers from the possibility of failing to satisfy demands.

In illustration 3 (see table 4), we dropped the DC-extension components (i.e. the fourth term of the left-hand side of the constraints in (2.2-7)) from the FOV, leaving only the fixed-and-inextensible- DC- space constraints. Like in illustration 2, the new LP problem obtained from this was not feasible until we changed all the "equality" constraints in (2.2-8), and (2.2-9) to "less-than-or-equal-to" constraints. The optimal solution shows that only 237667 goods were delivered to retailers. This is less than the quantity delivered under FOV in example 1 of section 3.2 (see table 1) by 61.10%. The optimal value of the objective function was $104,142, 864.00, which is less than the value produced by FOV in example 1 in table 1 by 53.63%. This shows that it is very advantageous if an SC model can automatically determine the optimal amount of extension needed in each DC.

The results obtained in illustration 4 when the plant inventory components were dropped show that the total quantity of goods delivered to retailers is still 611000, just as it is under FOV in example 1 of section 3.2. However, the total profit dropped by $17008.00. When the DC inventory component alone was dropped in illustration 5, the optimal value of the objective function was $224,585,648.00, which is the same as the value produced by FOV in example 1 (see table 1). Therefore, unlike the case of plant inventory, dropping the DC inventory component from the model makes no difference.

When we dropped both the plant and DC inventory components from the model in illustration 6, the optimal objective-function value was the same as the objective function value in illustration 4 (see table 4). The reason for this is that the optimal values of all the plant inventory variables here are the same with those of illustration 4 while the optimal values of the DC inventory variables are all zero.

In illustration 7, when the plant and DC inventory and subcontract components were drooped, the optimal value of the objective function was $220,384,304.00. This is less than the value obtained in four of the first five illustrations above. Only the objective-function value obtained in illustration 3 is less than this.

The above illustrations have shown the great necessity for integrating as many stages and key components of SC operations and managements as possible into SC models. The integration of these stages and key components into the models is not just to increase the models' sophistications, it is to improve their performance and the profitability of SC operations.

4. Conclusion

The IMILP model, with its two versions, developed in this paper is a very good and effective tool for planning and managing three-stage SCs. Numerical examples given to test and illustrate the two model-versions show that they work very well and that, under appropriate and applicable situations, the application of the FOV can be more financially beneficial than the application of the LFOV. We also use additional numerical examples to determine and illustrate the effects of integrating all the stages and key components of a three-stage SC. The illustrations show that the integrations of each and all of the key components and stages greatly improve SC's performance.

Among the things that distinguish this model from the existing models are its two versions – the FOV and the LFOV. Unlike any of the existing models, the two versions will make the model to be readily applicable under different conditions and situations that commonly occur in practice. (See section 2.4 for more details). We have also shown that the two model versions can be very useful in making make-or-buy decisions.

We find the LINDO and the Excel 2007 solver to be very good for solving the examples. After entering the input data into the model in each example, the LINDO solver produced optimal solution to it in less than a micro second.

The Excel 2007 solver produced optimal solution to each example within a time-range of 18 to 24 seconds.

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Table 1. Results of the numerical examples used for illustrating the two versions of the model

Model details	Numerical examples and solution parameters							
	Example 1		Example 2		Example 3		Example 4	
	FOV version	LFOV version	FOV version	LFOV version	FOV version	LFOV version	FOV version	LFOV version
Number of products	3	3	3	3	3	3	3	3
Number of plants	3	3	3	3	3	3	3	3
Number of DCs	2	2	2	2	2	2	2	2
Number of retailers	3	3	3	3	3	3	3	3
Number of periods	2	2	2	2	2	2	2	2
Number of constraints	124	136	124	136	118	130	118	130
Number of continuous variables	142	142	142	142	142	142	142	142
Number of binary variables	22	22	22	22	18	18	18	18
Number of iterations	553	365	467	430	368	398	446	430
Objective function value	$224585 648.00	$21922 2640.0	$22458 5648.0	$21922 2640.0	$22461 5648.0	$21925 2640.0	$22461 5648.0	$21925 2640.0
Cost of transportation from plants to DCs	$190600 0.00	$1905 000.00	$18890 00.00	$1905 000.0	$18960 00.0	$1885 000.00	$19460 00.00	$1889 000.00
Cost of transportation from DCs to retailers	$182100 0.00	$1640 000.00	$18210 00.00	$1640 000.0	$18210 00.00	$1640 000.0	$18210 00.00	41640 000.00
Total cost of transportation	3727000	$35450 00.00	$37100 00.00	$35450 00.00	$37170 00.00	$3525 000.00	$37670 00.00	$352 9000.0
Total goods transported to all retailers	611000	611000	611000	611000	611000	611000	611000	611000
Cost of inventories at the plants	$17000.0 0	$170 00.00	$170 00.00	$1700 0.00	$27000. 00	$2100 0.00	$27000. 00	$210 00.00
Cost of inventories at the DCs	00.00	0.0	0.0	0.0	0.0	$2000 0.00	0.0	$120 00.00
Total cost of inventories	$17000.0 0	$170 00.00	$170 00.00	$1700 0.00	$27000. 00	$430 00.00	$27000. 00	$330 00.00
Total goods subcontracted	14000	14000	14000	14000	14000	14000	14000	14000
Amount of DC extensions	371500	371500	371500	371500	371500	371500	371500	371500
Total costs of DC e extensions	$379800 0.00	$3882 0000.0	$37980 00.00	$3882 0000.0	$37980 00.00	$3882 0000.0	$37980 00.00	$38820 000.00

Table 2. The DC-demand quantities produced by FOV and the values specified as inputs into LFOV in example 1

Product	Optimal levels of DC-demand produced by FOV in example 1				Levels of DC-demands specified as inputs into LFOV in example 1			
	DC 1		DC 2		DC 1		DC 2ed	
	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
1	67000	31000	31000	88000	50000	59000	48000	60000
2	30000	0	57000	96000	45000	42000	42000	54000
3	63000	117000	31000	0	49000	61000	45000	56000
Total	160000	148000	119000	184000	144000	162000	135000	170000
Total	611000				611000			

Table 3. The Excel 2007 solver's solutions to the numerical examples

Model details	Example 1		Example 2		Example 3		Example 4	
	FOV	LFOV	FOV	LFOV	FOV	LFOV	FOV	LFOV
Number of products	3	3	3	3	3	3	3	3
Number of plants	3	3	3	3	3	3	3	3
Number of DCs	2	2	2	2	2	2	2	2
Number of retailers	3	3	3	3	3	3	3	3
Number of periods	2	2	2	2	2	2	2	2
Number of constraints	124	136	124	136	118	130	118	130
Number of continuous variables	142	142	142	142	142	142	142	142
Number of binary variables	22	22	22	22	18	18	18	18
Objective function value	$224585641.00	$219222641.0	$224585641.0	$219222641.0	$224615641.0	$219252641.0	$224615641.0	$219252641.0
Solution time in (seconds)	21	21	18	24	22	19	20	22
% deviation from LINDO solution, using LINDO solution as standard	0.00000312%	0.0%	0.00000312%	0.0%	0.00000312%	0.0%	0.00000312%	0.0%

Table 4. Results of the examples used to Illustrate the effects of the model's key components

Model details	Illustration 1	Illustration 2	Illustration 3	Illustration 4	Illustration 5	Illustration 6	Illustration 7	Illustration 8
Number of products	3	3	3	3	3	3	3	3
Number of plants	3	3	3	3	3	3	3	3
Number of DCs	2	2	2	2	2	2	2	2
Number of retailers	3	3	3	3	3	3	3	3
Number of periods	2	2	2	2	2	2	2	2
Number of constraints	FSP: 106 SSP: 30	118	124	124	124	124	118	118
Number of continuous variables	FSP: 106 SSP: 36	130	138	124	124	106	94	90
Number of binary variables	FSP: 22 SSP: 0	18	22	22	22	22	18	18
Number of iterations	FSP: 360 SSP: 27	408	528	366	393	427	325	393
Objective function value	Net profit: $219222640	$220401312.0	$104142864.0	$224568640.0	$224585648.0	$224568640.0	$220384304.0	$103334176.0
Cost of transportation from plants to DCs	$2625000.00	$1961334.00	$764002.00	$1923000.00	$1889000.00	$1923000.00	$1923334.00	$774670.00
Cost of transportation from DCs to retailers	$1640000.00	$1776666.00	$746334.00	$1821000.00	$1821000.00	$1821000.00	$1776666.00	$723668.00
Total cost of transportation	$4265000.00	$3738000.00	$1510336.00	$3744000.00	$371000.00	$3744000.00	$3700000.00	$1498338.00
Total goods transported to all retailers	611000	597000	237667	611000	611000	611000	597000	234334
Cost of inventories at the plants	$17000.00	$17000.00	0.0	N/A	$17000.00	N/A	N/A	N/A
Cost of inventories at the DCs	0.0	0.0	0.0	0.0	N/A	N/A	N/A	N/A
Total cost of inventories	$17000.00	$17000.00	0.0	$0.00	$17000.00	N/A	N/A	N/A
Total goods subcontracted	14000	0.0	10000	14000	14000	14000	N/A	N/A
Amount of DC extensions	371500	361500	N/A	371500	371500	371500	361500	N/A
Total costs of DC extensions	$38820000.00	$37050000.00	N/A	$38820000.00	$38820000.00	$38820000.00	$37620000.00	N/A