Simulation Model of Multiple Queueing Parameters: A Case of Vehicle Maintenance System

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Abstract

Often, taking strategic decisions in maintenance systems are very difficult and at times impossible, because the data required are either not available or not in the right format. This study has developed a computer queueing model with an integrated set of algorithms for simulation of multiple queueing parameters. It used ITC workshop in Nigeria as a case study, which has a single channel multi-server queueing system. The developed simulation model was validated with the standard mathematical model results and found to be reliable. The structure of the software package is flexible and robust enough to accommodate any value of maximum simulation time and number of crew size as the storage capacity of the computer allows. In addition to being user friendly, performing an experiment using the simulation model is more than forty million times faster than doing it with the case study. It is also equipped with post object oriented animation and digital tracing for each discrete step of the simulation run. Hence, it is an effective queueing model pedagogical tool. With the simulation model, decision taking is made easier, requiring less data and as fast and the simulation runs.

Keywords: Simulation, Queueing, Crew-size, Algorithm and Validation

1. Introduction

Simulation is a very important and flexible modelling tool in operations research studies. It amounts to imitating the performance of a real system in a controlled environment in order to estimate what its actual performance will be (Law, 1986). The experiment is done on the model rather than on the real system itself, because the latter would be too inconvenient, expensive, risky and time-consuming. The construction of a simulation model requires a thorough understanding of the *system.* This understanding can reveal subtle relationships which have important bearing on the interaction of variables. Without such revelations, the system may not be understood (Read, 1998). There have been numerous applications of simulation in a wide variety of contexts. It has been developed to the extent of using object oriented techniques (Shewchuck and Chang, 2001), (Liu, 2002).

Belky (2006) reported how difficult and time consuming it took knitting department to perform an experiment that estimated the optimum repair crew size. However, simulation can easily and effectively be applied in stochastic queueing systems like above with a view towards estimating the optimal solution to a given problem. In Alfredsson (1999), a proposal of OPRAL, a model for optimum spare allocation as well as repair facility allocation was made. The queue at the repair facility is modeled as *M/M/s* so that the expected waiting time for an available resource can be calculated. Jung (1993), presented a methodology for a repairable system with time-dependent demand by implementing discrete event simulation while Slay et al. (1996) proposed an aircraft sustainability model that can handle time-dependent demand rates but having infinite repair resources.

Different types of simulation algorithms for running an M/M/1 system have been developed. Brain (1996) and Ultima (1997) algorithms though having slight differences in there logic gave valid results when used. In 1998 an improvement was made by the development of Solver & Simulator in New York which not only has graphical user interface but has the capability of using a mathematical model and simulation side by side for easy validation.

Moreover, unlike a mathematical model, the uniqueness of the model to be simulated can effectively be integrated into an existing simulation algorithm, making it more flexible, like the simulation of a server uni-directional patrolling in Buffa's (1989) work in which walking time of the server has to be considered in determining the efficiency of the machines.

Queueing type situations that require decision-making arise in a wide variety of contexts. Designing a queue system typically involves making one or more combination of decisions. For instance, decisions regarding the amount of service capacity to provide usually are based primarily on two considerations. Firstly, the cost incurred in providing the service and the second is the cost in waiting for that service (Pooch and Wall, 1992). Providing too much service involves excessive cost, not providing enough service capacity causes the waiting line to become excessively long which is costly too. Therefore, the ultimate goal is to achieve an economic balance between the cost of service and the cost associated with waiting for that service (Pooch and Wall 1992), (Harvey, 1989).

The primary motivation behind this research is the challenge posed by poor maintenance record. This has made taking strategic decisions in maintenance systems very difficult and often times impossible. Even when such records are available, accessing, collating and analyzing them with a view towards taking a valid decision is often very difficult.

However, the development of a valid and reliable simulator with digital tracing for each discrete step of the simulation run, will make decision taking easier, requiring less data and as quick as the simulation runs. Simulation model with digital tracing and animation provides greater insight into the performance of the system for any given design, thereby allowing room for forecasting and adding credibility to the results of the simulated study.

2. Materials and Methods

2.1 Materials

In carrying out a simulation study, a model representing the system to be investigated is developed. This requires the analyst(s) to become thoroughly familiar with the operating realities of the system involved and the objectives of the study (Harvey, 1989). This will enable the researcher(s) to reduce the real system to a valid logical flow diagram and program. In view of the above, the maintenance workshop of Imo Transport Company Limited was used as a case study. The symbols or abbreviations and their full meanings used in this study is shown in Table 1.

In the selected maintenance workshop, the maintenance crew, drivers/customers, operating managers and the staff of accounts department of the company formed the target and the source of the relevant information. They provided information on the operational procedure of the company. They also provided relevant data which include salaries/allowances of the crew, job cards, log books, work orders, operational statistics, cost reports and hiring costs of the brand of vehicles repaired.

2.2 Arrival and Service Rate Determination

The time of each of the vehicles arrival and service were collected from were the inter-arrival time and cumulative service time were deduced. Time interval was used to group the observations (n), with the frequency of observations (F_n). Therefore, the arrival and service rates of the distribution were calculated thus

$$AR = \left[\frac{\sum_{n=0}^{\infty} nF_n}{\sum_{n=0}^{\infty} F_n} \right]_{AR} \quad \text{while} \quad SR = \left[\frac{\sum_{n=0}^{\infty} nF_n}{\sum_{n=0}^{\infty} F_n} \right]_{SR}$$

Stability is assumed when *AR* is less than *SR*. With the application of Sturgis rule, I = 1 + 3.3log x, the best time interval size was determined (Michel, 1993).

2.3 Random Test Using Chi-Square Goodness of Fit Test

Calculations of the chi-square statistics is done using the formula

$$\chi^2 = \sum_{n=0}^{\infty} \frac{(F_n - T_n)^2}{T_n}$$

where the theoretical Frequency T_n is

$T_n = P_n(\sum F_n)$ and P_n is the Poisson distribution using the probability distribution of the Poisson's random variable. For Arrival Rate (*AR*)

$$P_n = \frac{(AR^n e^{-AR})}{n!} \qquad n = 0,1,2,3, \ldots \infty \qquad \text{where } AR > 0$$

Thereafter, comparison of the χ^2_{value} with the critical value of the χ^2 distribution was carried out at a level of significance of 0.05 and the degree of freedom of 1.

2.4 Determination of the Total Maintenance Cost

Total Maintenance Cost was determined making use of the fact that it is equal to the cost of using maintenance crew and the cost of waiting of jobs.

$$TMC_{(CS)} = CUMC + CWJ \qquad CUMC = (CS)C_1$$

$$CWJ = C_2 ANJS_{(CS)} \qquad ANJS = \frac{UF}{(CS) - UF}$$

The appropriate costs of salaries/allowances of crew and the appropriate percentages of depreciation costs on tools, equipments, infrastructure and building were used in determining the average cost of using one member of a crew per day C_1. The cost of hiring or amount realized by making use of the job brought for maintenance was considered as the average cost of waiting for the job C_2.

2.5 Performing Manual Simulation of the Collected Data.

In the stochastic system under consideration, the arrival and service processes go on simultaneously although a customer cannot be served unless the customer is present. The events that change the state of the system are the arrival and service completion of the customer currently in service (if any). The state transition mechanism is thus:

$$N(t) = \begin{cases} N(t) + 1 & \text{if arrival occurs at time (t)} \\ N(t) - 1 & \text{if service completion occurs at time (t)} \end{cases}$$

Where $N_{(t)}$ is the number of jobs in the system at time (t). Manual simulation of the data collected was performed as an in-depth study of the system in readiness for the designing/development of the model itself.

2.6 Generating Random Variables

The process of generating random variables can be reduced to: The generation of random numbers and the transformation of the random numbers into the appropriate random variables. The inverse transform algebraic method was used in the simulation development.

The random phenomenon has a negative exponential density function:

$$F(t) = \lambda e^{-\lambda t} \qquad F(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t}$$

$$F(x) = 1 - e^{-\lambda t} \qquad e^{-\lambda t} = 1 - F(x)$$

$$-\lambda x = In\{1 - F(x)\} \qquad x = -(1/\lambda) InR(x)$$

where $R(x) = 1 - F(x)$

R(x) is a random number between zero and one and x is the variable (Burghardt, 1991).

2.7 Time Advancement of a Simulation Clock

Two types of time advancement exist, the fixed increment method and the next event method (Taha, 1987). The next event method was used in this project because discrete event simulation model was carried out. In this method, the clock is incremented by a variable amount, which is the time from the event that has just occurred until the next event of any kind occurs. In other words, the clock jumps from event to event and remains constant between events. For this simulation model, the computer needs to keep track of two future events, namely, the next arrival and the next service completion (If any customer is currently being served).

2.8 Development of the Algorithm

The summary of the algorithm works thus:

The computer generates a random number, which is transformed into a random variable representing next arrival time or the next service completion time as the case may be. Each time an arrival or service completion occurs, the computer determines how long it will be until the next time this event will occur, adds this time to the current clock time and then stores the sum in a computer file. If the service completion leaves no customer in the system, then the generation of the time of the next service completion is postponed until the next arrival occurs. To determine which event will occur next, the computer finds the minimum of the clock times stored in the file and expediate the housekeeping involved. This procedure goes on for the duration of the simulation.

2.9 Verification and Validation of the Model

Verification refers to the process of confirming that the simulation model is correctly translated into a computer program. In this model development, four techniques were used: A. Running it for a known situation where the result was easily calculated. B, *Traces*: Listing of the events and status of the model for each discrete step of the simulation. This work used digital tracing. C. *Animation*: In this model, post object oriented animation was integrated into the simulation run, allowing the user to interact with the simulation program while running. Indeed animation is a valuable tool to support verification of the logic of a simulation program. D. *Structured Walk Through*: Other Engineering minds knowledgeable about the model and the simulation language went through the original computer code in detail with the researcher.

Validation is the process of building an acceptable level of confidence that an inference about a simulated process is a correct or valid inference for the actual process. The process of testing and improving a model to increase its validity is commonly referred to as validation. The simulation model was compared with the mathematical model and with the result of the case study.

3. Results and Discussion

3.1 Results from the Case Study

The values of the major results obtained from the case study are:

AR = 0.8575 per day, SR = 1.0057 per day, C_1 = ₦639.38 per worker per day and C_2 = ₦8,300.00 per day. In addition, the arrival time distribution of jobs was tested and it follows Poisson distribution while service time follows exponential distribution (this implies that cumulative service time follows Poisson distribution). The tabulation of values for Chi-Square random test for the arrival time distribution of jobs is shown in Table 2.

$\chi^2_{v,\alpha} = \chi_{1,(0.05)} = 3.841.$

By comparing the results

$\chi^2_{\text{-value}} < \chi^2_{v,\alpha}$ that is $2.195 < 3.841$

This led to the conclusion that the hypothesis of the distribution came from a Poisson distribution. The same statistical tools were used in determining the service time distribution.

3.2 Manual Simulation of the Collected Data

The manual simulation parameters built up from the values obtained from the case study is shown in Table 3 and the actual manual simulation done in Figure 1.

The following results were obtained directly from the manual simulation of Fig 1. The time is measured in hours.

CLOCK= 211, TNJ = 25, TNJEQ = 14, TIDT = 42 TNC = 2

TWT = 71, TISF = CLOCK – TIDT = 169, TIS = TISF +TWT=240

ATIS = TIS / TNJ = 9.6 and ATEQ = TWT / TNJEQ = 5.07

3.3 Algorithm for Computer Model Simulation

Having carried out an in-depth study on the system and carried out manual simulation of the collected data. The algorithm for the computer model was developed and the flowchart of the computer model simulation is shown in Fig. 2.

3.4 The Developed Software Simulation Model

Fig. 3 is the input dialogue box for keying in the input parameters. The simulator window environment is shown in Fig. 4 when the simulation is in progress while Fig 5 is the result dialogue box for displaying the simulation results.

3.5 Validation of the Simulation Model Results

Table 4 shows the results obtained for both Mathematical or Deterministic Approach (DA) and Simulation or Stochastic Approach (SA). Comparing the results show that the simulated result is spectacularly valid. The input parameters that were used are C_1 = ₦3000, C_2 = ₦10000, AR = 0.555 and SR = 0.999

3.6 Determination of the Most Cost Effective Crew Size

The costs of using a maintenance crew, waiting of jobs and the total maintenance, were simulated and plotted in Fig 6. The above costs were obtained as the crew size varies from 1 to 10 workers. The input parameters used for this are AR = 0.605, SR = 0.999, C_1 = ₦2553.53 and C_2 = ₦10000

From the graph of Fig 6, as the number of crew increases, the cost curve of waiting of jobs tends to reduce geometrically while the cost of using maintenance crew increases linearly. The total maintenance cost initially reduces geometrically until it reaches a point when it smoothly changed its course and starts to increase thereafter forming a curve with a minimum value. Hence, the corresponding crew size at that approximate minimum value of TMC is the most cost effective crew size. In this case, it is two workers.

3.7 Effects of Varying the MST and UF on the computer processing time

Simulation runs were carried out at instances when the AR was 0.5, 1.0 and 1.5 (i.e. where the UF is 0.3, 0.6 and 0.9) and other input parameters kept constant. At each instance of AR, the maximum simulation time was varied uniformly five times. The actual time in seconds spent by the computer in carrying out these variations were determined. The results obtained were plotted in Fig 7.

Essentially, two factors affect the computer processing time. They are the maximum simulation time and the utilization factor and as any of this parameter increases, it increases the processing time. It was also found that performing the experiment using the developed simulator is more than 40 million time faster than carrying out the same experiment directly with the case study.

4. Conclusion

Simulation is not only the last resort as it looks like when compared with experiment and mathematical analysis in getting information of an objective reality. It can contribute very much to understanding of the system being analysed not only by supplying answers to the questions that were originally asked. Very often, creation of the simulation model is the first occasion where certain things are taken into account and the system extremes are explored without any risk attached. By well implementation of the results obtained from this model, analytical, judgmental and possibly simulation can be proffered for other problem areas. Simulators equipped with post object oriented animation and digital tracing indeed can be a very effective queueing model pedagogical tool. With this simulation model, strategic decision can be taken.

5. Recommendation

Future research can extend the analysis here in many directions. Although the researcher was motivated by a system without service differentiation to the vehicles that come for maintenance (all customers are treated equally), considering the possibility of having prioritized treatment of vehicles that come for maintenance work would be an appropriate extension. In addition, considering multi-channel and multi-server queueing system would be interesting.

Beyond that, Maintenance workshops should develop a data base management system (DBMS). This will help in creating, sorting, monitoring, displaying, collating etc of data/records. This will reduce significantly the information that will be very difficult or even impossible to retrieve or sort for management report or research work. In addition with this DMBS, input data may no longer be entered manually but can be interfaced with a developed simulation package like the one developed in this study.

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Table 1. List of Symbols and Abbreviations

Abbre.	**Meaning**	**Abbre.**	**Meaning**
AR	Arrival Rate (/day)	MQL	Maximum Queue Length (Vehicles)
AT	Arrival Time (Hours)	MST	Maximum Simulation Time (days)
ANJS	Average Number of Jobs in the system (Vehicles)	MSTE	Maximum Simulation Time Exceeded? (days)
		NAT	Next Arrival Time (days)
ATEQ	Average Time in Queue for all Jobs (days)	NDT	Next Departure Time (days)
		NIS	Current Number of Jobs in System (vehicles)
ATIS	Average Time in System (days)	NQ	Current Number of Jobs in Queue (vehicles)
ATEQ	Average Time of Waiting for Jobs that Entered Queue (days)	OA	Order of Arrivals
		SR	Service Rate capacity (/day)
C_1	Average cost of using one member of a crew per day (N)	ST	Service Time (hours)
		TIDT	Total Idle Time (days)
C_2	Average cost of waiting for jobs per day (N)	T in S	Time in Service (hours)
		T into S	Time into Service (hours)
CS	Crew Size (workers)	TIS	Total time for all jobs in the System (days)
CST	Cumulative Service Time (hours)	TISF	Total time In-Service of the Facility (days)
CUMC	Cost of Using a Maintenance Crew Gang	TMC	Total Maintenance Cost
		TMC_O	Minimum Total Maintenance Cost
CWJ	Cost of Waiting of Jobs (N)	TNC	Coincidences of Arrivals & Departures (vehi.)
DA	Deterministic Approach	TNJ	Total Number of Arrivals of Jobs (vehicles)
DT	Departure Time (hours)	TNJEQ	Total Number of Jobs that Entered Queue (vehicles)
EWT	Expected Waiting Time (days)		
F(X)	Random Number	TNJES	Total Number of Jobs that Entered Service (vehicles)
I-AT	Inter-Arrival Time (hours)		
I-CS	Inter-Crew Size (workers	UF	Utilization Factor

Table 2. Test Results for Randomness of Arrivals

n	P_n	F_n	T_n	$(F_n - T_n)^2 / T_n$
0	0.4795	14	16.3	0.352
1	0.3524	16	11.98	1.347
2	0.1295	3	4.403	
3	0.0317	1	1.078	0.496
4	0.0058	0	0.197	
Total	1	34	34	2.195

Table 3. Manual Simulation Parameters

OA	AT	T into S	ST	DT	WQ	T in S
1	0	0	2	2	0	2
2	2	2	3	5	0	3
3	5	5	6	11	0	6
4	10	11	13	24	1	14
5	18	24	6	30	6	12
6	23	30	5	35	7	12
7	25	35	4	39	10	14
8	38	39	9	48	1	10
9	50	50	7	57	0	7
10	59	59	6	65	0	6
11	64	65	14	79	1	15
12	85	85	13	98	0	13
13	92	98	10	108	6	16
14	103	108	18	126	5	23
15	114	126	8	134	12	20
16	143	143	2	145	0	2
17	144	145	8	153	1	9
18	146	153	5	158	7	12
19	151	158	4	162	7	11
20	159	162	6	168	3	9
21	164	168	1	169	4	5
22	175	175	8	183	0	8
23	187	187	3	190	0	3
24	199	199	1	200	0	1
25	204	204	7	211	0	7

Table 4. Results of UF, ANJS, CWJ and TMC using DA and SA

	DA	SA	DA	SA	DA	SA	DA	SA
CS	UF	UF	ANJS	ANJS	CWJ	CWJ	TMC	TMC
1	0.556	0.552	1.250	1.233	12500.00	12382.99	15500.00	15328.99
2	0.278	0.276	0.385	0.381	3846.15	3813.60	9846.15	9813.60
3	0.185	0.184	0.227	0.226	2272.73	2255.66	11272.73	11255.66
4	0.139	0.138	0.161	0.160	1612.90	1601.44	13612.90	13601.44
5	0.111	0.110	0.125	0.124	1250.00	1241.39	16250.00	16241.39
6	0.093	0.092	0.102	0.101	1020.41	1013.51	19020.41	19013.52
7	0.079	0.079	0.086	0.086	862.07	856.34	21862.07	21856.34
8	0.069	0.069	0.075	0.074	746.27	741.36	24746.27	24741.36
9	0.062	0.061	0.066	0.065	657.89	653.60	27657.89	27653.60
10	0.056	0.055	0.059	0.058	588.24	584.42	30588.24	30584.42

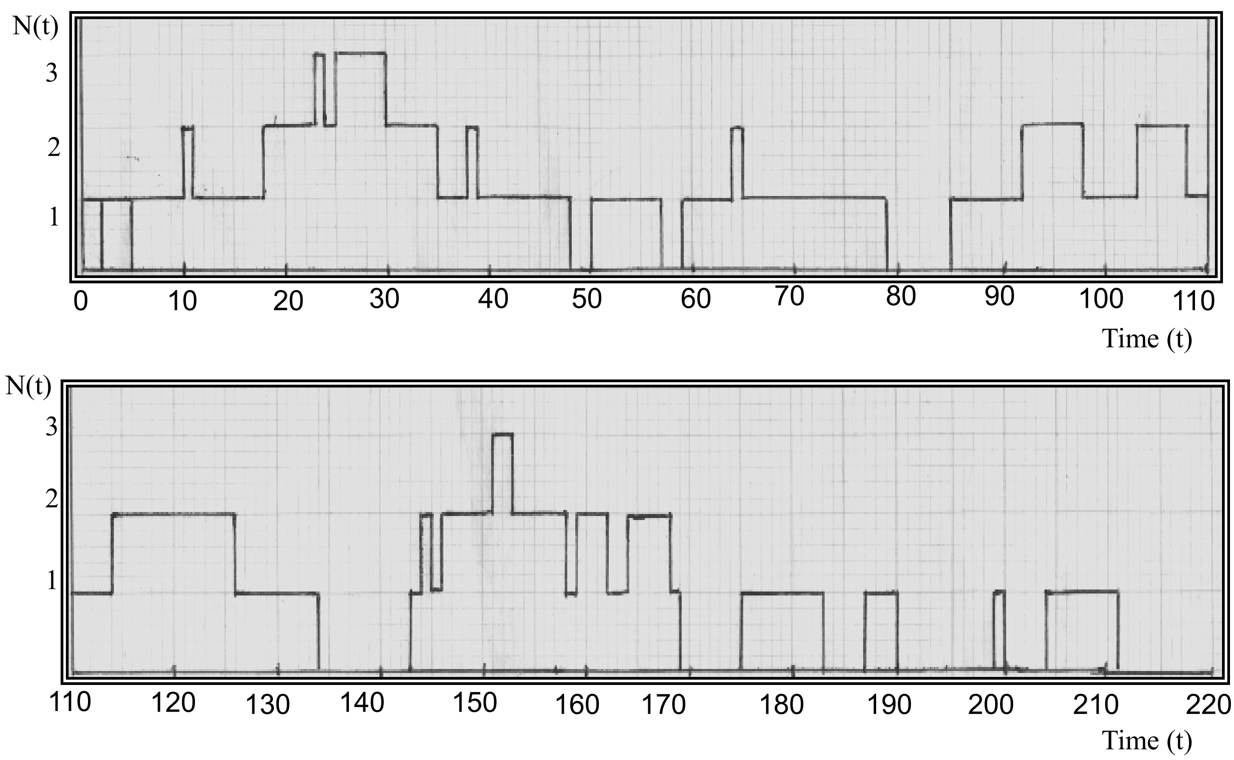


Figure 1. Manual Simulation Showing Number in the System

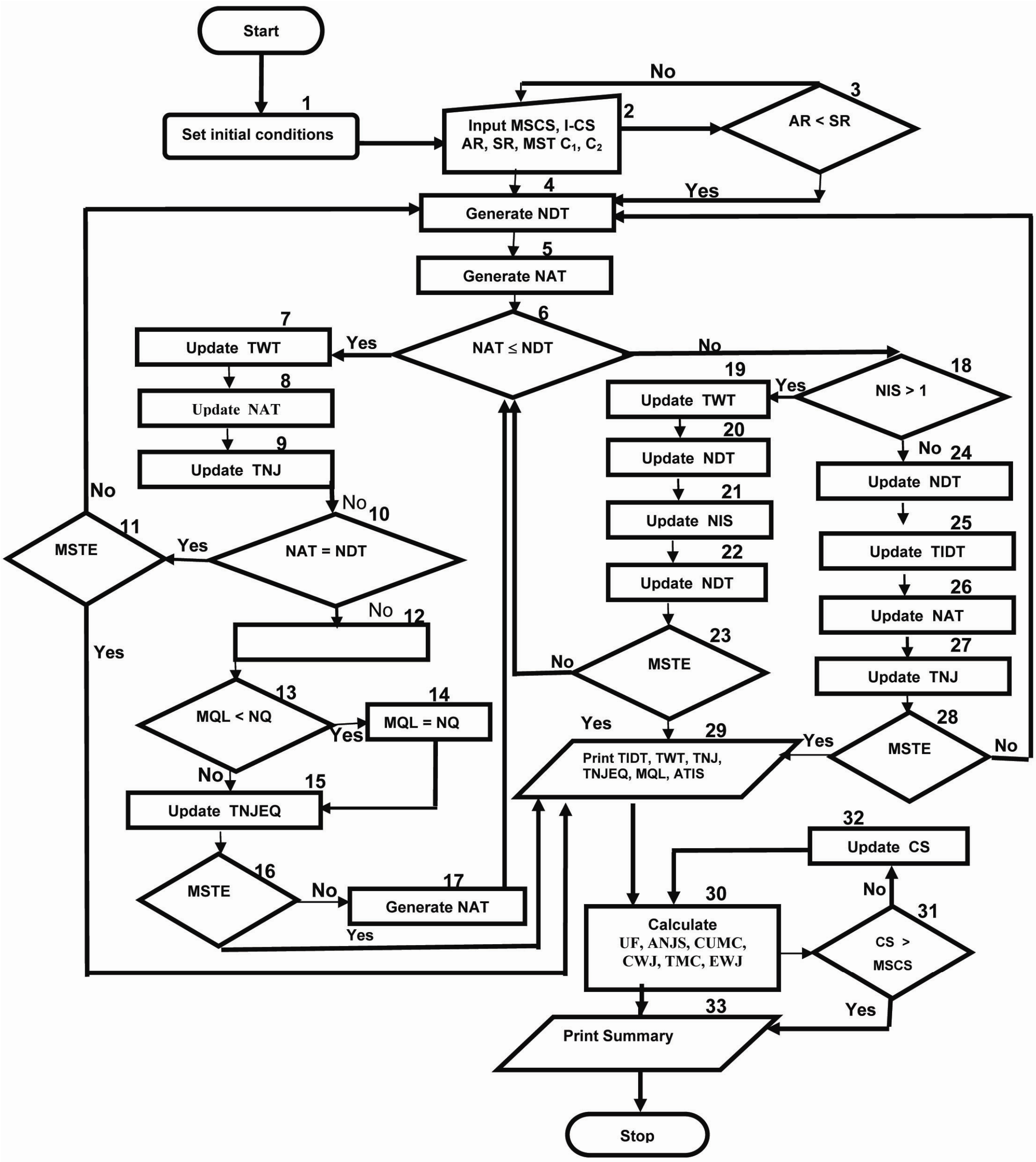


Figure 2. Flowchart for the Computer Model Simulation

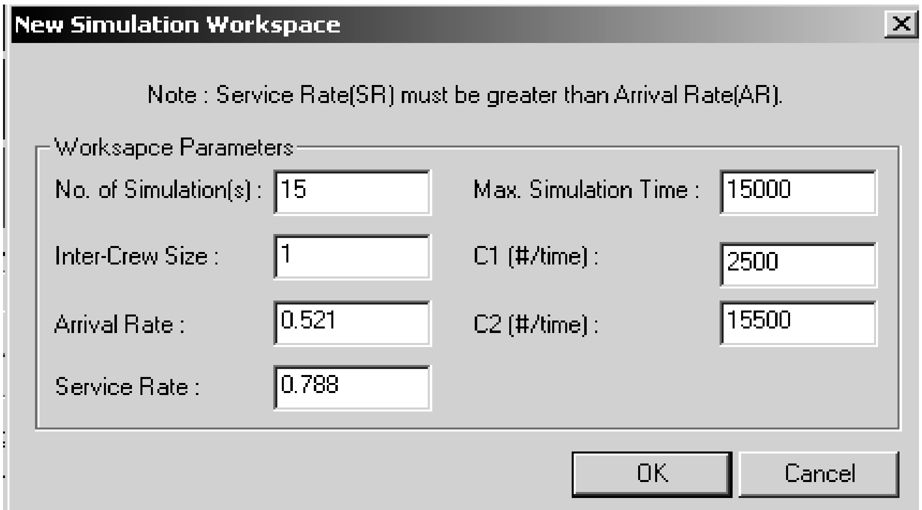


Figure 3. Simulation Input Dialogue Box

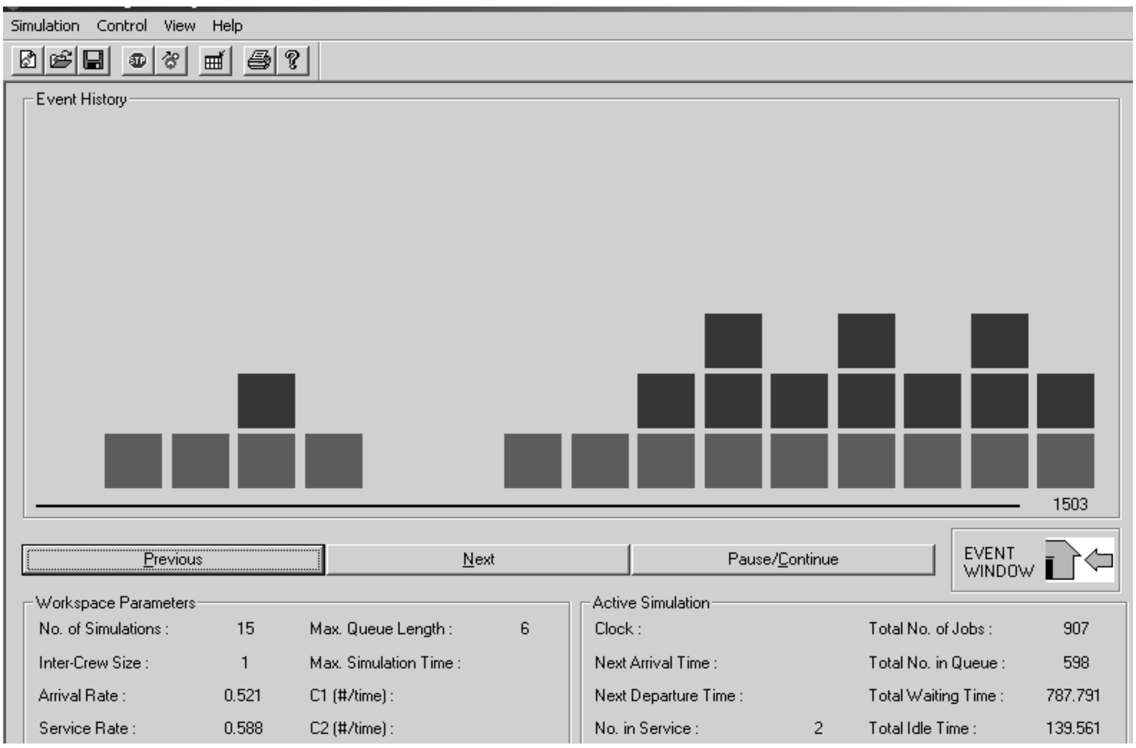
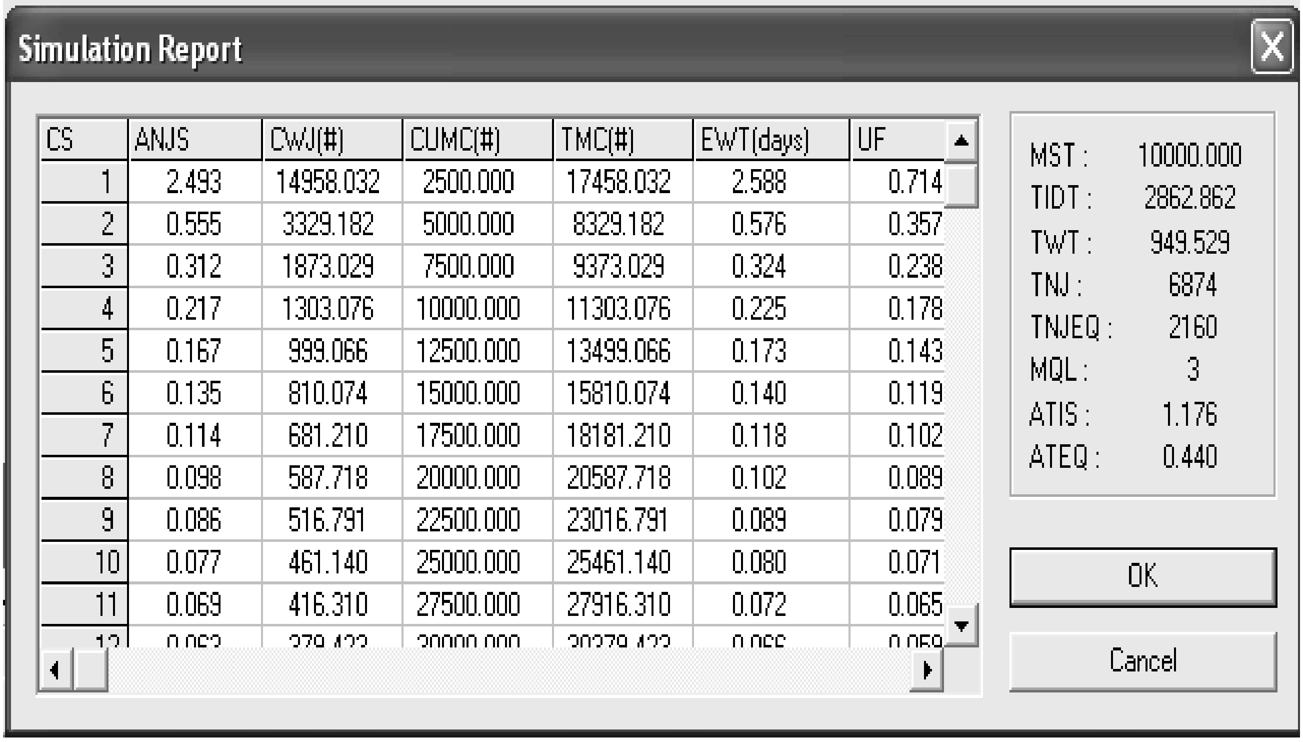


Figure 4. Simulation Window Environment



Simulation Report

CS	ANJS	CWJ(#)	CUMC(#)	TMC(#)	EWT(days)	UF
1	2.493	14958.032	2500.000	17458.032	2.588	0.714
2	0.555	3329.182	5000.000	8329.182	0.576	0.357
3	0.312	1873.029	7500.000	9373.029	0.324	0.238
4	0.217	1303.076	10000.000	11303.076	0.225	0.178
5	0.167	999.066	12500.000	13499.066	0.173	0.143
6	0.135	810.074	15000.000	15810.074	0.140	0.119
7	0.114	681.210	17500.000	18181.210	0.118	0.102
8	0.098	587.718	20000.000	20587.718	0.102	0.089
9	0.086	516.791	22500.000	23016.791	0.089	0.079
10	0.077	461.140	25000.000	25461.140	0.080	0.071
11	0.069	416.310	27500.000	27916.310	0.072	0.065
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

MST :	10000.000
TIDT :	2862.862
TWT :	949.529
TNJ :	6874
TNJEQ :	2160
MQL :	3
ATIS :	1.176
ATEQ :	0.440

OK

Cancel

Figure 5. Simulation Report Dialogue Box

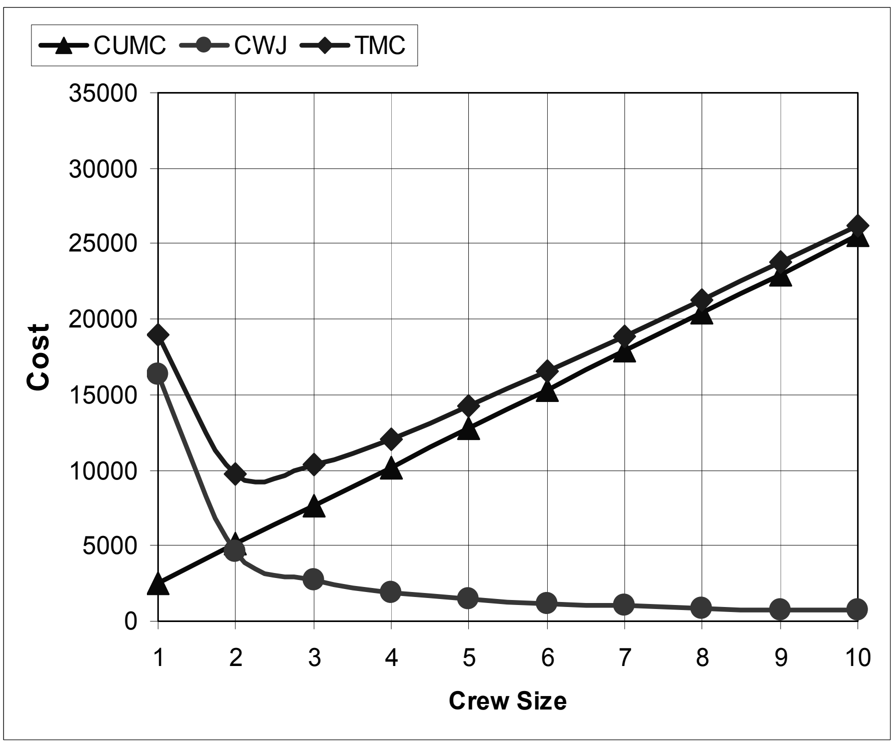


Figure 6. Composite Graph of CUMC, CWJ and TMC as CS Varies

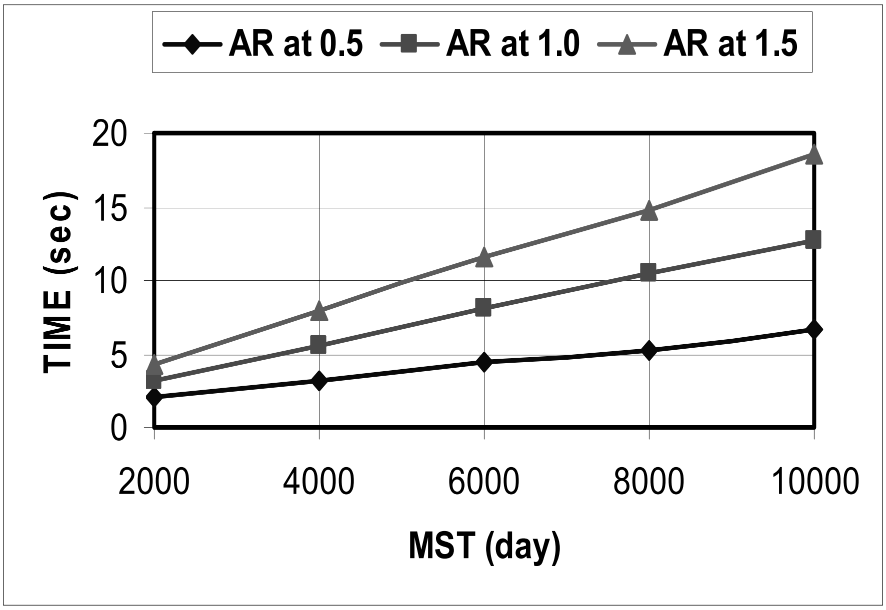


Figure 7. Model Processing Time at Varying MST and UF