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Modeling and Estimation of Volatility in the Indian Stock Market

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Abstract

The international financial markets turmoil, which started around mid-2007, has depreciated substantially since August 2008. The financial market crisis has led to the collapse of major financial institutions. Nevertheless, crashes and/or crises are not devoted to only developed markets and developing countries including India, are not excluded from this rule and it may face such a condition. The sharp decline of Sensex price index from its closing peak of 20 873 on January 8, 2008, to less than 10 000 by October 17, 2008, in line with similar large declines in other major stock markets is good reminders of this fact. Volatility as a measure of risk plays an important role in many financial decisions in such a situations. The main purpose of this study is to examine the volatility of the Indian stock markets and its related stylized facts using ARCH models. The BSE500 stock index was used to study the volatility in the Indian stock market over a 10 years period. Two commonly used symmetric volatility models, ARCH and GARCH were estimated and the fitted model of the data, selected using the model selection criterion such as SBIC and AIC. The adequacy of selected model tested using ARCH-LM test and LB statistics. The study concludes that GARCH (1, 1) model explains volatility of the Indian stock markets and its stylized facts including volatility clustering, fat tails and mean reverting satisfactorily

Keywords: India stock exchange, Volatility, Stylize facts, ARCH models

1. Introduction

Fluctuation of stock prices is not destructive per se and is a sign of market efficiency in stock markets. In an efficient market, stock price fully reflects all available information. Thus, stock price fluctuates in response to new information. The main problem with price fluctuation that affects the financial market efficiency is destructive excess volatility that ends up in crashes and/or crises in financial markets. In such a situation, difference between stock intrinsic value and its related market value is significant and has several consequences.

The turmoil in the international financial markets of advanced economies, that started around mid-2007, has exacerbated substantially since August 2008. The financial market crisis has led to the collapse of major financial institutions. Nevertheless, crashes and/or crises are not devoted to only developed markets and developing countries including India, are not excluded from this rule and it may face such a condition. Top-11 Indian stock market crashes include Apr 1992, May 2004, May 2006, April 2007, July 2007, Aug 2007 Oct 2007, Nov 2007,Dec 2007,Aug, 2007and particularly, Jan 2008 are good reminders of this fact. With the volatility in portfolio flows having been large during 2007 and 2008, the impact of global financial turmoil has been felt particularly in the Indian equity market. The BSE Sensex increased significantly from a level of 13 072 as at end-March 2007 to its peak of 20 873 on January 8, 2008 in the presence of heavy portfolio flows responding to the high growth performance of the Indian corporate sector. With portfolio flows reversing in 2008, partly because of the international market turmoil (*Mohan,2008*) the Sensex fell from its closing peak of 20 873 on January 8, 2008, to less than 10000 by October 17, 2008, in line with similar large declines in other major stock markets. In addition, Between January 1 and October 16 2008, the rupee fell by nearly 25 per cent, even relative to a weak currency like the dollar, from Rs 39.20 to the dollar to Rs 48.86 (*Chandrasekhar and Ghosh, 2008*).Hence, the study of financial asset volatility is important to academics, policymakers, and financial markets participants for several reasons. First, prediction of financial market volatility is important to economic agents because it represents a measure of risk exposure in their investments. Second, a volatile stock market is a serious concern for policymakers because instability of the stock creates uncertainty and thus adversely affects growth prospects. Evidence shows that when markets are perceived as highly volatile it may act as a potential barrier to investing. Third, the stock market volatility causes reduction in consumer spending. Fourth, pricing of derivative securities and pricing of call option is a function of volatility. Finally, stock return forecasting is in a sense volatility forecasting and this has created new job opportunities for the professionals those who are experts in volatility forecasting (*Onyeaso and Rogers, 2004*). Consequently, it can be seen that the study of stock market volatility and its characteristics is very important and can be helpful for formulation of economic policies and forming rules and regulations related to stock market.

While the volatility and its relationship with stock price in developed financial markets has been well studied, little concentration has been paid towards an extensive study of the volatility of the emerging stock market of India. It is now well known that equities from emerging capital markets have vastly different characteristics than equities from developed capital markets. There are at least four distinguishing features of emerging market returns: higher sample average returns, low correlations with developed market returns, more predictable returns, and higher volatility (*Bekaert and Wu, 2000*) .These differences may have important implications for decision making by investors and policy makers and put emphasis on developed markets finding may mislead policy makers in making proper decisions. Therefore, in line with developed markets studies, the main objective of this study is to investigate volatility and its related stylized facts in the Indian stock markets using ARCH models.

The rest of this paper is organized as follows. Section 2 deals with the volatility models considered for this paper. The review of literature is presented in section 3. The description of the BSE500 data and the methodology is presented in section4 .The results and discussions are presented in section 5 and finally section 6 concludes the paper.

2. Models of Volatility

ARCH models are capable of modeling and capturing many of the stylized facts of the volatility behavior usually observed in financial time series including time varying volatility or volatility clustering (*Zivot and Wang, 2006*).

The serial correlation in squared returns, or conditional heteroskedasticity (volatility clustering), can be modeled using a simple autoregressive (AR) process for squared residuals. For example, let y_t denote a stationary time series such as financial returns, then y_t can be expressed as its mean plus a white noise if there is no significant autocorrelation in y_t itself:

$$y_t = c + \varepsilon_t \tag{1}$$

where c is the mean of y_t, and ε_t is iid with mean zero. To allow for volatility clustering *or* conditional heteroskedasticity, assume that $\mathrm{Var}_{t-1}(\varepsilon_t^2) = \sigma_t^2$ with $\mathrm{Var}_{t-1}(.)$ denoting the variance conditional on information at time t-1, and

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots\cdots + \alpha_p \varepsilon_{t-p}^2 \tag{2}$$

Since ε_t has a zero mean, $Var_{t-1}(\varepsilon_t^2) = \sigma_t^2$, the above equtation can be rewritten as:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots\cdots + \alpha_p \varepsilon_{t-p}^2 + u_t \tag{3}$$

Where $u_t = \varepsilon_t^2 - E_{t-1}(\varepsilon_t^2)$ is a zero mean white noise process. The above equation represents an AR (p) process for ε_t^2, and the model in (1) and (2) is known as the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982), which is usually referred to as the ARCH(p) model. Before estimating a full ARCH model for a financial time series, it is necessary to test for the presence of ARCH effects in the residuals. If there are no ARCH effects in the residuals, then the ARCH model is unnecessary and misspecified.

Since an ARCH model can be written as an AR model in terms of squared residuals as in equation 3, A simple Lagrange Multiplier (LM) test for ARCH effects can be constructed based on the auxiliary regression as in equation 3. Under the null hypothesis that there is no ARCH effects:

$$H_0 = \alpha_1 = \alpha_2 = \cdots\cdots = \alpha_P = 0$$

the test statistic is

$$LM = T \cdot R^2 \sim \chi^2_{(P)}$$

where T is the sample size and R^2 is computed from the regression (3) using estimated residuals.

If P-value is smaller than the conventional 5% level, the null hypothesis that there are no ARCH effects will rejected. In other word, the series under investigation shows volatility clustering or persistence. If the LM test for ARCH effects is significant for a time series, one could proceed to estimate an ARCH model and obtain estimates of the time varying volatility σ^2 based on past history. However, in practice it is often found that a large number of lags P , and thus a large number of parameters, is required to obtain a good model fit. A more parsimonious model proposed by Bollerslev (1986) replaces the AR model in (equation 2) with the following formulation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \tag{4}$$

where the coefficients $\alpha_i (i = 0........, p)$ and $b_j (j = 1........., q)$ are all assumed to be positive to ensure that the conditional variance σ^2 is always positive. The model in (equation 4) together with (equation 1) is known as the generalized ARCH or GARCH (p, q) model. When $q = 0$, the GARCH model reduces to the ARCH model.

Under the GARCH (*p, q*) model, the conditional variance of ε_t, σ_t^2, depends on the squared residuals in the previous p periods, and the conditional variance in the previous q periods. Usually a GARCH (1, 1) model with only three parameters in the conditional variance equation is adequate to obtain a good model fit for financial time series (*Zivot and Wang, 2006*)

2.1 Arch Models Specification for BSE500

Before estimating ARCH models for a financial time series, taking two steps is necessary.First check for unit roots in the residuals and second test for ARCH effects.

The input series for ARMA needs to be stationary before we can apply Box-Jenkins methodology .The series first needs to be differenced until is stationary. This needs log transforming the data to stabilize the variance. Since the raw data are likely to be non-stationary, an application of ARCH test is not valid. For this reason, it is usual practice to work with the logs of the changes of the series rather than the series itself.

The presence of unit root in a time series is tested using Augmented Dickey- Fuller test. It tests for a unit root in the univariate representation of time series. For a return series R_t, the ADF test consists of a regression of the first difference of the series against the series lagged k times as follows:

$$\Delta r_t = \alpha + \delta r_{t-1} + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \varepsilon_t$$

Or

$$\Delta r_t = r_t - r_{t-1} ; r_t = 1n(R_t)$$

The null and alternative hypotheses are as follows:

$$\left| \begin{array}{l} H_0 : the\ series\ contains\ unit\ root \\ H_1 : the\ series\ is\ stationary \end{array} \right.$$

The acceptance of null hypothesis implies non-stationary. If the ADF test rejects the null hypothesis of a unit root in the return series, that is if the absolute value of ADF statistics exceeds the McKinnon critical value the series is stationary and we can continue to analyze the series.

Before estimating a full ARCH model for a financial time series, it is necessary to check for the presence of ARCH effects in the residuals. If there are no ARCH effects in the residuals, then the ARCH model is unnecessary and misspecified (Zivot and Wang, 2006).

2.1.1 Arch effect test process

Consider the k-variable linear regression model.

$$y_t = \beta_1 + \beta_2 x_{2t} + \cdots\cdots + \beta_k x_{kt} + u_t$$

In addition, assume that conditional on the information available at time (t-1), the disturbance term distributed as

$$u_t \sim \left[0, (\alpha_0 + \alpha_1 u_{t-1}^2) \right]$$

That is, u_t is normally distributed with zero mean and

$$Var(u_t) = (\alpha_0 + \alpha_1 u_{t-1}^2)$$

That is the variance of u_t follows an ARCH (1) process. The variance of u at time t is dependent on the squared disturbance at time (t-1), thus giving the appearance of serial correlation. The error variance may depend not only on one lagged term of the squared error term but also on several lagged squared terms as follows:

$$Var(u_t) = \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-1}^2 + \cdots\cdot + \alpha_p u_{t-p}^2$$

If there is no autocorrelation in the error variance, we have

$$H_0 : \alpha_1 = \alpha_2 = \cdots\cdots\cdot = \alpha_p = 0$$

In such a case, $Var(u_t) = \alpha_0$, and we do not have the ARCH effect.

Since we do not directly observe σ_t^2 Engle has shown that running the following regression can easily test the preceding null hypothesis:

$$\hat{u}_t^2 = \hat{\alpha}_0 + \alpha_1 u_{t-1}^2 + \hat{\alpha}_2 \hat{u}_{t-1}^2 + \cdots\cdot + \alpha_p u_{t-p}^2$$

Where u_t, as usual, denote the OLS variance obtained from the original regression model.

The null hypothesis can be tested by the usual F test but the ARCH-LM test of Engle 1982 is a common test in this regard. Under ARCH-LM test the null and alternative hypothesis for BSE500 stock index are as follows:

$$H_0 : \alpha_1 = 0 \ and \ \alpha_2 = 0 \ and \ \alpha_3 = 0 \ and \ \ldots\ldots\ldots\alpha_q = 0$$
$$H_1 : \alpha_1 \neq 0 \ and \ \alpha_2 \neq 0 \ and \ \alpha_3 \neq 0 \ and \ \ldots\ldots\ldots\alpha_q \neq 0$$

Null hypothesis in this case is homoskedasticity or equality in the variance. Acceptance of this hypothesis imply that, there is no ARCH effects in the under process series. In other word, the data do not show volatility clustering i.e. there is no heteroskadasticity or time varying variance in the data.

Since an ARCH model can be written as an AR model in terms of squared residuals as in

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + u_t$$

a simple Lagrange Multiplier (LM) test for ARCH effects can be constructed based on the auxiliary regression.

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + u_t$$

Under the null hypothesis that there are no ARCH effects:

The test statistic is as follows:

$$LM = T \cdot R^2 \sim \chi^2_{(P)}$$

Where T is the sample size R^2 is computed from the regression

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + u_t$$

using estimated residuals. That is in large sample TR^2 follows the Chi-square distribution with df equal to the number of autoregressive terms in the auxiliary regression.

The test statistic is defined as TR2 (the number of observations multiplied by the coefficient of multiple correlation) from the last regression, and it is distributed as a $\chi^2_{(q)}$ (Gujarati,2007).

Thus, the test is one of a joint null hypothesis that all q lags of the squared residuals have coefficient values that are not significantly different from zero. If the value of the test statistic is greater than the critical value from the χ2 distribution, then one can reject the null hypothesis. The test can also be thought of as a test for autocorrelation in the squared residuals. Alternatively, if P-value is smaller than the conventional α % level, the null hypothesis that there are no ARCH effects will rejected. In other word, the series under investigation shows volatility clustering or volatility persistence (Brooks, 2002).

If an ARCH effect is found to be significant, then the specification of an appropriate ARCH model is necessary. In order to identify the ARCH characteristics in BSE500, the conditional return should be modeled first; the general form of the return can be expressed as a process of autoregressive AR (p), up to (p) lags, as follows:

$$R_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 R_{t-1} + \varepsilon_t$$

This general form implies that the current return depends not only on (R_{t-1}) but also on the previous (p) return value (R_{t-p}).

The next step is to construct a series of squared residuals (ε_t^2) based on conditional return to drive the conditional variance. Unlike the OLS assumption of a constant variance of (ε_t, s), ARCH models assumes that(ε_t, s) have a non constant variance or heteroscedasticity, denoted by(h_t^2).After constructing time series residuals, we modeled the conditional variance in a way that incorporates the ARCH process of (ε^2) in the conditional variance with (q) lags.

The general forms of the conditional variance, including (q) lag of the residuals is as follows:

$$h_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_1 \varepsilon_{t-1}^2$$

The above equation is what Engle (1982) referred to as the linear ARCH (q) model because of the inclusion of the (q) lags of the (ε_t^2) in the variance equation. This model suggests that volatility in the current period is related to volatility in the past periods,

For example in the case of AR(1) model, If β_1 is positive ,it suggests that if volatility was high in the previous period, it will continue to be high in the current period, indicating volatility clustering .If β_1 is zero, then there is no volatility clustering.

To determine the value of q or the ARCH model order, we use the model selection criterion such as AIC (Akaike Information Criterion) and SBIC (Schwartz Bayesian Information Criterion). The decision rule is to select the model with the minimum value of information criterion. This condition is necessary but not enough because the estimate meets the general requirements of an ARCH model. The model to be adequate should have coefficient that all are significant. If this requirement meets then the specified model is adequate and fit the data well.

2.2 Garch model

The problem with applying the original ARCH model is the non-negativity constraint on the coefficient parameters of (βi's) to ensure the positivity of the conditional variance. However, when a model requires many lags to model the process correctly the non-negativity may be violated.

To avoid the long lag structure of the ARCH (q) developed by Engle (1982), Bollerslev (1986), generalized the ARCH model, the so-called (GARCH), by including the lagged values of the conditional variance. Thus, GARCH(p,q) specifies the conditional variance to be a linear combination of (q) lags of the squared residuals ε_t^2 from the conditional return equation and (p) lags from the conditional variance σ_{t-j}^2 .Then, the GARCH(p,q) specification can be written as follows:

$$h_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_1 \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_2 h_{t-j}^2 \qquad \forall \text{ j=1,......p and i=1,........q}$$

Where $\beta_1, \beta_2 > 0$ and $(\beta_1 + \beta_2) < 1$ is to avoid the possibility of negative conditional variance.

The above equation states that the current value of the conditional variance is a function of a constant and values of the squared residual from the conditional return equation plus values of the previous conditional variance.

To show the significance of the explanation of conditional variance of one lag of both ε_t^2 and h_t^2 , e.g. ε_{t-1} and h_{t-1}^2 ,the GARCH process should be employed by estimating the conditional return to drive ε_t^2 ,and then the estimation of the conditional variance by using equation below

$$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \alpha_1 h_{t-1}^2$$

The adequacy of the GARCH model can be examined by standardized residuals, $(\frac{\varepsilon}{\sigma})$, where (σ) is the conditional standard deviation as calculated by the GARCH model, and(ε) is the residuals of the conditional return equation.

$$R_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 R_{t-1} + \varepsilon_t$$

If the GARCH model is well specified, then the standardized residuals will be Independent and Identically Distributed (IID).To shows this, two-step test is needed. The first step is to calculate the Ljung-Box Q-Statistics (LB) on the squared observation of the raw data. This test can be used to test for remaining serial correlation in the mean equation and to check the specification of the mean equation. If the mean equation is correctly specified, all Q-statistics should not be significant.

The next step is to calculate the Q-statistics of the squared standardized residuals. This test can be used to test for remaining ARCH in the variance equation and to check the specification of the variance equation. If the variance equation is correctly specified, all Q-statistics should not be significant. Put another way, if the GARCH is well specified, then the LB statistic of the standardized residuals will be less than the critical value of the Chi-square statistic χ_{m-p-q}^2 (Alsalman.A.E.2002).

The test for mean equation specification can be thought of as a test for autocorrelation in the standardized residuals. The test is one of a joint null hypothesis that there is no autocorrelation up to order k of the residuals.

If the value of the test statistic is greater than the critical value from the Q-statistics, then the null hypothesis can be rejected. Alternatively, if p-value is smaller than the conventional significance level, the null hypothesis that there are no autocorrelation will be rejected. In other words, the series under investigation shows volatility clustering or volatility persistence. The same is true for variance equation .The only difference is that in this case the test will be done on squared standardized residuals.

In addition to Ljang-Box Q-statistics the ARCH-LM test also can be used to test the adequacy of Arch model. The procedure is same as ARCH model. To model selection, model selection criteria such as SBC criteria and AIC is used.

2. 3 Mean reversion

The high or low persistence in volatility is generally captured in the GARCH coefficient(s)of a stationary GARCH model.For a statinary GARCH model the volatility mean reverts to its long run level,at rate given by the sum of ARCH and GARCH coefficients,which is generally close to one for a financial time series. The average number of time periods for the volatility to revert to its long run level is measured by the half life of the volatility shock and it is used to forecast the BSE500 series volatility on a moving average basis (Banerjee and sarkar, 2006).

A covariance stationary time series $\{y_t\}$ has an infinite order moving avarage representation of the form

$$y_t = \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i}, \qquad \psi_0 = 1, \sum_{i=0}^{\infty} \psi_i^2 < \infty$$

The plot of the ψ_i against i is called the Impulse Response Function (IRF).The decay rate of IRF is sometimes reported as a half-life, denoted by L_{half}, which is the lag at which the IRF reaches $\frac{1}{2}$.

2. 3.1 Calculation of half-life of volatility shock for a stationary GARCH (1, 1) process

The mean reverting form of the basic GARCH (1 1) model is:

$$(\varepsilon_t^2 - \overline{\sigma}^2) = (\alpha_1 + \beta_1)(\varepsilon_{t-1}^2 - \overline{\sigma}^2) + u_t - \beta_1 u_{t-1}$$

where $\overline{\sigma}^2 = \alpha_0/(1-\alpha_1-\beta_1)$ is the unconditional long run level of volatility and $u_t = (\varepsilon_t^2 - \sigma_t^2)$. The mean reverting rate $\alpha_1 + \beta_1$ implied by most fitted models is usually very close to 1.The magnitude of $\alpha_1 + \beta_1$ controls the speed of mean reversion.The half life of a volatility shock is given by the formula

$$L_{half} = \ln(\frac{1}{2}) / \ln(\alpha_1 + \beta_1)$$

Measures the average time it takes for $\left|\varepsilon_t^2 - \overline{\sigma}^2\right|$ to decrease by one half.The closer $\alpha_1 + \beta_1$ is to one the longer is the half life of a volatility shock. If $\alpha_1 + \beta_1 > 1$, the GARCH model is nonstationary and the volatility will eventually explode to infinity (Banerjee and sarkar,2006).

3. Review of Literature

Stock prices volatility is an extremely important concept in finance for numerous reasons. The literature on stock price volatility agrees on one key phenomenon. There is evidence of sever movements in stock prices. In other words, dynamic nature of stock prices behavior is an accepted phenomenon and all participants in stock markets include regulators, professionals and academics have consensus about it. But, what causes stock prices volatility is a question that remains unsettled in finance field. Answer to this question, because of the great number of involved variables is not an easy task and up to now there is no consensus about it. However researchers in quest of answer this question has investigated the stock prices volatility from different angels. In this regards, from late twentieth century and particularly after introducing ARCH model by Engle (1982), as said by Bollerslev (1999) and Granger and Poon (2000) several hundred research that mainly accomplished in developed country and to some extent in developing countries has been done by researchers in this area using different methodology. Our objective in this section is to give the reader just a glimpse of these studies as follows:

Engle (1982) published a paper that measured the time-varying volatility. His model, ARCH, is based on the idea that a natural way to update a variance forecast is to average it with the most recent squired "surprise"(i.e. the squired deviation of the rate of return from its mean).While conventional time series and econometric models operate under an assumption of constant variance, the ARCH process allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. In the empirical application of the ARCH model a relatively long lag in the conditional variance equation is often called for, and to avoid problems with negative variance parameters a fixed lag structure is typically imposed.

Bollerslev (1986) to overcome the ARCH limitations introduced his model, GARCH, that generalized the ARCH model to allow for both a longer memory and a more flexible lag structure. As noted above, in the empirical application of the ARCH model, a relatively long lag in the conditional variance equation is often called for, and to avoid problems with negative variance parameters a fixed lag structure is typically imposed. In the ARCH process the conditional variance is specified as a linear function of past sample variance only, whereas the GARCH process allows lagged conditional variances to enter in the model as well.

Engle, Lilien, and Robins (1987) introduced the ARCH-M model by extending the ARCH model to allow the conditional variance to be determinant of the mean. Whereas in its standard form, ARCH model expresses the conditional variance as a linear function of past squired innovations in this new model they hypothesize that, changing conditional variance directly affect the expected return on a portfolio. Their results from applying this model to three different data sets of bond yields are quite promising. Consequently, they conclude that risk premia are not time invariant; rather they vary systematically with agent's perceptions of underlying uncertainty.

Nelson (1991) extended the ARCH framework in order to better describe the behavior of return volatilities. Nelson's study is important because of the fact that it extended the ARCH methodology in a new direction, breaking the rigidness of the G/ARCH specification. The most important contribution was to propose a model (EARCH) to test the hypothesis that the variance of return was influenced differently by positive and negative excess returns. His study found that not only was the statement true, but also that excess returns were negatively related to stock market variance.

Glosten, Jagannathan and Runkle (1993),to modify the primary restrictions of GARCH-M model based upon the truth that GARCH model enforce a symmetric response of volatility to positive and negative shocks, introduced GJR's (TGARCH) models. They conclude that there is a positive but significant relation between the conditional mean and conditional volatility of the excess return on stocks when the standard GARCH-M framework is used to model the stochastic volatility of stock returns. On the other hand, Campbell's Instrumental Variable Model estimates a negative relation between conditional mean and conditional volatility. They empirically show that the standard GARCH-M model is misspecified and alternative specifications provide reconciliation between these two results. When the model is modified to allow positive and negative unanticipated returns to have different impacts on conditional variance, they find that a negative relation between the conditional mean and the conditional variance of the excess return on stocks. Finally, they also find that positive and negative unexpected returns have vastly different effects on future conditional variance and the expected impact of a positive unexpected return is negative.

Engle and Ng (1993) measure the impact of bad and good news on volatility and report an asymmetry in stock market volatility towards good news as compared to bad news. More specifically, market volatility is assumed to be associated with the arrival of news. A sudden drop in price is associated with bad news on the other hand, a sudden increase in price is said to be due to good news. Engle and Ng find that bad news create more volatility than good news of equal importance. This asymmetric characteristic of market volatility has come to be known as the "leverage effect". The studies of Black (1976), Christie (1982), FSS (1987), Schwert (1990) and Pagan and Schwert (1989) also explain this volatility asymmetry with the" leverage effect". However, their models do not capture this asymmetry. Engle and Ng (1993) provide new diagnostic tests and models, which incorporate the asymmetry between the type of news and volatility, they advise researchers to use such enhanced models when studying volatility.

Batra [2004] in an article entitled" stock return volatility patterns in India” examined the time varying pattern of stock return volatility and asymmetric Garch methodology. He also examined sudden shifts in volatility and the possibility of coincidence of these sudden shifts with significant economic and political events both of domestic and global origin. Also, he examined stock market cycles for variation in amplitude, duration and volatility of the bull and bear phases over the reference period. His analysis revealed that liberalization of the stock market or the FII entry in particular does not have any direct implications for the stock returns volatility. No structural changes in the stock price volatility around any liberalization event or more importantly around the dates of breaks for volatility in FII sales and purchases in India were observed. The apparent link generally drawn between stock price volatility and the sudden withdrawal or heavy purchase by the FIIs i.e. the volatile FII investment in the stock market did not seem to hold true for India. In all the phases, as delineated by their structural break analysis, the period between 1991:05 and 1993:12 was the most volatile period with the standard deviation of stock returns exceeding that in the other periods. The study also showed that in general over the references period the bull phases are longer, the amplitude of the bull is higher and the volatility in the phases is also higher. He also concluded that the gains during expansions are larger than the losses during the bear phases of stock market cycles. The bull phase, in comparison with its pre liberalization character was more stable in the post liberalization phase. The results of their analysis also, showed that the stock market cycles have dampened in the recent past. Finally, the study showed that volatility has declined in the post liberalization phase for both the bull and bear phase of the stock market cycles.

Kumar [2006] in an article entitled "comparative performance of volatility forecasting models in Indian markets" evaluated the comparative ability of different statistical and economic volatility forecasting models in the context of Indian stock and forex markets. Based on the out of sample forecasts and the number of evaluated measures that rank a particular method as superior he concluded that it is possible to infer that EWMA will lead to improvements in volatility forecasts in the stock markets and the GARCH (5,1) will achieve the same in the forex market. As he concluded, his findings were contrary to the findings of Brailsford and Paff [1996] who found no single method as superior, but the results in stock market were similar to the findings of Akigray [1989] , McNillian [2001], Anderson and Bollerslev[1998] and Anderson et al [1999] in the Forex market.

Banerjee and Sarkar [2006] in an article entitled" long memory property of stock returns; evidence from India" examined the presence of long memory in asset returns in the Indian stock market. They found that although daily returns are largely uncorrelated, there is strong evidence of long memory in its conditional variance. They concluded that FIGARCH is the best-fit volatility model and it outperforms other Garch type models. They also observed that the leverage effect is insignificant in SenSex returns and hence symmetric volatility models turn out to be superior as they expected.

4. Methodology

The required data including 2108 daily closing observation for BSE500 price index covering the period 26/7/2000 through 20/01/2009 were obtained from the Bangalore Stock Exchange, and were based on daily closing prices. The BSE500 returns (r_t) at time t are defined in the logarithm of BSE500 indices (p), that is, $r_t = \log(p_t / p_{(t-1)})$ .Visual inspection of the plot of daily returns series of BSE500 proved very useful. It can be seen that from figure 1 that return fluctuates around mean value that is close to zero. Volatility is high for certain time periods and low for other periods. The movements are in the positive and negative territory and larger fluctuations tend to cluster together separated by periods of relative calm. The volatility was highest in 2004 and 2008 .Thus figure 1 shows volatility clustering where large returns tend to be followed by small returns leading to continuous periods of volatility and stability .Volatility clustering implies a strong autocorrelation in squared return.

The number of observation is 2108 .The mean daily return is1.53E-18 .The volatility (measured as a standard deviation) is 0.017142 .There is indication of negative skewness (Skw= -0.906) which indicates that the lower tail of the distribution is ticker than the upper tail, that is ,the index declines occur more often than its increases. The kurtosis coefficient is positive, having high value for the return series (Kurt = 8.293) that is the pointer of leptokurtosis or fat taildness in the underlying distribution. In fact, under the null hypothesis of normality the Jarque-Bera statistic asymptotically follows a Qi-squire distribution with 2 degree of freedom. The computed value of 2750 with P-value of zero rejects the normality assumption due to the high kurtosis indicating fat tail .Q-Q plot in figure 2 also confirm the non-normality of the returns series.

As table.1 shows ARCH-LM test is statistically significant which indicates the presence of ARCH effect in the residuals of mean equation of BSE 500.The ADF test statistics rejects the hypothesis of unit root in the returns series at 1% level of significance. A formal application of ADF test on log returns, rejects the null hypothesis of a unit root in the return series .There is rejection at 0.01 level of significance because absolute value of ADF statistics 19.66671 exceeds McKinnon critical value 3.4365. These properties of the BSE500 returns series are consistent with other financial times series.

The ARCH and GARCH models are estimated for BSE500 returns series using the robust method of Bollerslev-Wooldridge's quasi-maximum likelihood estimator (QMLE) assuming the Gaussian standard normal distribution. Next, we use information criteria such as AIC, SBIC values, and a set of model diagnostic tests (ARCH-LM test and Q-Statistics) to choose the volatility models which represent the conditional variance of the BSE500 returns series appropriately. We estimated the model using Eviews 4, Eviews 5.1 and S-plus 8.0.

5. Findings

To detect the presence of ARCH effect in the mean equation of BSE500 we use the ARCH-LM (Lagrange multiplier) test. We tested for ARCH-effect for higher order and found that coefficient of ε^2_{t-3}, ε^2_{t-5}, ε^2_{t-6} and ε^2_{t-8} found to be statistically insignificant .

ARCH-LM test is statistically significant which indicates the presence of ARCH effect in the residuals of mean equation of BSE 500[table1]. To determine which ARCH model is adequate for describing the conditional heteroscedasticity of the data at 5% significance level we apply sample ACF and PACF of the squared residuals which showed the existence of ARCH effects. The sample PACF indicated that an ARCH (4) model might be appropriate. Consequently, we specify the ARCH (4) model as follows:

$$r_t = \mu + \alpha_1 r_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-3}^2 + \alpha_4 \varepsilon_{t-4}^2$$

The results for the ARCH (4) for daily log returns of BSE500 are reported in table 2. As table 2 shows the estimates of $\alpha_1, \alpha_2, \alpha_3$ and α_4 are all statistically significant at the 5% level of significance. Therefore, the model need not to be simplified. Therefore, we choose ARCH (4) for our data set of BSE500. Using the AIC, SBIC and Loglikelihood model selection criteria we achieved same results.

To test the adequacy of the model we applied the ARCH-LM test up to four lag. The result has reported in the table 3. As table 3 indicates, both test statistics are statistically insignificants. It means no ARCH effects left in the model. Thus, we found that ARCH (4) can be possible representative of the conditional volatility process for daily return series of BSE500.Hence we obtain the following fitted model for mean and variance equations.

$$r_t = 0.001691 + 0.146755 r_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 8.01E - 05 + 0.237925\varepsilon_{t-1}^2 + 0.181726\varepsilon_{t-2}^2 + 0.167373\varepsilon_{t-3}^2 + 0.170113\varepsilon_{t-4}^2$$

5.1 Garch model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of an asset returns. Bollerslev (1986) proposes a useful extension known as the generalized ARCH (GARCH) model. The modeling process of ARCH models can also be used to build a GARCH model. However, specifying the order of GARCH model is difficult. For this reason only lower order of GARCH, models are used in most application. We fit the GARCH models with different orders (up to 5) to the daily returns. To select the order of GARCH model, we used SBC criteria. The model with lower value of SBC fits the data best. The results are presented in table 4. As table 4 shows, The SBIC value is lowest for p=1 and q=1. Therefore, we choose GARCH (1,1) for our data set of BSE500.Thus,we found that GARCH(1,1) can be possible representative of the conditional volatility process for daily return series of BSE500.Table5 reports the statistics regarding GARCH(1,1).To test the adequacy of GARCH (1,1) model we apply ARCH-LM test up to 10 lag. The results of ARCH –LM test are reported in table 6. As results show the F-statistic and Obs*R-squared statistic both are insignificance and indicating no arch effects left in the series.

Thus we employed GARCH (1,1) to model volatility .The model of volatility for BSE500 index using GARCH (1,1) are as follows:

$$r_t = 0.001526 + 0.131403 r_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 1.13E - 05 + 0.179646\varepsilon_{t-1}^2 + o.786714\sigma_{t-1}^2 + \varepsilon_t$$

As above model indicates the value of α is 0.179646 and the value of β is 0.786714. The sum of parameters is 0.97.The stationary condition $(\alpha + \beta < 1)$ is satisfied. The mean reverting rate $(\alpha + \beta) = 0.97$, implied by our fitted model is close to one. Therefore, the fitted GARCH model implies that conditional volatility is very persistent. A large value of GARCH lag coefficient β (0.786714) indicates that shocks to conditional variance takes a long time to die out, so the volatility is persistent. Low value of error coefficient α i.e. (0.179646) suggests that large market surprises induce relatively small revision in future volatility. $(\alpha + \beta) = 0.97$ is close to unity and implies that a shock at time t persists for many future periods. A high value of this kind implies a "long memory" in the stock market. Any shock will lead to a permanent change in all the future values of h_t , hence shocks to conditional variance are persistent.

5.1 Mean reversion

To test the null of non staionary series or no mean reversion in the BSE500 returns we applied two tests. First we used the unit root test. As it stated in the beginning of the chapter, the results of the ADF test showed that the series is stationary. In other words there was no evidence in favor of unit root in the data and we concluded that the data series is stationary. When the data series is stationary, it is mean reverting and volatility finally reverts to its long run average. Another way of testing mean reversion is using GARCH model. For a statinary GARCH model the volatility mean reverts to its long run level,at rate given by the sum of ARCH and GARCH coefficients,which is generally close to one for a financial time series. The average number of time periods for the volatility to revert to its long run level is measured by the half life of the volatility shock and it is used to forecast the BSE500 series volatility.here the sum of arch and garch term is nearly 0.97 which is close to 1. The mean reverting

rate $\alpha_1 + \beta_1$ implied by our fitted model is very close to 1.The magnitude of $\alpha_1 + \beta_1$ controls the speed of mean reversion. The half life of a volatility shock Measures the average time it takes for $\left|\varepsilon_t^2 - \bar{\sigma}^2\right|$ to decrease by one half the closer $\alpha_1 + \beta_1$ is to one the longer is the half life of a volatility shock.if $\alpha_1 + \beta_1 > 1$,the GARCH model is nonstationary and the volatility will eventually explode to infinity.In our case it is almost 22 or approximatly one calendar month.Therefore, the null hypothesis of unit root or no mean reversion is rejected and we accept the alternative hypothesis of staionary or mean reverting in the underlying series.

6. Conclusions

This study attempted to study the volatility and its stylized facts in the Indian stock market. The BSE500 index of Mumbai stock exchange is used as a proxy for the Indian market. The data used for analysis were 2108 daily observations for the period of 07/26/2000 to 01/20/2009. Empirical results showed that GARCH (1,1) model can adequately describe the BSE500 stylized facts. The results suggest that the volatility in the Indian stock market exhibits the persistence of volatility and mean reverting behavior. The conditional volatility of the BSE500 was found to be quite persistence. Within the ARCH family that used in this study, our results revealed that the GARCH (1,1) model satisfactorily explains volatility and is the most appropriate model for explaining volatility clustering, fat tails and mean reverting in the series under analysis. The results of the study have useful implications for regulator and policy makers in the Indian stock market. Given the inefficiency of traditional methods of calculating volatility such as Moving Average and EWMA in capturing stylized facts of stock market i.e. volatility clustering and mean reversion, using these methods in evaluating risk needs to be reviewed and using GARCH-type model should be considered in risk management decisions.

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Table 1. ARCH-LM test of BSE500 log returns series up to 10 lags

ARCH-LM TEST			
F-statistics	53.79811	Probability	0.0
Obs*R-Squared	429.9781	Probability	0.0

Table 2. ARCH (4) model parameters

ARCH (4) model parameters				
Mean equation				
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001691	0.000299	5.659764	0.0000
AR(1)	0.146755	0.024379	6.019606	0.0000
Variance Equation				
C	8.01E-05	8.14E-06	9.834135	0.0000
ARCH(1)	0.237925	0.044676	5.325500	0.0000
ARCH(2)	0.181726	0.046683	3.892787	0.0001
ARCH(3)	0.167373	0.048233	3.470131	0.0005
ARCH(4)	0.170113	0.038239	4.448684	0.0000

Table 3. Arch-Lm Test for Arch (4) Model Up to 4 Lag

ARCH(4) Test			
F-statistics	0.918461	Probability	0.514922
Obs*R-Squared	9.192570	Probability	0.513931

Table 4. SBIC for different Garch model

Comparisons of the SBC for the GARCH(p,q) model with different combinations of p and q for BSE500					
p	1	2	3	4	5
q					
1	**-5.637290**	-5.633765	-5.630554	-5.629541	-5.631872
2	-5.633735	-5.636361	-5.626771	-5.623496	-5.628710
3	-5.630212	-5.628160	-5.626771	-5.626950	-5.628535
4	-5.629460	-5.623036	-5.626453	-5.626464	-5.631176
5	-5.631211	-5.627702	-5.624387	-5.625789	-5.618241

Table 5. GARCH (1, 1) parameters

GARCH(1,1) Parameters				
Mean equation				
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001526	0.000294	5.190229	0.0000
AR(1)	0.131403	0.024166	5.437636	0.0000
Variance Equation				
C	1.13E-05	2.95E-06	3.819740	0.0001
ARCH(1)	0.179646	0.030620	5.867004	0.0000
GARCH(1)	0.786714	0.030442	25.84289	0.0000

Table 6. ARCH-LM test for Garch (1,1) model up to 10 lag

ARCH(10) test			
F-statistics	0.634337	Probability	0.785415
Obs*R-Squared	6.357489	Probability	0.784388

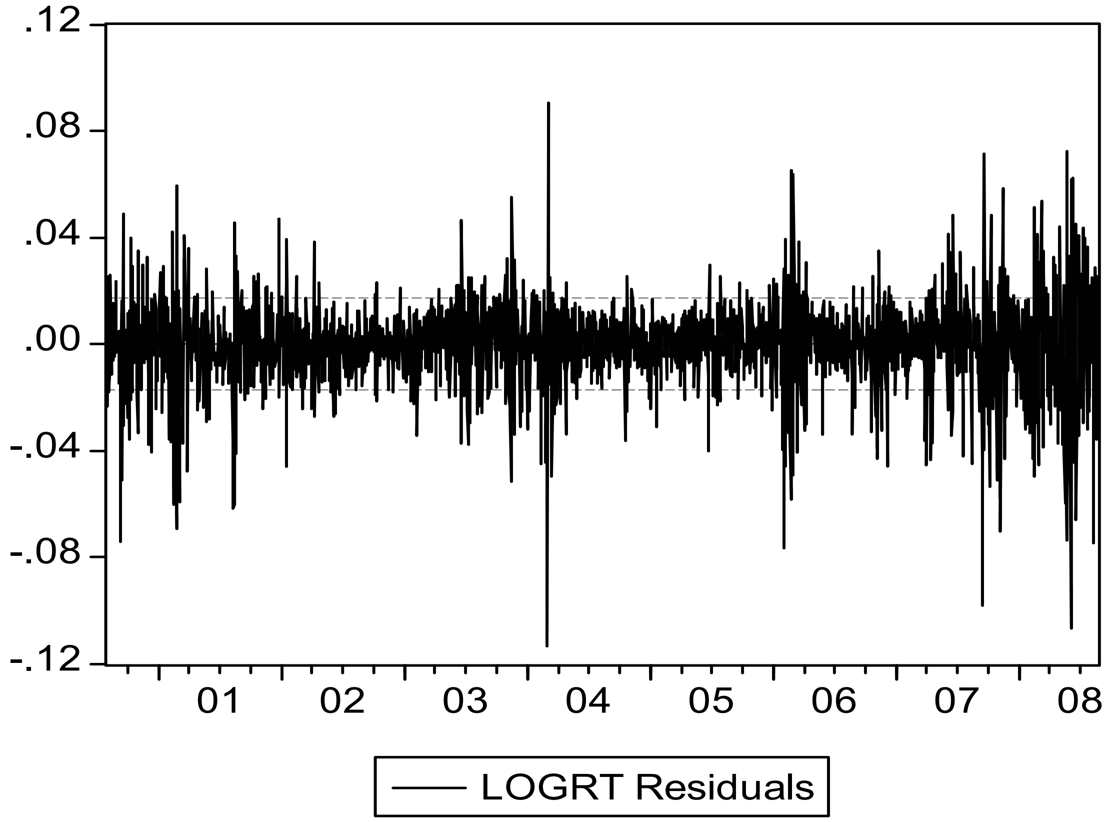

Figure 1. The Residuals of Bse500 Returns

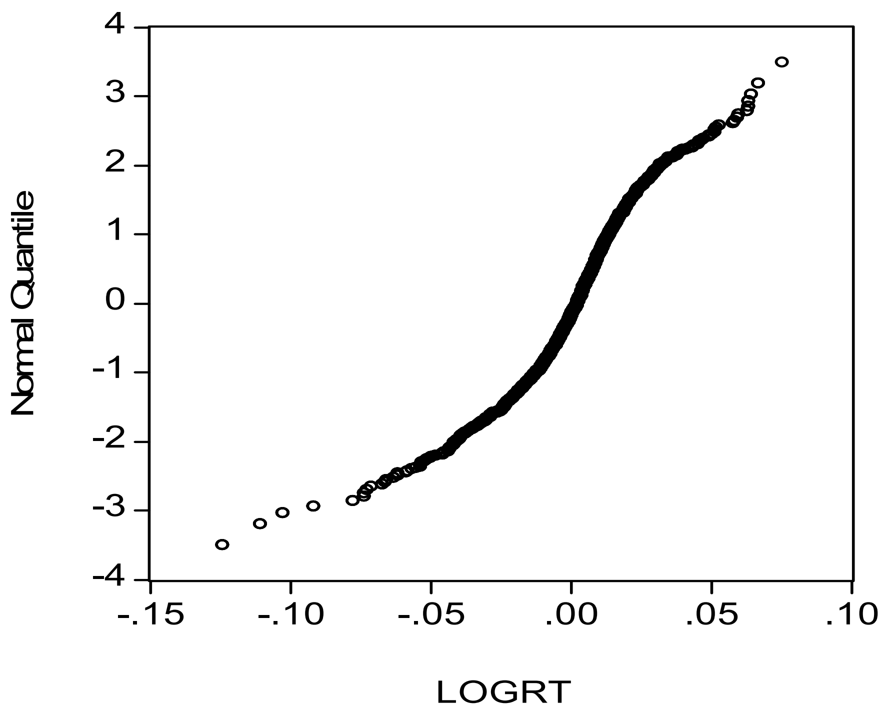

Figure 2. Q-Q Plot of BSE500 Daily Returns Series