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Modified Breusch-Godfrey Test for Restricted Higher Order Autocorrelation in Dynamic Linear Model – *A Distance Based Approach*

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Abstract

In business, dynamic models often provide valuable insights into the complex interactions between variables over time. But recent research contends that the lagged dependent variable specification is too problematic for use in most situations. More specifically, if residuals autocorrelation is present in a dynamic equation where lagged values of the dependent variable appear as regressors, Ordinary least squares (OLS) estimates are biased and generally inconsistent. For this reason it is important to have available tests against autocorrelation, particularly when it is a dynamic model. The Breusch-Godfrey (BG) test is the most appropriate test in the presence of stochastic regressors such as lagged values of the dependent variable for higher order autocorrelation, which is asymptotically equivalent to the Durbin-Watson h test for first order autocorrelation. But Durbin h test is not applicable for second or higher order autocorrelation. Moreover these existing tests are not suitable for one-sided higher order autoregressive schemes. Whenever the sign of the parameters are known of an econometric model, usual two-sided tests are no longer valid. In this situation, we propose a distance-based one-sided Lagrange Multiplier (DLM) test, a likelihood based test, to test one-sided alternative. Monte Carlo simulations are conducted to compare power properties of the proposed DLM test with the BG test. It is found that the DLM test shows substantially improved power than two-sided counterparts for most of the cases considered.

Keywords: distance-based LM test, Breusch-Godfrey test, restricted alternatives, Monte Carlo simulation, power

1. Introduction

Dynamic models are often needed in economics, business and other context. They can capture remarkably subtle feedback effects that are easily missed by static models. As a lagged dependent in ordinary least squares (OLS) regression is often used as a means of capturing dynamic effects in supporting process and as a method for ridding the model of autocorrelation. But the inference in the presence of lagged dependent variables is a long-standing problem in econometrics as well as in statistics. More specifically, if residuals autocorrelation is present, the lagged dependent variable causes the coefficients for explanatory variables to be biased and generally inconsistent. For this reason it is important to have available tests against autocorrelation, particularly when it is a dynamic model which is proposed to be estimated by OLS.

Because of the non-experimental nature of almost all economic data, economic models usually involve a large number of inferences. In the linear regression model, if the errors do not follow the assumptions of the classical linear regression model, then test based on this assumption are not appropriate and give misleading result. Different types of violations may be caused due to multicollinearity, heteroscedasticity, autocorrelation, etc. For example, If the disturbances of a linear model are autocorrelated, OLS estimates of the coefficient parameters are inefficient but unbiased.. Therefore, it is highly desirable to be able to perform diagnostic tests of the regression disturbances in the presence of autocorrelation and /or heteroscedasticity.

There is an extensive literature on testing of autocorrelation coefficients in the linear regression model, see for example Durbin and Watson (1950), Durbin (1970), Box and Pierce BP Q (1970), Wallis (1972), Ljung and

Box (1978), Breush and Godfrey BG (1978) etc. But all these standard tests are not valid when some of the regressors are lagged values of the dependent variable. Only the Breusch-Godfrey (BG) test is valid in the presence of stochastic regressors such as lagged values of the dependent variable for higher order autocorrelation. For first order autocorrelation, the test is asymptotically equivalent to the Durbin-Watson h statistic, which may be considered a special case of the Breusch-Godfrey test statistic. However, Durbin-Watson h test is not applicable for testing second or higher order autocorrelation in dynamic models. The BG test computes Lagrange multiplier test for nonindependence in the error distribution. For a specified number of lags p, the test's null of independent errors has alternatives of either $AR(p)$ or $MA(p)$.

But these tests are two-sided in nature and not suitable for testing one-sided alternatives. To overcome this situation one-sided tests are proposed by Majumder and King (1999); Basak, Rois and Majumder (2005), and Rois, Basak and Majumder (2008) using distance-based approach. Monte Carlo experiments reported by them show that the one-sided tests for autocorrelation rejects the null hypothesis more accurately than conventional two-sided counterparts. Unfortunately, all these one-sided tests are tested autocorrelation only in the linear regression model but not in the dynamic models.

Many econometric models provide us with prior information about some or all of their unknown parameters. Such information usually comes from economic theory, from previous empirical studies or from functional considerations such as variance always being nonnegative. For this reason, many econometric testing problems are potentially either strictly one-sided or partially one-sided. For example, the own price and income elasticity coefficients in demand analysis, and variances of error components in panel data model with individual and time error components can be expected to be positive, (Majumder, 1999). In practice, often the amount of data available to conduct a test is limited. In this situation, two-sided tests are no longer valid. So the one-sided and partially one-sided tests may be able to improve the quality of inferences. In such situation, the likelihood based Wald test can be improved by using distance-based approach for testing restricted higher order autocorrelation. So we expect better power performance of our one-sided likelihood-based Wald test in dynamic regression models.

Our particular focus in this paper is on the use of likelihood-based Wald test with distance-based approach (Majumder & King, 1999) for testing restricted higher order autocorrelation in the dynamic linear regression model. Since the most suitable test for testing higher order autocorrelation in dynamic models is the BG test. Thus to propose a new test in the context of dynamic models, we should consider the performance of the aforesaid standard test along with our newly developed test. Finally, make a comparative study of one-sided likelihood-based LM test (DLM) with two-sided BG test for testing restricted higher order autocorrelation in the context of dynamic linear regression model.

The organization of the paper is as follows. In section 2, we discuss the model and hypothesis uses to develop the test. Our proposed distance-base one-sided LM (DLM) test discuss in section 3. We also express some existing two-sided BG test in section 4. In section 5 we introduce the Monte Carlo simulation. A comparison is made between the powers of DLM test with two-sided BG test in section 6. Finally, section 7 contains concluding remarks.

2. Model and Hypothesis

Consider the following dynamic linear regression model,

$$y = Y_{-1}\alpha + X\beta + u, \quad u \sim N\left(0, \sigma^2\Omega(\theta)\right), \tag{2.1}$$

where, y and u are $(n-p)\times 1$ vectors, Y_{-1} and X are $(n-p)\times p$ and $(n-p)\times k$ matrices, α and β are $p\times 1$ and $k\times 1$ vector of parameters, respectively. $\Omega(\theta)$ is a positive definite matrix. Here we assume that the disturbance term follows a stationary $AR(p)$ process,

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \cdots + \rho_p u_{t-p} + \varepsilon_t, \text{ where, } \varepsilon_t \sim N(0,1). \tag{2.2}$$

We are interested in testing,

$$\text{H1} \quad H_0 : \rho = 0, \text{ against } H_1 : \rho > 0, \tag{2.3}$$

$$\text{H2} \quad H_0 : \rho = 0, \text{ against } H_1 : \rho < 0, \tag{2.4}$$

where, $\rho = (\rho_1, \rho_2, ..., \rho_p)'$ is a $(p\times 1)$ matrix. When errors are positively correlated in one quarter or negatively correlated in another, then the hypothesis may take the above forms. The alternative hypothesis is called strictly one-side, in such situation usual two-sided LM test is not totally suitable.

In this paper we developed strictly one-sided DLM test by considering the linear regression model (2.1) with the disturbance term u follows $AR(p)$ process (2.2) for the alternative hypothesis H1.

3. Distance-Based One-Sided LM (DLM) test

3.1 Distance-Based Approach

Distance-based approach suggests that we have to determine whether the estimated parameters under test likely to be closer to null hypothesis or to alternative hypothesis. Majumder's (1999) approach is outlined below for general testing problem:

Suppose we are interested in testing a hypothesis of a parametric model in which the parameter of interest, θ, is restricted under the alternative hypothesis. More specifically, we wish to test

$$H_0 : \theta = 0 \text{, versus, } H_a : \theta \in B \text{,} \tag{3.1.1}$$

based on the $n \times 1$ random vector y whose distribution has probability density function $f(y,\ \theta)$ where $\theta \in R^p$ is a subvector of an unknown parameter $\Theta \in R^s$ and B is a subset of R^p. Let $\hat{\theta}$ be a suitable estimate of θ such that $\hat{\theta}$ is asymptotically distributed as normal with variance-covariance matrix $cI^{-1}(\theta)$ where c is a constant and $I(\theta)$ is the information matrix. Following Shapiro (1988), Kodde and Palm's (1986) Majumder (1999) suggest that we should determine the closest point in the maintained hypothesis from the unconstrained point. This closest point is the solution of the following distance function or optimal function in the metric $cI^{-1}(\theta)$ of the parameter vector $\hat{\theta}$,

$$\left\|\breve{\theta} - \hat{\theta}\right\|^2 = \left(\breve{\theta} - \hat{\theta}\right)' I(\theta)\left(\breve{\theta} - \hat{\theta}\right), \text{ subject to } \breve{\theta} \in B. \tag{3.1.2}$$

The closest point or optimized $\breve{\theta}$ can be used in any appropriate two-sided tests to obtain the corresponding distance-based one-sided and partially one-sided tests. The asymptotic null hypothesis distribution generally follows a mixture of the corresponding two-sided distributions (Majumder, 1999).

3.2 Distance-Based One-Sided LM (DLM) Test

In the distance-based LM test, the optimum values of $\hat{\theta}$, the score vector $s(\hat{\theta})$ and the information matrix $I(\hat{\theta}) = E\left[\dfrac{d^2 \log L}{d\theta^2}\right]_{\theta=\hat{\theta}}$ are estimated according to the general formulation of distance-based approach, subject to the restrictions, H1 discussed in section 2. Thus the one-sided LM (DLM) statistic define as

$$DLM = s\left(\breve{\theta}\right)' I\left(\breve{\theta}\right)^{-1} s\left(\breve{\theta}\right), \tag{3.2.1}$$

where, $s\left(\breve{\theta}\right)$ and $I\left(\breve{\theta}\right)$ is the optimized value subject to the restriction, H1 (Basak, Rois & Majumder, 2005; Rois, 2005).

Under the null hypothesis the distribution of the statistic (3.2.1) follows asymptotically weighted mixture of chi-square distribution with p degrees of freedom (Kodde & Palm, 1986; Shapiro, 1988; Majumder, 1999).

4. Breusch-Godfrey (BG) Test

The Breusch-Godfrey test is the likelihood-based two-sided LM type test, which is the most appropriate test for detecting autocorrelation in dynamic models. The BG test developed under the null-hypothesis,

$$H_0 : \rho = 0 \text{,} \tag{4.1}$$

against, the alternative,

$$H_a : \rho \neq 0 \text{.} \tag{4.2}$$

Under the null hypothesis, the test statistic $(n-p)R^2$ is asymptotically distributed as chi-square with one degrees of freedom (Breusch, 1978; Godfrey, 1978; Gujarati, 2003; Johnston, 1997). R^2 is obtained from the regression,

$$\widehat{\mu_t} = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_k X_k + \hat{\rho}_1 \hat{\mu}_{t-1} + \hat{\rho}_2 \hat{\mu}_{t-2} + \cdots + \hat{\rho}_p \hat{\mu}_{t-p} + \varepsilon_t, \tag{4.3}$$

where, $\hat{u}_t$ is the Ordinary Least Squares (OLS) residual obtained from (2.1). The two-sided BG test is not totally suitable if the alternative hypothesis take one of the forms defined in Section 2.

5. Monte Carlo Simulation

Monte Carlo simulations are carried out to compare the powers of the usual two-sided BG test with one sided DLM test for detecting higher order autocorrelation of a dynamic regression model of the form (2.1). Here, we use real and artificially generated explanatory variables (X). In order to carry out Monte Carlo simulation we generate the following second order autocorrelated disturbance terms respectively,

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t\,, \tag{5.1}$$

where, $\varepsilon_t \sim N(0,1)$, and consequently we generate the model (2.1). We perform 20,000 replications to calculate (size corrected) simulated powers of the new and existing tests, when the error term follows second order autoregressive scheme (5.1).

5.1 Experimental Design

In order to compare the power properties of proposed DLM test with BG tests we use two different types of design matrices-real and artificially generated data. We use the following data sets:

D1: The per-capita income in constant dollars, average price for the single-family rate tariff electricity consumption in Summer, electricity consumption in winter of the San Diego Gas and Electric Company for the period 1972-1993, (Ramanathan, 1995, 502-503).

D2: Two regressor of stationary autoregressive time series data generated as, $X_{it} = 0.5X_{it-1} + \eta_{it}$, where, $\eta_{it} \sim NID(0,1)$, $t = -99, \ldots, 0, 1, \ldots, n$ and $X_i - 100 \sim N(0,1.333)$, (King, 1981).

D3: Age of the individuals, log of hourly wage in dollars, square of years of potential experience, weight of the individuals for the period 1959-1992, (Johnston, 1997).

D4: Quarterly Australian consumer price index covering the period 1959-1979, lagged one quarter of the variable.

For testing the maintained hypothesis, all the data sets D1, D2, D3 and D4 are employed. The following X design matrices are used in the experiment.

X1: A constant dummy, a lagged dependent variable and first one artificial variable of D2,

X2: A constant dummy, a lagged dependent variable and two artificial variables of D2,

X3: A constant dummy, a lagged dependent variable and first three real variables of D1,

X4: A constant dummy, a lagged dependent variable and all (four) real variables of D1,

X5: A constant dummy, a lagged dependent variable and first two real variables of D3,

X6: A constant dummy, a lagged dependent variable and last two real variables of D3,

X7: A constant dummy, a lagged dependent variable and all (two) real variables of D4.

We can perform our experiment for different values of the parameters ρ_i, $i = 1,2,\ldots,p$ (0 to 0.9). For the second-order autoregressive scheme (5.1) we estimate simulated powers for testing one-sided hypothesis (2.3) and the above X1, X2, , X4, X5, X6 and X7 matrices for $n = 50$. Here we use selected values of ρ_1 and $\rho_2 = 0(0.2)0.8$.

6. Results

This section compares the powers of the existing two-sided BG test and the newly proposed one-sided DLM test for testing H1, in the context of dynamic linear regression model (2.1). The estimated simulated powers of these tests are presented in Tables 1-2 for the design matrices defined in section 5.1. These tables represent the simulated powers of DLM test along with two-sided BG test for second order autoregressive scheme (5.1), when the alternative hypothesis is of the form H_1. In both tables, the estimated sizes of the two tests is 0.05 against AR(2) disturbances when asymptotic critical values at five percent nominal level are used. Thus all the tests have size-corrected power.

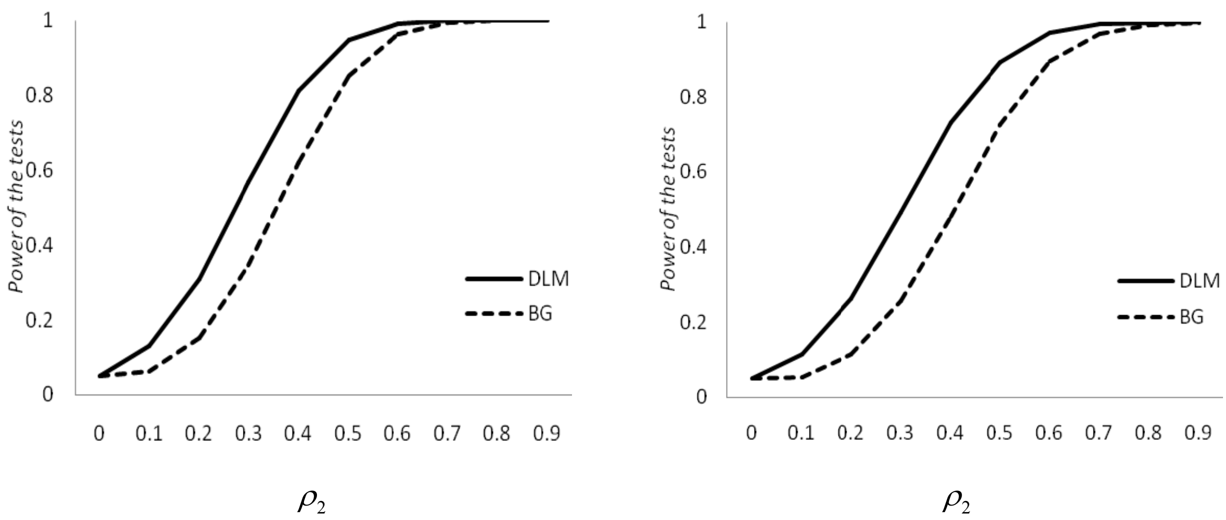
Table 1 reveals the estimated powers for artificially generated data of one-sided LM (DLM) and two-sided BG tests for $n = 50$ and design matrices $X1$ and $X2$, which contains one and two explanatory independent variable(s) excluding the lagged dependent variable. The performance of DLM and BG tests using real data sets with multicollinearity are also illustrated by design matrices $X3$ and $X4$ in table 1. We use four regressors in the design matrix $X3$, that is $k = 4$, which are a lagged dependent variable, per capita income, average price of the single-family rate tariff, electricity consumption in summer of the San Diego Gas and Electric Company for the period 1972-1993, (Johnston, 1997). In the design matrix $X4$ five regressors are used, that is $k = 5$, which are same as for the third set including the regressor electricity consumption in winter. We observe that the powers of DLM test are significantly higher than two-sided test near null value. For example, the power of the DLM and BG tests are 0.309 and 0.153, respectively, for $\rho_1 = 0.0, \rho_2 = 0.2$, $k = 2$ and $n = 50$.

Table 1. The powers of one-sided LM (DLM) and two-sided LM (BG) tests of hypothesis H1, for second-order autoregressive scheme, using stationary time series data and quarterly data on electricity demand for 50 observations

		$k = 2$ X1		$k = 3$ X2 $(r = 0.421)$		$k = 4$ X3 $(R^2 = 0.341)^*$		$k = 5$ X4 $(R^2 = 0.363)^*$	
ρ_1	ρ_2	DLM	BG	DLM	BG	DLM	BG	DLM	BG
0	0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	0.2	0.309	0.153	0.299	0.134	0.281	0.12	0.264	0.113
	0.4	0.812	0.62	0.795	0.574	0.77	0.511	0.731	0.48
	0.6	0.99	0.964	0.988	0.951	0.98	0.923	0.97	0.896
	0.8	1	0.999	1	0.999	0.999	0.996	0.998	0.991
0.2	0	0.277	0.135	0.261	0.112	0.248	0.086	0.25	0.093
	0.2	0.577	0.314	0.548	0.262	0.484	0.189	0.481	0.201
	0.4	0.907	0.754	0.89	0.695	0.826	0.563	0.813	0.56
	0.6	0.994	0.974	0.992	0.961	0.975	0.881	0.968	0.866
	0.8	1	0.998	1	0.996	0.997	0.983	0.996	0.975
0.4	0	0.773	0.564	0.745	0.499	0.72	0.453	0.727	0.473
	0.2	0.919	0.79	0.905	0.732	0.849	0.624	0.851	0.641
	0.4	0.988	0.959	0.984	0.942	0.945	0.829	0.944	0.834
	0.6	0.996	0.985	0.995	0.977	0.983	0.921	0.981	0.922
	0.8	1	1	1	1	1	1	1	1
0.6	0	0.983	0.937	0.977	0.908	0.967	0.885	0.97	0.893
	0.2	0.996	0.985	0.994	0.977	0.982	0.928	0.982	0.935
	0.4	0.997	0.991	0.996	0.986	0.986	0.945	0.987	0.949
	0.6	1	1	1	1	1	1	1	1
0.8	0	1	0.998	0.999	0.996	0.998	0.991	0.998	0.991
	0.2	1	0.998	0.999	0.997	0.997	0.984	0.997	0.986
	0.1	1	0.999	1	0.999	0.999	0.994	0.999	0.995
	0.4	1	1	1	1	1	1	1	1

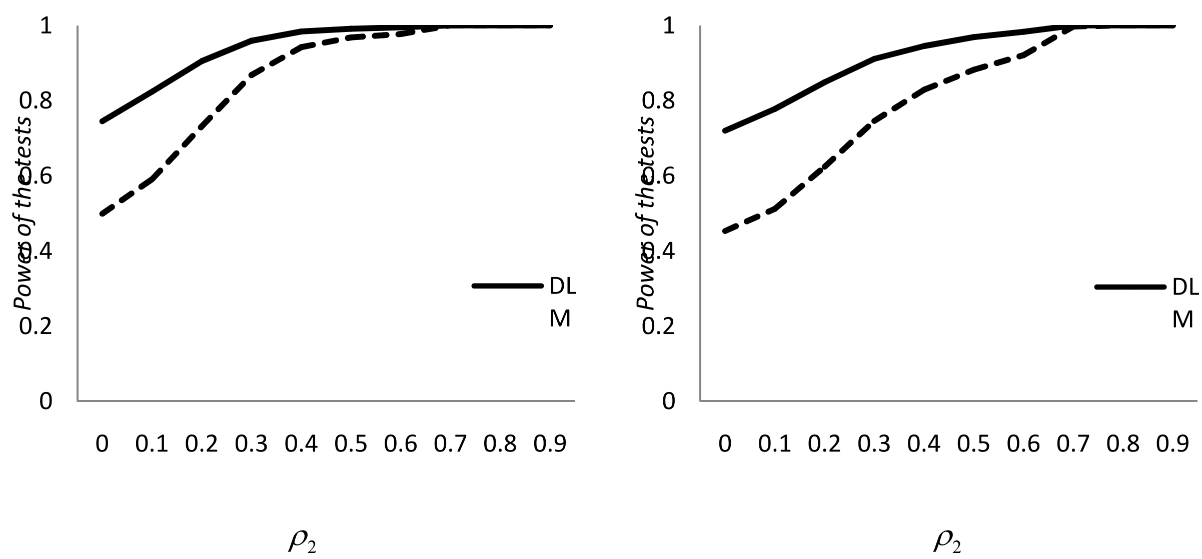
* Multicollinearity exists

Typically, powers of all tests are increases as ρ increases. For example, the power of the DLM test and BG test are 0.905 and 0.372, respectively, for $\rho_1 = 0.4, \rho_2 = 0.2$, $k = 3$ and $n = 50$ in table 1. In order to explore the performance of DLM test in the presence of multicollinearity, we consider X3 and X4 matrices for 50 observations. We observe from the Table 1 that the simulated powers of the DLM test are always higher than the BG test in all manifested cases. As an instance, the simulated power of the DLM test and BG test are 0.851 and 0.641, respectively, for $\rho_1 = 0.4, \rho_2 = 0.2$, $k = 5$ and $n = 50$.



(a) (b)

Figure 1. Power curves of DLM and BG tests of hypothesis H1 to detect second order autocorrelation, when $K = 2$ for artificially generated data in (a) and $K = 5$ for real data with multicollinearity in (b) with fixed $\rho_1 = 0.0$



(a) (b)

Figure 2. Power curves of DLM and BG tests of hypothesis H1 to detect second order autocorrelation, when $K = 3$ for artificially generated data in (a) and $K = 4$ for real data with multicollinearity in (b) with fixed $\rho_1 = 0.4$

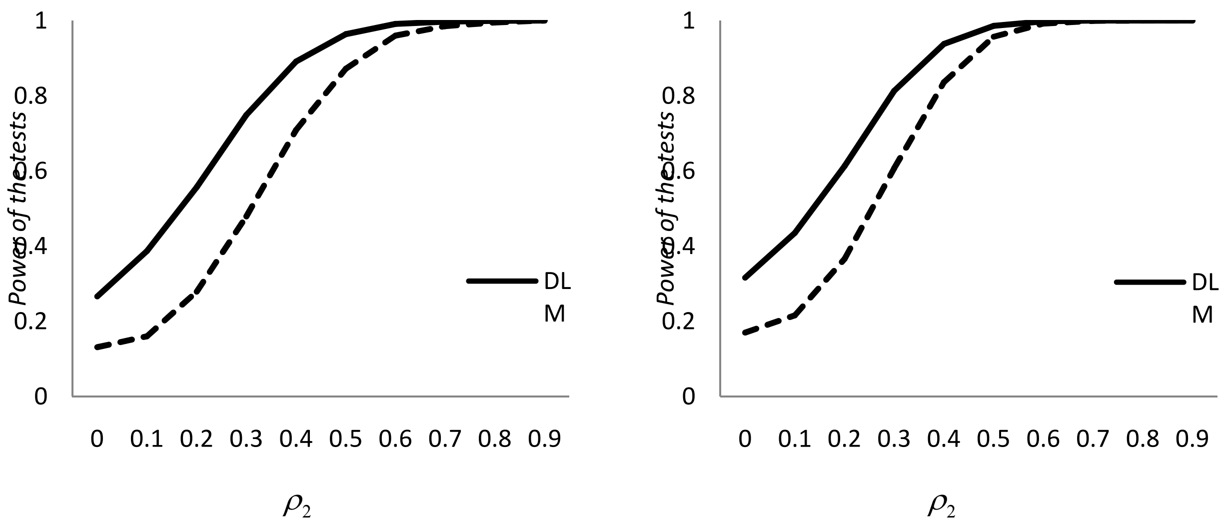
Power curves are very explicable practices to illustrate the performance of the proposed DLM test. In Figures 6.1 and 6.2 we exhibit two sets of power curves, set (a) for artificially generated data and set (b) for the real data with multicollinearity problem. Figure 1 explores this comparison near the null values for artificially generated design matrix X1 and the real design matrix X4, which also shows a strong performance of DLM test over the usual two-sided BG test with regressor(s) one and four excluding the lagged dependent variable in (a) and (b), respectively. Figure 2 also exhibit the influential performance of the newly developed DLM test over the conventional two-sided BG test.

Table 2. The powers of one-sided LM (DLM) and two-sided LM (BG) tests of hypothesis $H1$, for second-order autoregressive scheme in dynamic regression model, using correlated, uncorrelated, lagged data

		$k = 3$					
		X5 ($r = 0.9208$)		X6 ($r = 0.002$)		X7 ($r = 0.999$)	
ρ_1	ρ_2	DLM	BG	DLM	BG	DLM	BG
0	0	0.05	0.05	0.05	0.05	0.05	0.05
	0.2	0.29	0.14	0.299	0.157	0.314	0.169
	0.4	0.796	0.586	0.803	0.622	0.874	0.737
	0.6	0.987	0.954	0.988	0.959	0.997	0.991
	0.8	1	0.999	1	0.999	1	1
0.2	0	0.266	0.131	0.286	0.147	0.316	0.17
	0.2	0.557	0.279	0.605	0.357	0.613	0.366
	0.4	0.891	0.709	0.914	0.78	0.938	0.836
	0.6	0.991	0.96	0.993	0.975	0.998	0.992
	0.8	0.999	0.995	1	0.998	1	1
0.4	0	0.768	0.548	0.801	0.617	0.866	0.727
	0.2	0.91	0.756	0.934	0.828	0.956	0.878
	0.4	0.982	0.94	0.989	0.965	0.996	0.982
	0.6	0.993	0.974	0.997	0.988	0.999	0.997
	0.8	1	1	1	1	1	1
0.6	0	0.983	0.931	0.987	0.958	0.996	0.984
	0.2	0.994	0.978	0.997	0.99	0.999	0.996
	0.3	0.996	0.987	0.998	0.993	1	0.998
	0.4	0.996	0.983	0.997	0.992	1	0.998
0.8	0	0.999	0.997	1	0.999	1	1
	0.2	0.999	0.996	1	0.999	1	1
	0.4	1	1	1	1	1	1

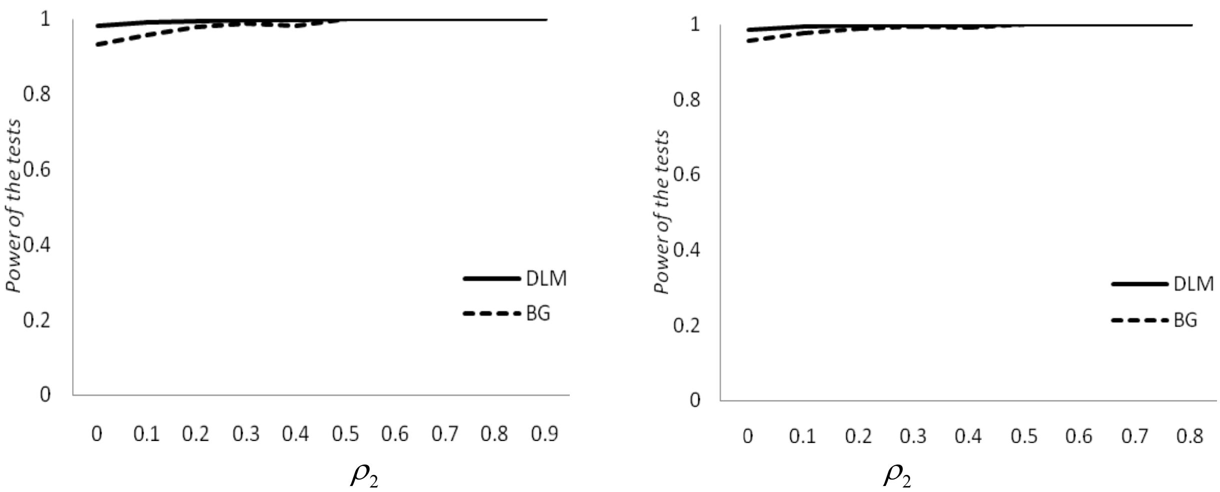
In order to investigate the power performances of DLM and BG tests in the case of correlated, uncorrelated and lagged data we use three sets of X-matrices, X5, X6, X7 each contains three explanatory variables including the lagged dependent variable for testing the hypothesis $H1$. Table 2 exhibits the simulated powers of DLM and BG tests. Hereafter, we explore that the simulated powers of DLM test are always superior to the usual two-sided test. For example, in the case of highly correlated data, the simulated powers of the DLM test and BG test are 0.290 and 0.140, respectively, for $\rho_1 = 0.0, \rho_2 = 0.2$, $k = 3$ and $n = 50$ (Table 2). In the case of uncorrelated and lagged data we also observe similar results. There is a clear tendency for DLM test to be more powerful for larger values of ρ. For example, the simulated powers of the DLM and BG tests are 1 and 0.999, respectively, for $\rho_1 = 0.8, \rho_2 = 0.2$, $K = 3$ and $n = 50$.

Figure 2 and 3 present similar dominance nature of DLM test over two-sided BG test near the null value. The most suitable BG test for detecting higher order autocorrelation in the context of dynamic linear model is not performed better than the one-sided DLM test. The powers of the DLM test leading in all situations (correlated, uncorrelated and lagged data) over the BG test, specially near the null value. Figure 4 presents the simulated power curves for the larger value of ρ. However, all the powers are very closer to 1 but the DLM performs significantly better than the BG test. For example, the simulated powers of the DLM and BG tests are 0.997 and 0.990, respectively, for $\rho_1 = 0.6, \rho_2 = 0.2$, $K = 3$ and $n = 50$.



(a) (b)

Figure 3. Power curves of DLM and BG tests of hypothesis H1 to detect second order autocorrelation, when $K = 3$ for correlated data in (a) and for lagged data in (b) with fixed $\rho_1 = 0.2$.



(a) (b)

Figure 4. Power curves of DLM and BG tests of hypothesis H1 to detect second order autocorrelation, $K = 3$ for correlated data in (a) and for uncorrelated data in (b) with fixed $\rho_1 = 0.6$

In the context of dynamic regression model, the DLM test performs better than BG test. Although BG test is the most popular test for testing higher order autocorrelation in dynamic model but the performance of this test is not satisfactory as one-sided DLM test. Figure 3 and 4 exhibit the performance of this two tests for different data types such as correlated, uncorrelated and lagged data with larger values of ρ. The power of one-sided DLM test is perform better in all cases specially around the larger value of ρ it gives power one persistently.

We observe from Monte Carlo simulation study that in all cases, i.e. correlated, uncorrelated, lagged or even when multicollinearity exists, our proposed DLM test gives higher power than two-sided BG test. All the figures and tables represent that the simulated power of distance-based one-sided LM (DLM) test is always superior to the usual two-sided test. In summary, if we use the distance-based one-sided test rather than the traditional counterpart does result typically more accurate in terms of power.

7. Conclusions

This paper develops distance-based one-sided DLM test for testing higher order autocorrelations of the disturbances in a dynamic linear regression model for the restricted one-sided alternatives. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the tests. Larger power is considered as a criterion of a good test. To illustrate the findings of the paper, an artificially generated data and three real data sets with different fact that is correlated, uncorrelated, lagged and even when multicollinearity exists are studied. Monte Carlo results indicate that the newly proposed distance-based one-sided LM (DLM) test performs better than two-sided BG test in all cases discussed in this paper. These improvements are very clear-cut in all the cases. All this evidence and the findings do suggest that there can be clear advantages in using one-sided DLM test in place of two-sided BG test for testing higher order autocorrelation in dynamic linear regression model.

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