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Optimization of Reorder Point Strategy of Assembly Manufacturer with Random Variables

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Abstract

For assembly system, randomness and variableness make it impossible to control the inventory accurately specially. Because the negative effect of them have ripple effects. So we investigate reorder point optimization strategy of assembly manufacturing system with random demand and random lead time. We seek the manufacturers order strategy for the minimum integration of the supply chain inventory cost. And we use scale benefits parameters of suppliers in this system to reflect actual influence of the lot's scale. Numerical example of two components assembly system is given to illustrate the effectiveness of the reorder point strategy. From the numerical simulation, it is can be seen backorder rate and order scale effect will impact the total supply chain cost dramatically.

Keywords: random leadtime, inventory optimization, supply chain

1. Introduction

In recent years, researches on order strategy with random demand and variable lead time are becoming more. Because the randomness and variableness make it difficult to predict orders accurately. And with randomness and variableness, phenomenon of high inventory level, long turnover and out of stock will become more and more. Also randomness makes more difficult to get the proper inventory control.

In fact, there are plenty of researches in variable demand and/or lead time in supply chain, especially since phenomenon of "bullwhip effect". So for enterprise's managemant, demand forecasting and information sharing and to eliminate the adverse effects of randomness in system are very important. Many research focused on the deterministic or controllable variables due to the stable environment of common commercial activities. But now moreand more research such as research of the demand variability and scheduled ordering policies in a supply chain facing stochastic demand, the effect of sharing stochastic demand and inventory data from the retailers to the supplier via information technology, single vendor and single buyer model with stochastic demand and variable lead time, and the lead time assumed varying linearly with the freight size are carried out.

And coordination is to manage production and inventory for different items on different levels to optimize assembly system inventory configuration. Appropriately planning the quantity and schedule of components reorder point leads to low inventory costs and the possibility to raise production profits. However, when uncertainty exists in the system, the coordination task is complex.

For assembly system, randomness and variableness make it impossible to control the inventory accurately specially. Because the negative effect of them have ripple effects. That is the shortage of one component will lead to not only the production halts but also the accumulation of others, the consumption of any component will be restrained by the storage level of others.

2. Literature Review

Uncertainties must be taken account into supply chain optimization and design. For the prior literature, it is discussed in detail about random demand and uncertain lead time (Zipkin, 2000). This book described the inventory control models under the conditions of specific patterns of demand, different ordering strategy and production capacity constraints. Inventory optimization strategy of a two-stage supply chain under uncertain condition is considered (Gérard P. Cachon & Paul H. Zipkin, 1999). It is proved the existence and uniqueness of the optimal expected inventory cost and the lead time, and found that the optimize strategy mainly depends on the fluctuation of lead time, and is less sensitive to other parameters. The theoretical approached are mainly queuing theory, Markov decision processes and other mathematical methods.

Managers have been under increasing pressure to decrease inventories as supply chains attempt to become leaner. The goal is to reduce inventories. There are main two factor bring the uncertainty to system, including the demand uncertainty and the lead time uncertainty. In some cases, lead time uncertainties have essentially no effect and therefore can be ignored or included in lead time demand modelling.Most often, however, lead time fluctuations strongly degradesystem performance. This is the case for the componentinventory control in assembly systems with random component procurement times i.e. lead times. This is because assembly systems are particularly sensitive to the lead time of components (all components must be present to begin the assembly). Therefore, for assembly systems, the lead time uncertainties must be considered precisely.

For recent articles on random variable of order strategy, a dynamic inventory and pricing problem with period of validity is considered(Pang 2011). An integrated model and a modified solution method for solving supply chain network design problems under uncertainty is proposed ,(Mohammadi Bidhandi, Mohd Yusuff 2011). The stochastic supply chain network design model is provided as a two-stage stochastic program where the two stages in the decision-making process correspond to the strategic and tactical decisions. Models of multi-item stochastic inventory system with backordered shortages when estimation of marginal backorder cost is available, and payment is due upon order arrival. The budget constraint can easily be converted into a storage constraint (Ghalebsaz-Jeddi, Shultes et al. 2004; Bera, Rong et al. 2009). Probabilistic lead-time is considered and shortages are allowed In any business, placement of an order is normally connected with the advance payment(Maiti, Maiti et al. 2009).

Reorder point order quantity inventory model where the demand and the lead time are independently and identically distributed random variables (Hayya, Harrison et al. 2009). Study deals with a multi-item mixture inventory model in which both demand and lead time are random. A budget constraint is also added to this model. The optimization problem with budget constraint is then transformed into a multi-objective optimization problem (Bera, Rong et al. 2009).

So we can see the recent studies deal with random in whole supply chain and some constraints. The stochastic supply chain network design model is provided as a multi-stage stochastic program (Çakanyildirim, Bookbinder et al. 2000; Huang and Küçükyavuz 2008; Rossi, Tarim et al. 2010).

This paper considers the optimal strategy of collaborative order and inventory with one assembly manufacturer and multi-suppliers with stochastic demand and variable lead time. That is to decide the manufacturer ordering strategy for minimizing supply chain whole cost with considering the ratio of raw materials in the assembly manufacturing production. And the manufacturer’s order scale parameter is used to model the impact of economies of scale in the actual operation of the supply chain.

3. Optimization Model with Random Demand and Lead Time

3.1 Problem Description and Notation

We consider a two-stage replenishment system of multi-components and single manufacturer with the assumption that both lead time and demand are random variables. Each supplier provides just one kind of component, and the proportion is 1: 1. The shortage of any component will make the assembly impossible. (Q, r) strategy is adopted by the manufacturer, as shown in Figure 1.

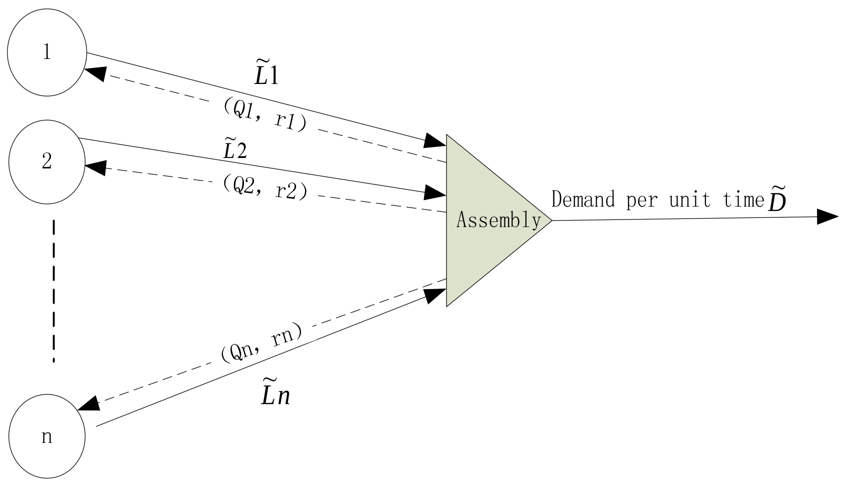

Figure 1. Multi-components assembly system

When the inventory level of component $i(i = 1,2\cdots n)$ is below the reorder point r_i, an order of Q_i will be required to the i th supplier. $\widetilde{L}_i$ means the lead time, and it's random. It can be known from the practice that greater the order quantity, more attention to the order supplier pay to, and so more efficient corresponding services, the shorter lead time.

With above assumption, lead time is variable, and is related to the order quantity Q.

$$\widetilde{L}_i(Q_i) = Q_i^{\theta}\widetilde{T}, \theta \geq 0 \tag{1}$$

Where θ means the affection degree of economic quantity on process time, which can be divided into two situations: $\theta = 1$ means there's no economic scale benefit; and $\theta = 0.5$ means that economic scale benefit is positive. Random variable $\widetilde{T}$ presents the unit lead time of component i. T follows exponential distribution, and its density function is given as Eq.(2).

$$f(\widetilde{T}_i) = \begin{cases} \lambda_i e^{-\lambda_i \widetilde{T}_i}, \widetilde{T}_i > 0 \\ 0, \quad \text{other} \end{cases} \tag{2}$$

The demand of finished product follows a normal distribution (μ, σ^2), and its density function is given as Eq. (3).

$$f(\widetilde{D}) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\widetilde{D}-\mu)^2}{2\sigma^2}} & \widetilde{D} > 0 \\ 0 & \text{other} \end{cases} \tag{3}$$

The market demand of component i during lead time $\widetilde{L}_i$ is $\widetilde{X} = \widetilde{L}\widetilde{D} = Q_i^{\theta}\widetilde{T}_i\widetilde{D}$, where variable $\widetilde{T}_i$ and $\widetilde{D}$ are independent. The mean is given as $\mu_{\tilde{X}} = \int_0^{\infty}\int_0^{\infty} Q_i^{\theta}\tilde{T}_i\tilde{D}f(\tilde{T}_i)f(\tilde{D})d\tilde{T}_i d\tilde{D} = Q_i^{\theta}\mu_{\tilde{D}}\mu_{\tilde{T}} = \frac{Q_i^{\theta}\mu}{\lambda_i}$, and the variance is given as Eq.(4).

$$\begin{aligned} Var(\widetilde{X}) &= E\left\{\left[\widetilde{X} - E(\widetilde{X})\right]^2\right\} \\ &= \mu_{\widetilde{L}_i}\sigma_{\widetilde{D}}^2 + \mu_{\widetilde{D}}^2\sigma_{\widetilde{L}_i}^2 \\ &= \frac{Q_i^{\theta}}{\lambda_i}\sigma^2 + \mu^2\frac{Q_i^{2\theta}}{\lambda_i^2} \end{aligned} \tag{4}$$

Notations:

s_i : order cost of component i per unit time;

μ : mean of market demand per unit time;

σ : standard deviation of market demand per unit time;

λ_i : exponential distribution parameter of i supplier;

$\widetilde{L}_i$: actual lead time of component i ;

$\widetilde{X}_i$: market demand during $\widetilde{L}_i$;

β : backorder coefficient;

π : shortage cost per unit time;

A_i : setup cost of supplier i per unit time;;

h_{mi} : unit inventory cost of component i on the manufacturer per unit time;

h_i : unit inventory cost of component i on supplier per unit time;;

$[g]^+$: $Max(g,0)$.

Decision variables:

Q_i : purchasing quantity of component i ;

r_i : reorder point of component i .

3.2 Cost Analysis of Manufacturer

The total cost per unit time of the manufacturer is composed of holding cost, shortage cost, and order cost.

(1) Holding cost. For component i, demand during the lead time $\widetilde{L}_i$ is $\widetilde{X}_i$, and $\widetilde{X}_i = \widetilde{D}\widetilde{L}_i$, reorder point $r_i = \mu_{\widetilde{X}_i} + k_i\sigma_{\widetilde{X}_i}$ .The average shortage of component i is $B(r_i)$, abbreviations for B_i .

$$\begin{aligned} B_i &= E[\widetilde{X}_i - r_i]^+ \\ &= \int_{r_i}^{\infty}(\widetilde{X}_i - r_i)f(\widetilde{X})d\widetilde{X} \\ &= \int_0^{\infty}\int_0^{\infty} Max(Q_i^{\theta}\widetilde{T}_i\widetilde{D} - r_i, 0)f(\widetilde{D})f(\widetilde{T}_i)d\widetilde{D}d\widetilde{T}_i \\ &= \int_0^{\infty}\int_{\frac{r_i}{Q_i^{\theta}\widetilde{T}_i}}^{\infty}(Q_i^{\theta}\widetilde{T}_i\widetilde{D} - r_i)f(\widetilde{D})f(\widetilde{T}_i)d\widetilde{D}d\widetilde{T}_i \end{aligned} \tag{5}$$

When component j is in shortage, the assembly will be stopped, which will result in overstock of component i. In the condition of cross-shortage, the shortage lost will be calculated according to the component which is in the largest shortage $MAX(B_i)$; otherwise, the shortage lost is the sum of shortages of all components $\sum_{i=1}^{n} B_i$. In this paper, we take the latter into consideration. So the overstock of component i because of the shortage of component j is shown in figure 2:

$$\frac{B_j}{\frac{Q_i}{\mu}} = \frac{\mu}{Q_i}B_j \tag{6}$$

The average inventory of component i in the manufacturer per unit time is I_i as Eq. (7).

$$\begin{aligned} I_i &\le \frac{1}{2}\left[Q_i + (r_i - \mu_{\widetilde{X}_i}) + (1-\beta)B_i + (r_i - \mu_{\widetilde{X}_i}) + (1-\beta)B_i\right] + \sum_{j \ne i}\frac{\mu}{Q_j}B_j \\ &= \frac{1}{2}Q_i + (r_i - \mu_{\widetilde{X}_i}) + (1-\beta)B_i + \sum_{j \ne i}\frac{\mu}{Q_j}B_j \end{aligned} \tag{7}$$

(2) The shortage cost of the manufacturer caused by the shortage of component i is given as Eq.(8).

$$SC_i = \frac{\mu}{Q_i}\pi(1-\beta)B_i \tag{8}$$

(3) Ordering cost OC: s_i presents the ordering cost of component i each time, Q_i represents the ordering quantity, and the ordering cost per unit time is $s_i \frac{\mu}{Q_i}$.

So total cost of the manufacturer is given as Eq.(9).

$$\begin{aligned} ECM &\le \sum_{i=1}^{n}(OC_i + h_m I_i + SC_i) \\ &= \sum_{i=1}^{n}(s\frac{\mu}{Q_i} + \frac{1}{2}Q_i h_m + k_i\sigma_{\tilde{X}_i}h_m + (1-\beta)B_i h_m + \sum_{j\neq i}\frac{\mu}{Q_j}B_j h_m + \frac{\mu}{Q_i}\pi(1-\beta)B_i \end{aligned} \tag{9}$$

3.3 Cost Analysis of Suppliers

The cost of supplier i includes two parts: setup cost and holding cost.

(1) Setup cost. A_i represents the setup cost of each batch, Q_i means product quantity of each batch of supplier i, μ represents the average demand per unit time, and the length of product cycle is $T_m = \frac{Q_i}{\mu}$. So, the expected setup cost per unit time of supplier i is $\frac{\mu}{Q_i}A_i$.

(2) Holding cost. Since Q_i means product quantity of each batch, the average inventory level of supplier i is $h_i\frac{Q_i}{2}$. The expected total cost per unit time of supplier i is given as follows:

$$ECS_i = \frac{\mu}{Q_i}A_i + h_i\frac{Q_i}{2} \tag{10}$$

3.4 Joint Cost of Suppliers and Manufacturer

$$\begin{aligned} EC &\le ECM + \sum_{i=1}^{n}ECS_i \\ &= \sum_{i=1}^{n}(s\frac{\mu}{Q_i} + \frac{1}{2}Q_i h_m + k_i\sigma_{\tilde{X}_i}h_m + (1-\beta)B_i h_m + \sum_{j\neq i}\frac{\mu}{Q_j}B_j h_m + \frac{\mu}{Q_i}\pi(1-\beta)B_i + \cdots \\ &+ \sum_{i=1}^{n}\frac{\mu}{Q_i}A_i + h_i\frac{Q_i}{2} \end{aligned} \tag{11}$$

In the system of a supply chain, different agents have different optimization target, what can we get from a decentralized decision making system is just a local optimal solution. In order to reach the globally optimal solution, we should integrate these benefit antinomical agents, decide Q_i and r_i on the basis of joint optimal inventory cost.

4. Solution Algorithm Analysis

The centralized model is considered i.e., that the goal is to minimize the joint cost of the two-stage supply chain per unit time. As $\mu_{\tilde{X}_i}$ is given by equation $r_i = \mu_{\tilde{X}_i} + k_i\sigma_{\tilde{X}_i}$ for $i = 1,2\cdots n$, to decide the order point for every component's safety factor k_i and the manufacturer's order quantity Q_i of every component. It is nonlinear relationship between the object function EC and decision variable. And the decision variable k_i and Q_i are both subjected to boundary constraints. So the model of this paper is a nonlinear optimization with boundary and nonlinear constraints. Sequential quadratic programming (SQP) is an iterative method for nonlinear optimization. SQP methods are used on problems for which the objective function and the constraints are twice continuously differentiable. SQP methods solve a sequence of optimization subproblems, each of which optimizes a quadratic model of the objective subject to a linearization of the constraints. If the problem is unconstrained, then the method reduces to Newton's method for finding a point where the gradient of the objective vanishes. If the problem has only equality constraints, then the method is equivalent to applying Newton's method to the first-order optimality conditions, or Karush–Kuhn–Tucker conditions, of the problem.

So we use SQP algorithm to solute this model. In order to achieve the global optimal solution as far as possible, few initial points are start to obtain the local optimal solution, and then compare each partial solution, take the minimum as the objective function.

5. Numerical Examples

According to the above optimization model, 8 group data are tested the calculation as shown in table 1-2. First four data are use to observe impact of backorder coefficient and affection degree of economic quantity on process time and standard deviation of market demand per unit time on the optimization result. And the latter foure data are used to observe impact of order cost of component i per unit time and shortage cost per unit time and unit inventory cost of component i on the manufacturer per unit time on the optimization result.

In Table 1, let s_1=8, s_1=9, $\mu = 100$, h_{M1}=8, h_{M1}=10, $\pi = 40$, $\sigma = 4$, A_1=40, A_2=40, $\lambda_1 = 20$, $\lambda_2 = 30$. Backorder rate β and order scale factor θ_1, θ_2 are changed to analysis the effect for supply chain of the backorder order scale. In Table 2, let $\theta_1 = \theta_2 = 0.5$, $\beta = 0.5$ and order cost of component i per unit time s_i, shortage cost per unit time π,unit inventory cost of component i on the manufacturer h_{mi}. Then according to the above optimization model and its solving method, the calculation results were shown in table 2.

From the tables, it can be seen that the minimal joint cost per unit time is equivalent to 364.3. This is because the backorder rate is a hundred percent, so the end customer's requirements can be delay to maximum limit. And the greater order quantity is the more economic scale benefit is. And with demand uncertainty increasing, the inventory cost will increase.

Table 1. Optimization results with data set 1-4

Parameter value	Q1	K1	R1	Q2	K2	R2	Supplier cost	Manufacturer	Joint cost
β=1θ=0.5	333	0.95	17.7	381	0.85	12.03	22.59	341.7	364.3
β=0.5θ=1	210	1	210	264	0.9	16.7	34.19	1075.86	1110.1
β=1θ=0.5 σ=7	402	0.95	19.6	427	0.88	12.94	19.33	426.32	445.65
β=1θ=0.5 σ=9	433	0.98	19.6	457	0.9	13.6	20.35	446.84	467.19

Table 2. Optimization results with data set 5-8

Parameter value	Q_1	K_1	r_1	Q_2	K_2	r_2	Supplier cost	Manufacturer	Joint cost
$s_1 = 10$ $s_2 = 15$	452.7	0.8	19.2	476.9	0.75	12.7	21.5	465.5	487.1
$s_1 = 20$ $s_2 = 25$	509.1	0.6	18.1	533.9	0.55	11.9	23	508.9	531.9
$\pi = 50$	501.1	0.98	22.2	507.3	0.9	14.3	21.8	532	553.8
$h_{m1} = 10$ $h_{m2} = 10$	482.9	0.9	20.9	484.6	0.8	13.2	20.7	591.4	620

Note: θ=θ1=θ2 in table 1-2.

(1) The joint cost of the two-stage supply chain per unit time is the minimal value equivalent to 364.3. And the backorder rate is a hundred percent, so the end customer's requirements can be delay to maximum limit, the inventory quantity and reorder point r is also reduced to maximum limit. And the greater order quantity, the more economic scale benefit is.

(2) When the backorder rate is 50% and there is no order scale effect, the total supply chain cost increase dramatically. When there is economics of scale, the backorder rate is 50%, the joint cost value is 1110.1. While there are no economies of scale, backorder rate is 100%, the total cost value is 364.3. So, with the assumption of

the data, more benefits can be obtained with the backorder equivalent to 50% than the scale parameter of 0.5 in supply chain

(3) When we change manufacturer's order cost s_i, increase s_i from 8 to 10 and 20, and increase s_2 from 9 to 15 and 25, Q_i increases while r_i decreases consequently.

(4) When the components' holding cost and shortage cost per unit time in manufacturer increase, Q_i and r_i will decrease, the cost of manufacturer will reduce, the cost of the supplier will increase, and the total supply chain cost increases consequently.

6. Conclusion

This paper investigates supply chain optimization of assembly manufacturing consisting of multiple independent suppliers and a manufacturer with stochastic demand and variable lead time. From the numerical simulation, it is can be seen backorder rate and order scale effect will impact the total supply chain cost dramatically. And the uncertainty of supply chain is important factor for decision too. The bigger backorder rate and order scale effect, the lower supply chain system cost. And the total supply chain costs and input parameter is non-linear relationship, when a unit cost increases, the total system cost will be increased to different degree.

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References

Bera, U. K., Rong, M., Mahapatra, N. K., & Maiti, M. (2009). A multi-item mixture inventory model involving random lead time and demand with budget constraint and surprise function. *Applied Mathematical Modelling, 33*(12), 4337-4344. http://dx.doi.org/10.1016/j.apm.2009.03.025

Çakanyildirim, M., Bookbinder, J. H., & Gerchak, Y. (2000). Continuous review inventory models where random lead time depends on lot size and reserved capacity. *International Journal of Production Economics, 68*(3), 217-228. http://dx.doi.org/10.1016/S0925-5273(99)00112-7

Ghalebsaz-Jeddi, B., Shultes, B. C., & Haji, R. (2004). A multi-product continuous review inventory system with stochastic demand, backorders, and a budget constraint. *European Journal of Operational Research, 158*(2), 456-469. http://dx.doi.org/10.1016/S0377-2217(03)00363-1

Hayya, J. C., Harrison, T. P., & Chatfield, D. C. (2009). A solution for the intractable inventory model when both demand and lead time are stochastic. *International Journal of Production Economics, 122*(2), 595-605. http://dx.doi.org/10.1016/j.ijpe.2009.06.001

Huang, K., & Küçükyavuz, S. M. (2008). On stochastic lot-sizing problems with random lead times. *Operations Research Letters, 36*(3), 303-308. http://dx.doi.org/10.1016/j.orl.2007.10.009

Maiti, A. K., Maiti, M. K., & Maiti, M. (2009). Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. *Applied Mathematical Modelling, 33*(5), 2433-2443. http://dx.doi.org/10.1016/j.apm.2008.07.024

Mohammadi Bidhandi, H., & Mohd Yusuff, R. (2011). Integrated supply chain planning under uncertainty using an improved stochastic approach. *Applied Mathematical Modelling, 35*(6), 2618-2630. http://dx.doi.org/10.1016/j.apm.2010.11.042

Pang, Z. (2011). Optimal dynamic pricing and inventory control with stock deterioration and partial backordering. *Operations Research Letters, 39*(5), 375-379. http://dx.doi.org/10.1016/j.orl.2011.06.009

Rossi, R., Tarim, S. A., Hnich, B., & Prestwich, S. (2010). Computing the non-stationary replenishment cycle inventory policy under stochastic supplier lead-times. *International Journal of Production Economics, 127*(1), 180-189. http://dx.doi.org/10.1016/j.ijpe.2010.05.011

Zipkin, P. H. (2000). *Foundations of Inventory Management*. McGraw-Hill: New York.