Max Covering Phasor Measurement Units Placement for Partial Power System Observability

Vahidhossein Khiabani[1] & Kambiz Farahmand[1]

[1] Department of Industrial and Manufacturing Engineering, North Dakota State University, Fargo, USA

Correspondence: Vahidhossein Khiabani, Department of Industrial and Manufacturing Engineering, North Dakota State University, NDSU Dept. 2485, P.O.Box 6050, Fargo, ND 58108-6050, USA. Tel: 1-701-231-5290. E-mail: vahid.khiabani@outlook.com

Abstract

Phasor Measurement Units (PMUs) are key elements in secure monitoring and control operations of the power network. Most existing algorithms consider the full power grid observability for the PMU placement problem with the objective of minimizing the cost assuming the availability of the resources. This paper presents a PMU placement approach to ensure maximum coverage of the power network for the case where the resources are limited and full observability is not achievable. Given the limited number of PMUs the model aims to find the best placement in order to reach the maximum coverage and if possible the full power system observability. The problem formulated as an integer linear programming (ILP) model and solved for optimality. The optimization model is solved for IEEE 14, 30, 57, 118 and 2383 standard test systems incorporating the zero-injection buses. The reliability of the power system has been evaluated for the placement results reaching full network observability. The results show that with sufficient resources in hand the developed maximum covering model is comparable to the existing algorithms for full power network observability.

Keywords: phasor measurement unit, power system, max cover, integer linear programming

1. Introduction

The wide Area Measurement System (WAMS) has a crucial role in electrical power system monitoring and control. Secure operation of the power system is highly dependent to the WAMS's robustness. Phasor Measurement Unit (PMU) is a monitoring device, which provides Global Positioning System (GPS) based time synchronized measurement of voltage and current phasors in power systems (Phadke & Thorp, 2008). This property of the time stamped estate estimation makes the PMUs the key elements of WAMS and consequently one of the most important measurement devices in power system protection, security and control (Phadke, 1993). A bus or node is an electrical conductor serve as a conducting pathway for continuous connection of the loads and the sources of electric power between different parts of a power network. Transmission between buses is made through lines or branches in the power network.   A bus is called observable when the voltage phasor at that bus is known and the power system is called to be observable if all the buses are observable. A PMU placed on a bus yields the voltage phasor at that bus and current phasors of all branches that are incident to the bus. Since the voltage phasor of the bus and the entire branch currents are known, the voltage phasor at adjacent buses can be calculated using the Ohm's law. Therefore, the presence of a PMU on a bus makes that particular bus and all of its immediate neighboring busses observable (Dongjie, Renmu, Peng & Tao, 2004; Denegri, Invernizzi, & Milano, 2002)

The use of PMUs installed on each bus will lead to a simplified linear state estimator (Phadke and De Moraes, 2008). Several algorithms have thus been proposed for optimal placement of PMUs to ensure observability. A graph theoretic procedure to find a minimal PMU placement was proposed in (Baldwin, Mili, Boisen, & Adapa, 1993). Niqui & Phadke (2005) proposed a simulated annealing model for PMU placement problem considering both complete and incomplete observability of the power system. An integer linear programing (ILP) approach to solve the minimal PMU placement was proposed in (Xu & Abur, 2004). It has been extended by (Gou, 2008) to address the cases of redundancy, partial observability and pre-existing conventional measurements. A systematic ILP approach for phasing of PMU placement considering failure of single PMU and modeling zero-injection busses was proposed in (Dua, Dhambhare, Gajbhiye, & Soman, 2008). The PMU placement

problem is shown to be NP complete (Brueni & Heath, 2005). Kavasseri and Srinivasan (Kavasseri and Srinivasan, 2010) considered reducing the total number of PMUs required for system observability through judicious placement of the conventional power flow measurements. A hybrid discrete particle swarm optimization approach for the optimal placement of PMUs in power grid presented in (Alinejad-Beromi, Ahmadi, Rezai. Soleymanpour, 2011).

System reliability is the probability that the system will perform its intended function for a given period of time under pre specified operating conditions. Moreover, for a system to perform its intended functions, it is important that all components and sub-systems contained in the system are highly reliable and able to perform specified functions within given requirements. Therefore the reliability of power network highly depends on the reliability and arrangement of PMUs. With a placement approach which is optimal with respect to cost, it is highly important to compute the reliability of the arrangement. Clearly protecting the power grid against loss of observability under failures such as transmission line outages, bus faults, outages, or metering failures will require increased level of redundancy of the PMUs. A reliability estimation model for a single PMU was proposed in (Yang, Wenyuan, & Jiping, 2009). The model develops a series-parallel structure for a single PMU viewed as a collection of seven subcomponents and identifies the most critical component within a PMU. Khiabani, Yadav, & Kavasseri (2012) developed a reliability based optimal PMU placement problem as a two-stage optimization model.

The existing PMU placement models consider the minimizing number of PMUs to reach full power network observability. However, the reliability based placement models consider the minimizing the number of PMUs to reach full system observability maintaining a pre-specified level of reliability both relaxing the existence of limited number PMUs. However in practice the resources are limited because of the high price of the purchasing and installing the PMU. In this case the decision maker will decide to allocate the limited recourses either to the strategic locations or to cover maximum possible buses.

This motivates to consider the PMU placement problem from a maximum covering standpoint. In the proposed model, the number of existing PMUs is factored as inputs into the model. The maximum coverage thus dictated by this input subject to the system topology. In case that the number of the PMUs is sufficient for full system observability the observability constraint added to the model. The problem is formulated as an integer linear programming (ILP) model with the objective of maximizing the network coverage and reaching the full network observability in case possible. The solution thus achieves maximum coverage with complete observability or incomplete observability depending on the availability of the recourses. Then the reliability evaluation method presented in (Khiabani, Yadav and Kavasseri, 2012) is used to evaluate the reliability of the resulting placement. To demonstrate the application of the proposed max covering model, the integer linear programming model is solved for IEEE 14, 30, 57, 118 and 2383 test system.

2. Method

PMU placement on a bus enables direct measurement of voltage magnitude and phase angle at that bus and computation of the voltage phasors at immediate neighboring buses. To build the optimization model we need to transform the Power network into the mathematical form. Let A denote the binary connection matrix obtained directly by transforming the bus admittance matrix into the binary form. Let the binary variables $A_{i,j}$, 1 if i and j are identical or i is adjacent to j and 0 otherwise. Then $A_{i,j}$ can be defined by:

$$A_{i,j} = \begin{cases} 1 & \text{if } i \text{ is adjacent to } j \\ 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Consider the IEEE 14 standard system shown in Figure 1 then the binary connection matrix for IEEE 14 bus system is described as follows:

$$A = \begin{bmatrix} 11001000000000 \\ 11111000000000 \\ 01110000000000 \\ 01111010100000 \\ 11011000000000 \\ 00000100001110 \\ 00010011100000 \\ 00000011000000 \\ 00010010110001 \\ 00000000111000 \\ 00000100011000 \\ 00000100000110 \\ 00000100000111 \\ 00000000100011 \end{bmatrix} \qquad (2)$$

Figure 1. IEEE 14 standard bus system

Let X denote the binary decision matrix. Let the binary variables x_i, either 0 or 1, denoting the absence and presence of a PMU at bus i. Then x_i, can be defined as:

$$x_i = \begin{cases} 1 & \text{if a PMU is present at bus i} \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

To define the total number of PMUs covering a specific bus, let f_i be the total number of PMUs covering bus i and defined as:

$$f_i = \sum_{j=1}^{n} A_{i,j} x_j \qquad (4)$$

where n is the total number of buses in the network. Referring to the IEEE 14 bus system in Figure 1, the total number of buses covering bus i, where n=14, are defined as:

$$f_1 = x_1 + x_2 + x_5$$

$$f_2 = x_1 + x_2 + x_3 + x_4 + x_5$$

$$f_3 = x_2 + x_3 + x_4$$

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9$$

$$f_5 = x_1 + x_2 + x_4 + x_5$$

$$f_6 = x_6 + x_{11} + x_{12} + x_{13}$$

$$f_7 = x_4 + x_7 + x_8 + x_9 \tag{5}$$

$$f_8 = x_7 + x_8$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14}$$

$$f_{10} = x_9 + x_{10} + x_{11}$$

$$f_{11} = x_6 + x_{10} + x_{11}$$

$$f_{12} = x_6 + x_{12} + x_{13}$$

$$f_{13} = x_6 + x_{12} + x_{13} + x_{14}$$

$$f_{14} = x_9 + x_{13} + x_{14}$$

The Maximum covering placement model has been formulated as an integer linear programming problem. The main objective is to maximize the coverage of the buses in the power network through assigning the limited number of PMUs available to the strategic buses. Clearly the resource limitation would not always allow reaching the complete observability of the power network. However in the case of existence of the sufficient number of PMUs, the observability constraint will be added to the optimization model. The addition of an extra constraint may result in reduced coverage but will maintain the full system observability. The integer linear programming model formulated as a maximum covering is as follows:

$$Max \sum_{i=1}^{n} f_i$$

$$s.t. \tag{6}$$

$$\sum_{i=1}^{n} x_i \leq c$$

$$x_i \in \{o \ or \ 1\} \quad i = 1, 2, 3, ..., n$$

where f_i and x_i are given in (3) and (4) respectively. Here, c is the number of the PMUs available. The objective function in (6) is to maximize the coverage of the power system. In case a limited number of PMUs is sufficient to reach complete system observability the following constraint can be added to the model:

$$\sum_{i=1}^{n} f_i \geq 1 \tag{7}$$

Decision maker may need to cover some of the strategic buses in the system. To do this if the number of PMUs is not sufficient for the full system observabiliity, then only the ith element of the constraint (7) could be added to the optimization problem to make sure bus i is covered. The model can be modified to incorporate both zero injection buses (Dua, Dhambhare, Gajbhiye and Soman, 2008) and flow measurement cases (Kavasseri and Srinivasan, 2011) for further reduction in the total number of PMUs needed for full system observability. The model developed in (Dua, Dhambhare, Gajbhiye, & Soman, 2008) has modified for the proposed max covering problem to incorporate the zero injection buses in the system. Also the model developed in (Khiabani, Yadav and Kavasseri, 2012) is adopted to evaluate the reliability of the power network for the placement results. The reliability evaluation portion of the reliability based placement model presented in (Khiabani, Yadav and Kavasseri, 2012) is briefly presented here. As mentioned before placement of a PMU at a given bus allows direct measurement of voltage phasor at that bus and calculation of the voltage phasors at immediate neighboring buses. Thus, the entire power system will be fully observable if all buses in the network are covered with at least a PMU. Therefore buses are connected in series from a reliability point of view. In case no redundant PMU existed,

the failure of either of the PMUs would result in loss of observability. Thus reliability of observability of the entire system is given as:

$$R = \prod_{i=1}^{n} r_i \tag{8}$$

where r_i represents the reliability level for i^{th} bus, R is the overall system reliability and n is the total number of the buses in the power system. In case bus i is covered with more than a PMU then the redundant PMUs covering bus i will treated as parallel connected from the reliability standpoint. Thus i^{th} bus will no longer be observable if all PMUs covering it fail simultaneously. Hence we can define bus reliability of observability (r_i) as:

$$r_i = 1 - \prod_{j=1}^{f_i} q_j = 1 - q_j^{f_i} \tag{9}$$

where q_j denotes the probability of failure of jth PMU and f_i is the total number of PMUs covering i^{th} bus. Hence $\prod_{j=1}^{f_i} q_j$ denotes probability of failure of all PMUs observing i^{th} bus.

3. Results

The proposed maximum covering placement model is solved for the IEEE 14, 30, 57, 118 and 2383 bus standard test systems. The observability constraint added where complete power system observability was possible. The reliability of the placement solutions has been calculated. The computations were performed with Wolfram Mathematica 8.0. on a 2.66 GHz Intel(R) Core™ 2 Quad CPU with system memory of 2.96 GB. Results are reported with PMU reliabilities assumed to be 0.99 for all cases for both incorporating zero-injection buses and without zero-injection buses. The comparison plots have been done using Matlab. A Mathematica code using a For loop has been applied for the all sets of possible inputs for all IEEE standard bus systems.

3.1 Max Covering Placementt

The results for IEEE 14, 30, 57, 118 and 2383 standard bus system are shown in Tables 1-5 for the number of PMUs given, the total coverage and overall system reliability achieved. The overall system reliability has been calculated after and based on the optimization problem results and the PMU reliabilities assumed to be 99%. The 99% level of PMU reliability assumed since PMU reliabilities are near 98%.

Table 1. Placement results for IEEE 14 bus system

#PMU	Cover	R
1	6	0
2	11	0
3	16	0
4	18	0.90

Table 2. Placement results for IEEE 30 bus system

#PMU	Cover	R
1	8	0
2	15	0
3	21	0
4	26	0
5	31	0
6	36	0
7	41	0
8	45	0
9	49	0
10	52	0.83

Table 3. Placement results for IEEE 57 bus system

#PMU	Cover	R
1	7	0
2	14	0
3	20	0
4	26	0
5	31	0
6	36	0
7	41	0
8	46	0
9	51	0
10	56	0
11	60	0
12	64	0
13	68	0
14	72	0
15	76	0
16	80	0
17	69	0.62

Table 4. Placement results for IEEE 118 bus system

#PMU	Cover	R
1	10	0
2	19	0
3	27	0
4	35	0
5	42	0
6	49	0
7	56	0
8	63	0
9	70	0
10	77	0
11	83	0
12	89	0
13	95	0
14	101	0
15	107	0
16	113	0
17	119	0
18	125	0
19	131	0
20	137	0
21	143	0
22	148	0
23	153	0
24	158	0
25	163	0
26	168	0
27	173	0
28	178	0
29	183	0
30	188	0
31	193	0
32	164	0.45

Table 5. Placement results for IEEE 2383 bus system

#PMU	Cover	R
1	10	0
2	20	0
3	30	0
4	40	0
5	50	0
6	60	0
7	69	0
8	78	0
9	87	0
10	96	0
11	105	0
12	114	0
13	123	0
14	132	0
15	141	0
16	150	0
17	159	0
18	167	0
19	175	0
20	183	0
21	191	0
22	199	0
23	207	0
24	215	0
25	223	0
26	231	0
27	239	0
28	247	0
29	255	0
30	263	0
31	271	0
32	279	0
⋮	⋮	⋮
⋮	⋮	⋮
745	3714	0
746	3288	$3.90705*10^{-8}$

The results for IEEE 14, 30, 57, 118 standard bus systems has been summarized and shown in Figures 2 & 3. The Figures show the Cover, number of buses with PMUs installed on and the evaluated reliability. The results for IEEE 2383 standard bus system have been shown in Figure 4.

Figure 2. Comparison of coverage between IEEE 14 & 30 bus system

Figure 3. Comparison of coverage between IEEE 57 & 118 bus system

Figure 4. Coverage for IEEE 2383 bus system

3.2 Max covering Incorporating Zero Injection Buses

The results for IEEE 14, 30, 57, 118 standard bus systems considering the zero injection buses are shown in Tables 6-9 for the number of PMUs given, the total coverage and overall system reliability achieved. The overall system reliability has been calculated after and based on the optimization problem results and the PMU reliabilities assumed to be 99%.

Table 6. Placement results for IEEE 14 bus system incorporating zero injection buses

#PMU	Cover	R
1	7	0
2	13	0
3	15	0.88

Table 7. Placement results for IEEE 30 bus system incorporating zero injection buses

#PMU	Cover	R
1	13	0
2	23	0
3	33	0
4	43	0
5	52	0
6	61	0
7	57	0.86

Table 8. Placement results for IEEE 57 bus system incorporating zero injection buses

#PMU	Cover	R
1	9	0
2	18	0
3	26	0
4	33	0
5	40	0
6	47	0
7	54	0
8	61	0
9	68	0
10	75	0
11	82	0
12	88	0
13	72	0.65

Table 9. Placement results for IEEE 118 bus system incorporating zero injection buses

#PMU	Cover	R
1	12	0
2	24	0
3	36	0
4	46	0
5	56	0
6	66	0
7	76	0
8	85	0
9	94	0
10	103	0
11	112	0
12	120	0
13	128	0
14	136	0
15	144	0
16	152	0
17	160	0
18	167	0
19	174	0
20	181	0
21	188	0
22	195	0
23	202	0
24	209	0
25	215	0
26	221	0
27	227	0
28	184	0.47

The results for IEEE 14, 30, 57, 118 standard bus systems considering zero injection buses has been summarized and shown in Figures 5 & 6. The figures show the Cover, number of buses with installed PMUs and the evaluated reliability.

Figure 5. Comparison of coverage between IEEE 14 & 30 bus systems

Figure 6. Comparison of coverage between IEEE 57 & 118 bus systems

4. Discussion

The usefulness of the proposed Max covering optimization model investigated, comparing the results to the PMU placement results in (Kavasseri and Srinivasan, 2011). Comparison results are shown in Table 10 for IEEE 14, 30, 57 and 118 bus systems. The comparison of the results shows that the models reach the same output with minor difference. However the proposed model has less complexity and also can consider the cases where reaching the full observability is not feasible. This ability of the proposed model will empower the decision maker through availability of more options for the case of limited resources. Almost each section in output is the same in Table 10 except the placement buses for the PMUs, this is trivial for the placement problems because of the existence of the alternative optimal solutions. However each alternative optimal solution will result in a different reliability levels. This can be seen in Table 10 for the IEEE 30 bus system.

Table 10. Comparison of placement results with traditional PMU placement problem results

IEEE System	Max cover Placement	Max cover #PMU	Max cover R	PMU placement Placement	PMU placement # PMU	PMU placement R
14	2,6,7,9	4	0.90	2,6,7,9	4	0.9
30	2,4,6,9,10,12, 15,18,25,27	10	0.83	1, 2, 6, 9, 10, 12, 15, 18, 25, 27	10	0.84
57	1,4,9,13,19,22 ,25,26,29,32,3 6,39,41,45,47, 50,53	17	0.62	1, 4, 9, 10, 19, 22, 25, 26, 29, 32, 36, 39, 41, 44, 46, 49, 53	17	0.62
118	3,5,9,12,15,17 ,20,23,28,30,3 4,37,40,45,49, 52,56,62,64,6 8,71,75,77,80, 85,86,90,94,1 01,105,110,11 4	32	0.45	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56,62, 63,68, 70, 71, 76, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	32	0.44

To further investigate the usefulness of the proposed model, we compared the results to the reliability based placement results in (Khiabani, Yadav, & Kavasseri, 2012). Since the results for the (Khiabani, Yadav, & Kavasseri, 2012) analyzed for PMU reliability of 0.95 therefore for this comparison only we run the Max covering model and evaluate the reliability with the PMU reliability of 0.95 with the selection of results reaching the overall system reliability of at least 0.90. Comparison results are shown in Table 11 reaching the minimum system wide reliability level of 0.90 and in Table 12 with the same number of PMUs for IEEE 14, 30, 57 and 118 bus systems. The results show that the comparison between reliability based placement model and the Max cover model derived from optimization procedure result in more cover as compared to reliability based placement in the literature. On the other hand reliability based placement model reached higher system wide reliability level compared to Max covering problem.

Table 11. Comparison of placement results with reliability based placement problem results with R=0.90

IEEE System	Max cover #PMU	Max cover Cover	Max cover R	Reliability #PMU	Reliability Cover	Reliability R
14	11	44	0.94	9	37	0.98
30	28	108	0.90	21	85	0.95
57	55	203	0.90	57	207	0.99
118	117	474	0.93	115	470	0.99

Table 12. Comparison of placement results with reliability based placement problem results with same #PMUs

IEEE System	Max cover #PMU	Max cover Cover	Max cover R	Reliability #PMU	Reliability Cover	Reliability R
14	9	38	0.85	9	37	0.98
30	21	88	0.69	21	85	0.95
57	57	207	0.99	57	207	0.99
118	115	470	0.84	115	470	0.99

In this paper the optimal PMU placement problem is solved by Max covering model considering both partial and full observability of the power network. The Max covering based PMU placement was considered using an integer linear programming approach. The main contribution of the paper is to make the PMU placement model feasible for the optimal PMU placement problem to solve for limited number of PMUs and calculating the reliability evaluation for the model. The proposed model is solved for the IEEE 14, 30, 57, 118 and 2383

standard bus systems with the number of PMUs available as input to the optimization model. The results compared with two existing methods in the literature, PMU placement and reliability based PMU placement. The comparison results show that with sufficient resources in hand the developed maximum covering model is comparable to the existing algorithms for full power network observability. This connection between max coverage and system reliability could be potentially useful and insightful in building large and complex electrical power networks.

References

Alinejad-Beromi, Y., Ahmadi, A., & Rezai, S. H. (2011). Optimal PMU Placement Considering Contingencies by Using Hybrid Discrete Particle Swarm Optimization Technique. *International Review of Electrical Engineering-IREE, 6*(4), 1927-1938.

Baldwin, T. L., Mili, L., Boisen, M. B., & Adapa, R. (1993). Power system observability with minimal phasor measurement placement. *IEEE Transactions on Power Systems, 8*(2), 707-715. http://dx.doi.org/10.1109/59.260810

Brueni, D. J., & Heath, L. S. (2005). The PMU placement problem. *SIAM J. Discrete Math, 19*(3), 744-761. http://dx.doi.org/10.1137/S0895480103432556

Dongjie, X., Renmu, H., Peng, W., & Tao, X. (2004). Comparison of several PMU placement algorithms for state estimation. In *Proc. Inst. Elect. Eng. Int. Conf. Develop. Power Syst. Protection,* 32-35. http://dx.doi.org/10.1049/cp:20040056

Denegri, G. B., Invernizzi, M., & Milano, F. (2002). A security oriented approach to PMU positioning for advanced monitoring of a transmission grid. In *Proc. IEEE Int. Conf. Power Syst, 2*, 798-803. http://dx.doi.org/10.1109/ICPST.2002.1047509

Dua, D., Dhambhare, S., Gajbhiye, R. K., & Soman, S. A. (2008). Optimal multistage scheduling of PMU placement: An ILP approach. *IEEE Transactions on. Power Delivery, 23*(4), 1812-2388. http://dx.doi.org/10.1109/TPWRD.2008.919046

Gou, B. (2008). Generalized integer linear programming formulation for optimal PMU placement. *IEEE Trans. Power Systems, 23*(3), 1099-1104. http://dx.doi.org/10.1109/TPWRS.2008.926475

Kavasseri, R., & Srinivasan, S. K. (2010). Joint optimal placement of PMU and conventional measurements in power systems. *Circuits and Systems (ISCAS), Proceedings of 2010 IEEE International Symposium*, 3449-3452. http://dx.doi.org/10.1109/ISCAS.2010.5537855

Kavasseri, R., & Srinivasan, S. K. (2011). Joint placement of Phasor and Flow Measurements for Observability of Power Systems. *IEEE Transactions on Power Systems, 8*(2), 707-715. http://dx.doi.org/10.1109/TPWRS.2011.2130544

Khiabani, V. H., Yadav, O. P., & Kavasseri, R. (2012). Reliability-based placement of Phasor measurement Unit in Power Systems. *Proc. Institution of Mechanical Engineers (IMechE), Part O: Journal of Risk and Reliability, 226*(1), 109-117. http://dx.doi.org/10.1177/1748006X11417959

Nuqui, R. F., & Phadke, A. G. (2005). Phasor measurement unit placement techniques for complete and incomplete observability. *IEEE Transactions on Power Delivery*, *20*(4), 2381-2388. http://dx.doi.org/10.1109/TPWRD.2005.855457

Phadke, A. G., (1993). Synchronized phasor measurements in power systems. *IEEE Comp. Appl. in Power Systems, 6*(2), 10-15. http://dx.doi.org/10.1109/67.207465

Phadke, A. G., & Thorp, J. S. (2008). *Synchronized Phasor Measurements and Their Applications*. New York: Springer. http://dx.doi.org/10.1007/978-0-387-76537-2_1

Phadke, A. G., & de Moraes, R. M. (2008). The wide world of wide-area measurement. *IEEE Power Energy, 6*(5), 52-65. http://dx.doi.org/10.1109/MPE.2008.927476

Xu, B., & Abur, A. (2004). Observability analysis and measurement placement for systems with PMUs. *IEEE Power Systems Conf. Expo, 2*, 943-946. http://dx.doi.org/10.1109/PSCE.2004.1397683

Wang, Y., Li, W., & Lu, J. (2009). Reliability Analysis of Phasor Measurement Unit Using Hierarchical Markov Modeling. *Electric Power Components and Systems. Taylor & Francis*, *37*(5), 517-532. http://dx.doi.org/10.1080/15325000802599361