



Optimal Investment Decision on Open-end Funds

Wei Cheng

Institute of Business Administration, Shenyang University, Shenyang 110044, China

Postdoctoral Working Station, Dong Bei University of Finance and Economics, Dalian 116023, China

E-mail: chengw523@163.com

Guifang Ren

Institute of Business Administration, Shenyang University, Shenyang 110044, China

Hailong Wang

Institute of Business Administration, Shenyang University, Shenyang 110044, China

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Abstract

The study of open-end fund is conducted in this paper in terms of the theory of Random process and the theory of Sequential Decision, which based on the benefit of investors and the cost of transaction (commission occurred in the transaction). In addition the thesis introduces the method of factor of random discounting, by which investors can choose the optimal way of investment, which is calculated in an analogue case.

Keywords: Umbrella funds, Open-end funds, Sequential decision, Ergodicity

1. Introduction

In 2001 Open-end fund was introduced into China. The fund is characterized by refund, less risk and more income. But the problem is that in China the fund is only used in few financial products and manipulated in a similar way. The way of investment is all alike. The introduction of the "Securities Investment Fund Law" and QFII (qualified foreign institutional investors scheme) will undoubtedly promote the function of the orderly market, and enforce regulations and law, which provides a good opportunity for fund development. The new variety of fund will emerge continuously; the type of product design will be more varied. The fund companies such as Xiang Cai Hefeng, China Merchants, Jiashi, Cathay Pacific and a number of fund management companies have designed Umbrella Fund that is also adapted to the demand of the market. This problem has been studied previously (Yao Er-qiang, 1998, pp.455). Umbrella Fund refers to a mother in a number of sub-funds established under the Fund. They are of independent management. In the conversion between the funds the cost is lower or zero. Of Umbrella funds in Europe, the United States and Hong Kong, the proportion is as high as 60%. It is expected to reach 70% -80% in China. It can meet the demand of different investors in the investing objective, direction and areas. The products under the same brand can appeal to investors with different preferences, enjoy the advantages of the scale of economy and lower the cost of administration. Also will some new types of open-end fund such as (SR recorded as LOF), index funds, secured capital funds, futures funds and other fund come out. Among them, investors have to be faced with the dilemma of choosing to which one is the most profitable. That makes up one of the subjects worth studying.

First, investors can choose fund in terms of its performance. Because Securities Investment Fund has just been introduced into China, the approach of valuation adopted in the west is not suitable in China. And the valuation organizations in China are not yet recognized as the just authorities and performance of fund is so fluctuated that speculation is wild. This problem has been studied previously (Wu Qi-fang, 2003, pp.33). Second, the anticipated profit that fund management company promise is not deducted from the proceeds related to the cost which occurs in liquidity of fund which reported by Liu Hailong (2003, pp.217) and Edelen, R. (1999, pp.439-466). Because investors would purchase and redempt fund, conversion and pay higher transaction costs. All fund management companies are competing for rates and take rates progressive decline, back-end fees, the redemption rate or conversion rate of 0, and other measures Managerial experience from Europe and the United States, high-yield can bring in high costs and high costs come from the investors transaction costs. So from a long-term development, competitive rates will be maintained at a certain level. In addition, China's foreign fund compared with the size of a far cry from, but it is closer to the level of rates, leading fund management companies operating difficulties. Therefore, the choice of investment fund investors must consider transaction costs.

In this paper, we make a research on the medium and small investors, because the transactions rates are relatively fixed for

them, and transaction costs directly affect the profit number. In the course of the study, with discounts to the sequential theory-based decision-making process, and in accordance with open-end fund investment existence of two rates are different from the past, the discount factor random formula, the establishment of a model, through calculating, the optimal draw more accurate decision-making.

2. Sequential basic tenets of the decision-making process

Markov process with the assumption that a state of N, from the state *i* to the state *j* has shifted to step k ways, each way recorded as 1,2 k K, the probability of occurrence for the k transfer p_{ij}^k , for the corresponding profit r_{ij}^k ,

$v_i(n)$ said from the beginning *i* of the transfer *n* of maximum profit, and the following recursive relationship

between $v_i(n+1)$ and $v_j(n)$

$$v_i(n+1) = \max_k \sum_{j=1}^N p_{ij}^k [r_{ij}^k + v_j(n)] \quad i = 1,2,\dots,N \tag{1}$$

If

$$q_i^k = \sum_{j=1}^N p_{ij}^k r_{ij}^k \tag{2}$$

Then (1) can be written as

$$v_i(n+1) = \max_k [q_i^k + \sum_{j=1}^N p_{ij}^k v_j(n)] \tag{3}$$

Above q_i^k ground state *i* with the first k form of a transfer of the average profit (profit expectations).

Now considering the process of transfer of part of the loss, the introduction of random discount factor α_i^k, λ_j^k . From

(3) losses can be divided into two parts: one part state *i* with the first k form of a transfer of the average to state j,

Have to pay the cost ratio is the ratio recorded as losses λ_j^k , then (3) can be changed into

$$v_i(n+1) = \max_k [(1 - \alpha_i^k)q_i^k + \sum_{j=1}^N (1 - \lambda_j^k)p_{ij}^k v_j(n)] \tag{3'}$$

$v_i(n+1)$ means started from the state *i* that the transfer of the n +1 times. α_i^k means from the state *i* of the k

form of a transfer to loss ratio of *j*. λ_j^k is the N-step transfer of the remaining loss ratio.

We know alignment of the Markov chain has the following theorem. (G. B. Gnedenko, 1956 and Wang Zi-kun, 1976).

Theorem 1: With a S+1 finite state of the homogeneous Markov chain $\{\xi_n\}$, if the existence of positive integers n_0 ,

made of all $i, j = 0,1,2,\dots,s$ exist $p_{ij}(n_0) > 0$, this Markov chain is through the state, that is, when the number

$n \rightarrow \infty$ transfer probability exists Limit

$$\lim_{n \rightarrow \infty} p_{ij}(n) = \pi_j \tag{4}$$

The beginning has nothing to do with the state of *i*, π_j meet equations

$$\pi_j = \sum_{i=0}^s \pi_i p_{ij} \quad (j = 0,1,2,\dots,s)$$

And

$$\pi_j > 0 \quad \sum_{j=0}^s \pi_j = 1 \tag{5}$$

is the only solution.

Now considering all the states through the N -state Markov process. Assuming that the transfer process can be long-term, and considering the process depends on the total expected profit frequency of the transfer system, the total number of transfers n profit with the increase of constantly changing. Because the process is after all the states, there is a probability π_i limit state has nothing to do with the initial state. Basing (3) where each transfer with a way to use the k , can be removed from the right side of great symbolic value \max_k , Anything dealing with the two sides of the Z -transform from the literature available:

$$v_i(n) = ng_i^k + v_i \tag{6}$$

This is that abscissa is n , Y-axis equation is $v_i(n)$ for coordinate surface point $(n, v_i(n))$ changing an artwork when

$n \rightarrow \infty$, which is the approximation line. v_i is the approximation line nodal increment on the axis of bank, at present

$$g_i^k = \sum_{j=1}^N \pi_j q_j^k \tag{7}$$

g_i^k : from state i begins to change n times, and average changing under k kinds of metastasis way every time in mansion earns profit when pretty big. q_j^k : Be that many times we use k kinds of way to begin to change from state j , the transferred average earns profit every time. When thinking that process is chain only when one cycles and every state gets through, so it is having same earning profit g after all beginning state changes many times suitable, see R. A. Howard in 1963.

$$g = \sum_{j=1}^N \pi_j q_j \tag{8}$$

Now (6) becomes

$$v_i(n) = ng + v_i \quad i = 1, 2, \dots, N \tag{9}$$

Put (9) style into (3) style is got on the right,

$$v_i(n+1) = \max_k [q_i^k + ng + \sum_{j=1}^N p_{ij}^k v_j]$$

Be that (10) style is write be accomplished

$$v_i(n+1) = \max_k w_i^k + ng \tag{10}$$

Can use as the checkout number that every state asks to maximization, n in style (4) and style (9) exchange into $n+1$, we get

$$v_i(n+1) = (n+1)g + v_i \quad i = 1, 2, \dots, N$$

Which right holds substitution arriving at

$$v_i(n+1) = [q_i^k + \sum_{j=1}^N p_{ij}^k v_j(n) + ng]$$

Get a set of equations

$$g + v_i = q_i^k + \sum_{j=1}^N p_{ij}^k v_j \quad i = 1, 2, \dots, N \tag{11}$$

Considering the loss part when the process is changing, introducing random discount factor, (11) style become:

$$g + v_i = (1 - \alpha_i^k) q_i^k + \sum_{j=1}^N (1 - \lambda_j^k) p_{ij}^k v_j \quad i = 1, 2, \dots, N \tag{11'}$$

3. Order passes through the decision process algorithm step

Order birthplace decision process algorithm as follows:

First step:

Order $t = 1$, $v_j(0) = 0$ Calculate the checkout number $w_i^k = (1 - \alpha_i^k)$, $q_i^k = \sum_{j=1}^N (1 - \alpha_i^k) p_{ij}^k r_{ij}^k$. when

$$w_i^l = \max_{1 \leq k \leq K} w_i^k, \text{ order } l = d_i^1, i = 1, 2, \dots, N.$$

Be once optimized decision-making $D^1 = (d_1^1, d_2^1, \dots, d_N^1)^T$. calculate $q_i^{d_i^1} = \sum_{j=1}^N (1 - \alpha_i^{d_i^1}) p_{ij}^{d_i^1} r_{ij}^{d_i^1}$ (12)

Solve a set of equations,

$$g + v_i = (1 - \alpha_i^{d_i^1}) q_i^{d_i^1} + \sum_{j=1}^N (1 - \lambda_j^{d_i^1}) p_{ij}^{d_i^1} v_j, i = 1, 2, \dots, N.$$

Second step: order t=2

Calculate the checkout number $w_i^k = (1 - \alpha_i^k) q_i^k + \sum_{j=1}^N (1 - \lambda_j^k) p_{ij}^k v_j$, when $w_i^l = \max_{1 \leq k \leq K} w_i^k$, order

$$d_i^2 = l, i = 1, 2, \dots, N.$$

Have to optimize decision-making the 2nd time $D^2 = (d_1^2, d_2^2, \dots, d_N^2)^T$.

As above recursion until the mansion t step, calculate

$$q_i^{d_i^t} = \sum_{j=1}^N p_{ij}^{d_i^t} r_{ij}^{d_i^t} \tag{13}$$

Solve a set of equations

$$g + v_i = (1 - \alpha_i^{d_i^t}) q_i^{d_i^t} + \sum_{j=1}^N (1 - \lambda_j^{d_i^t}) p_{ij}^{d_i^t} v_j \quad i = 1, 2, \dots, N \tag{14}$$

Gain that g, v_1, v_2, \dots, v_N .

Calculate $\lambda = |D^t - D^{t-1}|$, when $\lambda \leq \epsilon$, stop order birthplace iteration, g is to be that the maximum earns profit. Otherwise, order $t = t + 1$, shift to the mansion t step.

In the arbitrarily ϵ is small positive number which is being appointed as to shifting to an earlier date in algorithm,

$|\cdot|$ expresses the vector model. Reason is demonstrable $v_N = 0$ does not affect the person optimizing result in untying

set of equations process middle order. And reason is demonstrable, if tactics B earning profit is $slap g^B$, iteration tactics

of time as soon as order birthplace A earning profit is $slap g^A$, then $g^A \geq g^B$, which reported by R. A. Howard in

1963.

4. Open-end funds investment decision calculates an example

Following ascertains the open-ended fund investment decision considering transaction costs according to iteration method of tactics.

4.1 Fundamental hypotheses

- (1). Assumes that the investor invests in a single day, redeems or changes a fund, but calls back a fund no forever or.
- (2). Assumes that the fund management company wants to long range to continue to manage, and every kind of type fund there exists in long range.

4.2 Variables explanation

Since our country fund manages lower open-ended fund of company flag great majority at present, that every fund manages the lower fund of company flag only when are one future may be a various type's affirmatively. Think that the event composing condition's has pure probability in the process of Markov. Consider a series of random experiments, at every time the experiment is only limited or countable be the fundamental event more infinitely $A_i (i=1, 2, 3, \dots)$ may happen. Say system is in state right away A_i . This problem moves downwards in fundamental hypothesis premise, we can manage a

company being regarded as state with every fund, buying different type fund under the flag being regarded as metastasis way, the stock fund, bond fund, balance fund wait for instance. Then thinking that every fund manages company's every kind of type fund is investment, some funds which the investor is invested in surely manage some company's kind of type funds. Umbrella shape fund is appearing, feasible fund type enriches especially, choice foundation of investor is

more. Change probability being able to do estimation from history data, difference of earning profit being able to win front and back two funds avails according to metastasis process asks for. Random discount factor α_i^k representing to buy the kind of type k fund the mansion i fund manage company mansion buys rate, λ_j^k representing surplus n changes every step transferred rate, it can give to out from underneath formula

$$1 - \lambda_j^k = (1 - \alpha_j^p)(1 - \beta_i^m) \quad i = 1, 2, \dots, N \quad j = 1, 2, \dots, N \quad (15)$$

When $i \neq j$, β_i^m representing to redeem the i fund managing the m kind of type fund rate, α_j^p representing to buy the kind of type p fund from the j fund manage company the buying rate. When $i=j$, β_i^m representing redeems rate to be 0; ($m \neq p$) α_j^p representing the changing rate which is buying another type of the same company fund, It can be that 0 also can be not 0; ($m=p$) $\alpha_j^p = \beta_i^m = 0$, Be that the fund not being in progress changes. $1 - \beta_i^m$ representing to gain which remove the redemption rate which is the i fund management company m types funds, $1 - \alpha_j^p$ representing to remove the rate which is from the purchase of the first j fund management company first p types of funds, $1 - \lambda_j^k$ can use $(1 - \alpha_j^p)(1 - \beta_i^m)$ that be removed after the transaction costs of the actual proceeds.

4.3 Simulation examples are given below

As shown in table 1 data:

Table 1. Open-end fund initial investment decision-making data tables

State	Mode	Application rates	Transfer rates	Transfer probability		Profit		Step transfer Profit
				p_{i1}^k	p_{i2}^k	r_{i1}^k	r_{i2}^k	
i	k	α_i^k	λ_j^k	p_{i1}^k	p_{i2}^k	r_{i1}^k	r_{i2}^k	$(1 - \alpha_i^k)q_i^k$
1	1	1.5%	2%	0.5	0.5	9	3	5.91
	2	2%	3%	0.8	0.2	4	4	3.92
2	1	1%	1%	0.4	0.6	3	-7	-2.97
	2	1.5%	0.5%	0.3	0.7	1	-19	-12.805
	3	3%	3%	0.7	0.3	6	8	6.402

According to the data in table 1 which can be optimal investment strategy

$D = [1, 3]^T$. That is a long-term shift, the first fund management company's first type of fund benefit the most, the second fund management company third types of fund profit-largest. To calculate here is simple, relatively few data is used only on the first iteration concluded. Each iteration is in fact resulting in a corresponding transfer rate, which will eventually come to the fund of the largest fund company profits to every fund company. Such investors can choose one or more funds according to their personal preferences for investment.

4.4 The problem that should be stated

(1). Although we do not directly consider the scale of the fund putting its impact on investment decisions, but to some extent transaction costs has been reflected.

(2). In the process of calculating a fund company which has two or more types of funds is optimal strategy, the choice of different types of funds can be the biggest profit.

(3). In considering fund proceeds at the same time, we have to consider the level of risk (risk large returns, but the risks small receipts small). Because of all the different types of risks associated with the fund which is established, investors can be different risks according to their preference which will combine the optimal decision-making for investment.

5. Conclusion

In this paper the issue of open-end fund issue of the optimal has been analyzed in terms of Sequential Principle investment decision to analyze, and using random discount factor to describe transaction costs, and giving the different random discounts compare to the past with the Markov process model, which is deriving from the decision-making so that more accurate. This paper provides an investment decision-making method for investors, while provides a basis to investment fund management company to increase capacity and improve management to provides, that is, when the fund management company operate only one type of fund in long-term which has the largest profit. It also should strengthen management for other types of funds in order to improve yield rate.

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