Fuzzy Data Decision Support in Portfolio Selection: a Possibilistic Safety-first Model

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Abstract

Vast pools of historical financial information are available on economies, industry, and individual companies that affect investors' selection of appropriate portfolios. Fuzzy data provides a good tool to reflect investors' opinions based on this information. A possibilistic mean variance safety-first portfolio selection model is developed to support investors' decision making, to take into consideration this fuzzy information. The possibilistic-programming problem can be transformed into a linear optimal problem with an additional quadratic constraint using possibilistic theory. We propose a cutting plane algorithm to solve the programming problem. A numerical example is given to illustrate our approach.

Keywords: Portfolio selection, Fuzzy data; Safety-first, Possibility theory, Cutting plane algorithm

1. Introduction

Portfolio selection regards asset selection which maximizes an investor's return and minimizes her risk. In 1952, Markowitz (1952, 1959) published his pioneering work and laid the foundation of modern portfolio analysis. The core of the Markowitz mean variance model is to take the expected return of a portfolio as investment return and the variance of the expected return of a portfolio as investment risk. The main input data of the Markowitz mean variance model are expected returns and variance of expected returns of these securities. However, Markowitz's mean variance framework has been criticized due to several drawbacks (Korn,1997.). This framework employs the variance of the portfolio return as the only security risk measure. Controlling (Minimizing) the variance imposes bounds on both downside and upside deviation from the expected return, which may limit possible gains. A large literature on Markowitz's mean variance framework exists; see a review from Damodaran (1996) and Copeland (2000).

Another popular portfolio selection model is safety-first portfolio models originated from Roy (1952). This set of models helps investors to look at only a low-risk portfolio that offers some modest growth potential(Ortobelli, Rachev, 2001). There are also vast numbers of portfolio models using safety-first models (Copeland, Koller, Murrin, 2000). Both Markowitz's mean variance framework and safety-first portfolio models and a great deal of extensions are based on probability theory, where the theory of expected utility is usually used derived from a set of axioms concerning investor behaviour as regards the ordering relationship for deterministic and random events in the choice set. In other words, it is assumed that a probability measure can be defined on the random outcomes. However, if the origins of such random events are not well known, then the probability theory becomes inadequate because of a lack of experimental information (Wu, Zhang, Olson, 2009).

In the information age, vast pools of historical financial datas are available on the economy, industry, and individual companies which can all affect investors' selection of appropriate portfolios. In fact, investors are faced with so much possibly useful data that they find it difficult, often impossible, to process all. Their opinion of this information is often fuzzy, which motivates the utilization of fuzzy set theory in portfolio selection. It is traditionally supposed that data regarding the expected return for financial instrument is random or deterministic. However, since there is statistic error in estimating the return, the return variable might be introduced as a fuzzy number given the uncertainty inherent in financial markets. It is also apparent the fuzzy determination of financial risks is reasonable since risk concept can be very vague. Therefore, it is reasonable to solve the portfolio selection problem under assumption of fuzzy data(Olson,Wu,2006).

Since the 1960s, fuzzy set theory has been widely used to solve many problems in financial risk management. By using fuzzy approaches, experts' knowledge and investors' subjective opinions can be better integrated into a portfolio selection model. Bellman and Zadeh (1978) proposed fuzzy decision theory. Ramaswamy (1998) presented a fuzzy bond portfolio selection, where fuzzy return-risk tradeoff is analyzed for an assumed market scenario. A similar approach for portfolio selection was proposed in Leon et al(2002) by use of the fuzzy decision theory. Ostermark(1998) proposed a dynamic portfolio management model where the fuzzy decision principle can be combined. Watada(2001) presented another type of portfolio selection model based on the fuzzy decision principle. This model is directly related to the mean-variance model, where the goal rate for an expected return and the corresponding risk are described by logistic membership functions. Tanaka et al(2000) formulate fuzzy decision problems based on probability events. Carlsson et al(2002) studied a portfolio selection model in which the rate of return of securities follows the possibility distribution. Enriqueta et al(2007) presented a fuzzy downside risk approach for managing portfolio problems in the framework of risk-return trade-off using interval-valued expectations. Lacagnina and Pecorella (2006) proposed a multistage stochastic soft constraints fuzzy model to solve a portfolio management problem. Liu and Wang (1998) constructed a pair of two-level mathematical programming models, based on which the upper bound and lower bound of the objective values were obtained. Recent advances of portfolio selection model considered integration of various muti-criteria decision making models such as fuzzy AHP in Tiryaki and Ahlatcioglu (2009) and expert systems such as Smimou et al(2008). Xiaoxia(2007) gave a new defnition of risk for random fuzzy portfolio selection, Weiguo(2007) proposed two kinds of portfolio selection models based on lower and upper possibilistic means and possibilistic variances. Very limited research has been attempted on possibilistic mean-variance safety-first portfolio models.

In this paper, we assume the securities which has fuzzy rate of return and develop a possibilistic mean-variance safety-first portfolio model. Vague input data can be specified for these quantitative risk factors using historical data based on historical data quantile described in existing work such as Zmeskal(2001) and Wu et al(2009). Using the possibilistic means and variances, the possibilistic programming problem can be transformed into a linear optimal problem with an additional quadratic constraint by possibilistic theory. For such problem there are no special standard algorithms, we propose a cutting plane algorithm to solve them.

Using our proposed approach, fuzzy variance and covariance are derived directly from fuzzy numbers, which are different from the probability theory where variance and covariance are derived from a great deal of historical data such as Markowitz's mean variance framework and the safety-first portfolio model. This will, on the one hand, reduce the computation complexity and on the other hand, overcome the hurdle of semi-positive covariance matrix as required in a great deal of portfolio models based on the probability theory.

The rest of the paper is organized as follows. In Section 2, we briefly introduce possibilistic mean variance approach and possibilistic mean safety-first approach and present a possibilistic mean variance portfolio selection model with safety-first. In Section 3 we present the proposed possibilistic mean variance safety-first portfolio selection model and the solution approach. In Section 4, a example is given to illustrate the proposed model. The last section concludes the paper.

2. Mean variance portfolio selection model with safety-first

The expected losses, conditional on the states where there are large losses, may be higher sometimes. The mean-variance approach encourages risk diversification, but the mean safety-first approach discourages risk diversification sometimes. The mean-variance approach not only controls the downside risk of security return, but also bounds the possible upside gains. In contrast, the mean safety-first approach only controls the downside risk of security return. Another limitation of both approaches is that the underlying distribution of the rate of return is not well understood, and there are no higher degree information is utilized except means, covariances (variances), so we propose the following mean variance safety-first portfolio selection model:

$$
(MVSF)\quad \begin{cases} \max f(x) = \sum_{j=1}^{n} r_j x_j \\ s.t.\quad \mathrm{P}(\sum_{j=1}^{n} r_j x_j \le \mathrm{V}) \le \beta \\ \qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \le w \\ \qquad \sum_{i=1}^{n} x_i = 1 \\ \qquad 0 \le x_i \le \alpha_i (i = 1, 2, \ldots, n) \end{cases}
$$

3. Possibilistic Mean Variance safety-first Portfolio Selection Model

3.1 Possibility theory

Possibility theory was proposed by Zadeh(1978) and advanced by Dubois and Prade (1998) where fuzzy variables are associated with possibility distributions in a similar way that random variables are associated with probability distributions in probability theory. The possibility distribution function of a fuzzy variable is usually defined by the membership function of the corresponding fuzzy set. We call a fuzzy number $\tilde{a}$ of any fuzzy subset R with membership function $\mu_a : R \to [0,1]$. Let $\tilde{a}, \tilde{b}$ be two fuzzy numbers with membership function $\mu_a(x), \mu_b(x)$, respectively. Based on the concepts and techniques of possibility theory founded by Zadeh(1978), we consider in this paper the trapezoidal fuzzy numbers which are fully determined by quadruples $\tilde{r} = (r_1, r_2, r_3, r_4)$ of crisp numbers such that $r_1 \le r_2 \le r_3 \le r_4$. Their membership functions can be denoted by:

$$\mu(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1}, & \mathrm{r}_1 \le x < r_2 \\ 1, & \mathrm{r}_2 \le x \le r_3 \\ \dfrac{x - r_4}{r_3 - r_4}, & \mathrm{r}_3 \le x \le r_4 \\ 0, & orthewise \end{cases}$$

We note that the trapezoidal fuzzy number is a triangular fuzzy number if $r_2 = r_3$.

3.1.1 Fuzzy numbers and operation.

The sum of two trapezoidal fuzzy numbers is also a trapezoidal fuzzy number, and the product of a trapezoidal fuzzy number and a scalar number is also a trapezoidal fuzzy number. The sum of $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ is defined as $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$. We also have

$$k\tilde{a} = \begin{cases} (ka_1, ka_2, ka_3, ka_4), k > 0 \\ (ka_4, ka_3, ka_2, ka_1), k < 0 \end{cases}.$$

Now let us consider two trapezoidal fuzzy numbers $\tilde{r} = (r_1, r_2, r_3, r_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$, then the possibility value of the first trapezoidal fuzzy number being no larger than the second is defined as (Liu, Iwamura,1998):

$$Pos\{\tilde{r} \le \tilde{b}\} = \begin{cases} 1, & \mathrm{r}_2 \le b_3; \\ \dfrac{b_4 - r_1}{b_4 - b_3 + r_2 - r_1}, & \mathrm{r}_2 \ge b_3, r_1 \le b_4; \\ 0, & \mathrm{r}_1 \le b_4. \end{cases}$$

Specifically, when $\tilde{b}$ takes a crisp value of 0, the definition is is simplified as

$$Pos\{\tilde{r} \le 0\} = \begin{cases} 1, & \mathrm{r}_2 \le 0; \\ \dfrac{r_1}{r_1 - r_2}, & \mathrm{r}_1 \le 0 \le r_2; \\ 0, & \mathrm{r}_1 \ge 0. \end{cases}$$

The following lemma holds:

Theorem 1. Assume the trapezoidal fuzzy number $\tilde{r} = (r_1, r_2, r_3, r_4)$, then for any given confidence level $\beta(0 \le \beta \le 1)$, $\mathrm{Pos}\{\tilde{\mathrm{r}} \le 0\} \le \beta$ if and only if $(1 - \beta)\mathrm{r}_1 + \beta r_2 \ge 0$.

Proof: If $\mathrm{Pos}\{\tilde{\mathrm{r}} \le 0\} \le \beta$ then we have either $\mathrm{r}_1 \ge 0$ or $\dfrac{r_1}{r_1 - r_2} \le \beta$. If $\mathrm{r}_1 \ge 0$ then $r_2 > r_1 \ge 0$, so we have $(1-\beta)r_1+\beta r_2 \geq 0$; if $\frac{r_1}{r_1-r_2} \leq \beta$ then $r_1 \geq \beta(r_1-r_2)$ by the fact that $r_1 < r_2$. Hence we have $(1-\beta)r_1+\beta r_2 \geq 0$ for all cases.

If $(1-\beta)r_1+r_2 \geq 0$, the argument breaks down into two cases when $r_1 \geq 0$, we have $\mathrm{Pos}\{\tilde{r} \leq 0\}=0$ which implies that $\mathrm{Pos}\{\tilde{r} \leq 0\} \leq \beta$,when $r_1 < 0$, we have $r_1 - r_2 < 0$. We can rearrange $(1-\beta)r_1+\beta r_2 \geq 0$ as $\frac{r_1}{r_1-r_2} \leq \beta$, i.e., $\mathrm{Pos}\{\tilde{r} \leq 0\} \leq \beta$.

3.1.2 Possibilistic mean value and variance

The α-level set of a fuzzy number $\tilde{r}=(r_1,r_2,r_3,r_4)$ is a crisp subset of R and is denoted by $[\tilde{r}]^\alpha = \{x \mid \mu(x) \geq \alpha, x \in R\}$. For the trapezoidal fuzzy number,

$$[\tilde{r}]^\alpha = \{x \mid \mu(x) \geq \alpha, x \in R\} = [r_1+\alpha(r_2-r_1), r_4-\alpha(r_4-r_3)].$$

Carlsson et al(2001) introduced the notation of crisp possibilitic mean value of continuous possibility distributions, which are consistent with the extension principle. Let $[\tilde{r}]^\alpha = [a_1(\alpha), a_2(\alpha)]$, then the crisp possibilistic mean value of $\tilde{r}=(r_1,r_2,r_3,r_4)$ is computed as

$$E(\tilde{r}) = \int_0^1 \alpha(a_1(\alpha)+a_2(\alpha))d\alpha.$$

It is easy to see that if $\tilde{r}=(r_1,r_2,r_3,r_4)$ is a trapezoidal fuzzy number then

$$E(\tilde{r}) = \int_0^1 \alpha(r_1+\alpha(r_2-r_1)+r_4-\alpha(r_4-r_3))d\alpha = \frac{r_2+r_3}{3}+\frac{r_1+r_4}{6},$$

giving the crisp possibilistic variance value of $\tilde{r}=(r_1,r_2,r_3,r_4)$ as $\sigma(\tilde{r}) = \frac{1}{2}\int_0^1 \alpha(a_2(\alpha)-a_1(\alpha))^2 d\alpha$.

Then the crisp possibilistic covariance value of $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$ can be computed as $\mathrm{cov}(\tilde{a},\tilde{b}) = \frac{1}{2}\int_0^1 \alpha(a_2(\alpha)-a_1(\alpha))(b_2(\alpha)-b_1(\alpha))d\alpha$. Based on this, it is easy to see that if $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers, then we have:

$$\mathrm{cov}(\tilde{a},\tilde{b}) = \frac{(a_3-a_2)(b_3-b_2)}{8}+\frac{(a_4-a_1)(b_4-b_1)}{24}+\frac{(a_3-a_2)(b_4-b_1)}{24}+\frac{(a_4-a_1)(b_3-b_2)}{24},$$

and

$$\sigma(\tilde{a}) = \frac{(a_3-a_2)^2}{8}+\frac{(a_4-a_1)^2}{24}+\frac{(a_3-a_2)(a_4-a_1)}{12}.$$

3.2 Model formulation

In standard portfolio models uncertainty is handled in the form of randomness using probability theory. One of the main difference between the possibility and probability measures is that probability is additive whereas possibility is subadditive, which means for the possibility measure that the possibility of an event being partitioned into smaller events, is less than or equal to the sum of the possibilities of the smaller events. The subadditive property of the possibility measure fits the requirements of risk metrics in financial theory. Moreover, in probability, estimation of prior probability distributions of parameters such as mean and variance are usually obtained from judgment. Determination of the probability distribution of the parameters is difficult(Leon,Liern, Vercher,2002). Using probability theory can hardly account for the uncertainty in the probability distribution of the uncertain variables. In contrast, measurement of the uncertainty in a possibilistic model can be done by the sum of the possibilities of an event and its complement minus one.

So we will assume that the rates of return on assets are modeled by possibility distributions rather than probability distributions. Applying possibilistic distribution may have two-fold advantages (Inuiguchi,1992): first, the knowledge of the expert can easily be introduced to the estimation of the return rates; Second, the reduced problem is more tractable than the result of the stochastic programming approach. The rate of return on the j[th] asset will be represented by a fuzzy number $\tilde{r}_j$ in our method, and we will consider only trapezoidal possibility distributions for simplicity. In addition, we denote the disaster level by the trapezoidal fuzzy number $\tilde{b}=(b_1,b_2,b_3,b_4)$. Thus we use the shortfall possibility constraint instead of the shortfall probability constraint and formulate our possibilistic mean variance safety-first portfolio selection model as follows.

$$(FMVSF)\quad \begin{cases} \max f(x)=\sum_{j=1}^{n}\tilde{r}_j x_j \\ s.t.\quad \mathrm{P}(\sum_{j=1}^{n}\tilde{r}_j x_j \le \tilde{V}) \le \beta \\ \sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{j=1}^{n}\tilde{\sigma}_{ij}x_i x_j \le w \\ \sum_{i=1}^{n}x_i = 1 \\ 0 \le x_i \le \alpha_i (i=1,2,\ldots,n) \end{cases}$$

Theorem 1 provides a simplified deterministic form of Model (FMVSF).

Theorem 2: Solving (FMVSF) is equivalent to solving the following problem:

$$(FMVSF)\quad \begin{cases} \max f(x)=\dfrac{\sum_{j=1}^{n}r_{j2}x_j+\sum_{j=1}^{n}r_{j3}x_j}{3}+\dfrac{\sum_{j=1}^{n}r_{j1}x_{j1}+\sum_{j=1}^{n}r_{j4}x_j}{6} \\ s.t.\quad (1-\gamma)(\sum_{j=1}^{n}r_{j1}x_j - v_4)+\gamma(\sum_{j=1}^{n}r_{j2}x_j - v_3)\ge 0 \\ \sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{j=1}^{n}(\dfrac{(r_{i3}-r_{i2})(r_{j3}-r_{j2})}{8}+\dfrac{(r_{i4}-r_{i1})(r_{j4}-r_{j1})}{24}+\dfrac{(r_{i3}-r_{i2})(r_{j4}-r_{j1})}{24}+\dfrac{(r_{i4}-r_{i1})(r_{j3}-r_{j2})}{24})x_i x_j \le w \\ \sum_{j=1}^{n}x_j = 1 \\ 0 \le x_j \le \alpha_j (j=1,2,\ldots,n) \end{cases}$$

Proof: From 3.1.2, we have

$$E(\tilde{r}x_j)=\frac{\sum_{j=1}^{n}r_{j2}x_j+\sum_{j=1}^{n}r_{j3}x_j}{3}+\frac{\sum_{j=1}^{n}r_{j1}x_{j1}+\sum_{j=1}^{n}r_{j4}x_j}{6}$$

From **Theorem 1**, we can get that

$\mathbf{Pos}(\sum_{j=1}^{n}\tilde{r}_j x_j < \tilde{V}) \le \boldsymbol{\beta}$ is equivalent to $(1-\beta)(\sum_{j=1}^{n}r_{j1}x_j - v_4)+\beta(\sum_{j=1}^{n}r_{j2}x_j - v_3)\ge 0$, which completes the proof.

Remark: An investor yields his optimal portfolio by giving the value of $\tilde{V}$, γ, w and solving the resulting model (FMVSF).

3.3 Cutting plane algorithm

Problem (FMVSF) is a linear optimal problem with an additional quadratic constraint. For such problems there are no special standard algorithms. Of course, one could treat this problem with general methods of nonlinear optimization, but this would lead to local solutions. In this paper, we propose to solve Problem (FMVSF) using a cutting plane algorithm, which was first introduced by Kelley (1960) and Cheney and Goldstein(1959) for solving convex programming problems.

Let

$$g(x) = w - \sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\frac{(r_{i3}-r_{i2})(r_{j3}-r_{j2})}{8} + \frac{(r_{i4}-r_{i1})(r_{j4}-r_{j1})}{24} + \frac{(r_{i3}-r_{i2})(r_{j4}-r_{j1})}{24} + \frac{(r_{i4}-r_{i1})(r_{j3}-r_{j2})}{24}\Big)x_i x_j$$

where g(x) is a concave function on R^n. Let $G = \{x : g(x) \ge 0\}$ and

$T = \{x : \sum_{j=1}^{n} x_j = 1, (1-\beta)(\sum_{j=1}^{n} r_{j1}x_j - v_4) + \beta(\sum_{j=1}^{n} r_{j2}x_j - v_3) \ge 0, 0 \le x_j \le \alpha_j (j = 1,2,\ldots,n)\}$, the cutting plane algorithm proceeds from Step 1 to 3.

Step 1. Solve the linear programs:

$$(FMVSF0)\begin{cases} \max f(x) = \dfrac{\sum_{j=1}^{n} r_{j2}x_j + \sum_{j=1}^{n} r_{j3}x_j}{3} + \dfrac{\sum_{j=1}^{n} r_{j1}x_{j1} + \sum_{j=1}^{n} r_{j4}x_j}{6} \\ x \in T \end{cases}$$

Let x^0 be the optimal solution of (FMVSF0). If x^0 is contained in the set $G = \{x : x \in T, g(x) \ge 0\}$, an optimum of (FMVSF) has been achieved and stop. Otherwise let $k = 0$ and go to Step 2.

Step 2. Solve the linear programs:

$$(FMVSFk)\begin{cases} \max f(x) = \dfrac{\sum_{j=1}^{n} r_{j2}x_j + \sum_{j=1}^{n} r_{j3}x_j}{3} + \dfrac{\sum_{j=1}^{n} r_{j1}x_{j1} + \sum_{j=1}^{n} r_{j4}x_j}{6} \\ s.t \quad \tilde{g}(x, x^k) = g(x^k) + \Delta g(x^k)^T (x - x^k) \ge 0 \\ x \in T \end{cases}$$

Step 3. Let x^{k+1} be the optimal solution of the preceding linear program. If $x^{k+1} \in G$, stop. Otherwise, set $k = k+1$ and return to step 2.

Let us state and prove the convergence of the cutting plane algorithm. Denoted by S_k the feasible set of linear program solved in step 2 of Iteration k. These sets are nested, i.e., $S_k \subset S_{k-1} \subset \cdots \subset S_0$

Theorem 3 Let g be closed concave function on the compact convex set $T \subset R^n$ such that at every point $x \in T$, the sets of subgradient $\Delta g(x)$ are nonempty and there exists a K such that

$$\sup\{\|\xi\| : \xi \in \Delta g(x), x \in T\} \le K,$$

Further assume that G the feasible set of (FMVSF), is nonempty and contained in T. let

$$S_k = S_{k-1} \cap \{x : \tilde{g}(x, x^k) \ge 0\}$$

Where $S_0 = T$. If $x^{k+1} \in S_k$ is such that $f(x^{k+1}) = \max\{c^T x : x \in S_k\}$ then the sequence $\{x^k\}$ contains a subsequence that converges to an optimal solution of (FMVSF).

Proof: First we observe from $S_k \subset S_{k-1} \subset \cdots \subset S_0$ that $\{f(x^k)\}$ is monotonically decreasing. Hence if $\{x^k\}$ contains a subsequence that converges to a point $x^* \in G$, then $\{f(x^k)\}$ converges to $\{f(x^*)\}$ and $x^* \in G$ solve (FMVSF). Suppose now that x^k does not have a subsequence converging to a point in G. Then there exists an $\alpha > 0$ such that $g(x^k) \le -\alpha$. If x^{k+1} maximizing $c^T x$ on S_k, then $x^{k+1} \in T$ and

$g(x^h) + g(x^k)^T (x^{k+1} - x^h) \ge 0, h = 0,1,\cdots,k.$

From the last two relations and the Schwarz inequality, it follows that

$$\alpha \le -g(x^h) \le g(x^h)^T (x^{k+1} - x^h) \le K \left\| x^{k+1} - x^h \right\|.$$

Hence for every subsequence $\{k_p\}$ of indices we have $\left\| x^{k_p} - x^{k_q} \right\| \ge \frac{\alpha}{K}, q < p.$ This means $\{x^k\}$ does not have a Cauchy subsequence, which contradicts that $\{x^k\} \subset T$ is bounded.

4. Numerical example

In this section,we present a numerical examples to demonstrate our proposed approach: for a 3-security practical problem which allows us to show a step-by-step computation using the proposed approach.

We first consider a market risk manager's decision of choosing 3-securities: IBM, GE and MSFT. The manager structuring an equity portfolio only has vague views regarding equity return scenarios described as "bullish", "bearish" or "neutral". The manager forms such views as a result of the subjective or intuitive opinion of the decision-maker on the basis of information available at a given point in time. It is recognized that a fuzzy set can be used to characterize the range of acceptable solutions to the portfolio selection problem under this circumstance.

The manager may specify the following possibility distribution for expected rates:

$$\tilde{r}_1 = (0.12, 0.15, 0.21, 0.24), \tilde{r}_2 = (0.12, 0.16, 0.22, 0.26), \tilde{r}_3 = (0.20, 0.28, 0.38, 0.40)$$

The above trapezoidal fuzzy data can also be yielded by fuzzifying historical stochastic data. The approach for stating vague input data using historical data is similar to an interesting and practically applicable method based on historical data quantile employed in Zmeskal(2001) and Wu et al(2009). For parameters such as return, risks, and skewness are derived from standard error of historical values and normal density function of error is assumed as an approximation. For example, based on probability theory, suppose the mean and standard error (S.E.) of the first security are $\mu = 18\%$ and $\sigma = 3\%$. The following formula is used to transform this historical stochastic data into Trapezoidal fuzzy data: $a_1 = \mu - 2\sigma, a_2 = \mu - \sigma, a_3 = \mu + \sigma$, and $a_4 = \mu + 2\sigma$, where μ and σ are mean and S.E. standard error of related historical data. The deviation of one times S.E. for a_2 and a_3 corresponds to 34.1% quantile and 2 times S.E. for a_1 and a_4 corresponds to 47.7% quantile. Results are generated in 3 seconds using Matlab software.

The covariance matrix of three securities are calculated as:

$$\begin{pmatrix} 0.0017 & 0.0024 & 0.0037 \\ 0.0024 & 0.002 & 0.0041 \\ 0.0037 & 0.0041 & 0.0046 \end{pmatrix}$$

Here, variance and co-variance are derived directly from fuzzy numbers, which is different from the probability theory where variance and co-variance are derived from a great deal of historical data such as Markowitz's mean variance framework and the safety-first portfolio model. Therefore, comparing to existing research based on probability theory, computation complexity is reduced. Moreover, the problem of semi-positive co-variance matrix is handled.

We continue to our computation and let $\alpha_j = 0.5, \gamma = 0.01, \tilde{b} = (0.11, 0.115, 0.12, 0.123)$, $w = 0.0025$ .Substituting parameter values into (FMVSF) we obtain:

$$\begin{aligned} &\max f(x) = 0.18x_1 + 0.19x_2 + 0.32x_3 \\ &s.t\ 0.0017x_1^2 + 0.0048x_1x_2 + 0.0074x_1x_3 + 0.002x_2^2 + 0.0082x_2x_3 + 0.0047x_3^2 \le 0.0025 \\ &\quad 0.1203x_1 + 0.1204x_2 + 0.2008x_3 - 0.123 \ge 0 \\ &\quad x_1 + x_2 + x_3 = 1 \\ &\quad 0 \le x_j \le 0.5, (j = 1, 2, 3). \end{aligned}$$

Denote by

$$T=\{x: x\in R^3, \sum_{j=1}^{3} x_j =1, 0.1203x_1+0.1204x_2+0.2008x_3-0.123\geq 0, 0\leq x_j\leq 0.5, (j=1,2,3)\}$$

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$$g(x)=0.0025-0.0017x_1^2-0.0048x_1x_2-0.0074x_1x_3-0.002x_2^2-0.0082x_2x_3-0.0047x_3^2$$

Now we apply the cutting plane algorithm in Section 3.3. We first solve the linear program of maximizing $f(x)$ subject to $x\in T$. The optimal solution is $x^0=(0,0.5,0.5)$, where $f(x^0)=0.25165, g(x^0)=-0.0012<0$. We then construct the linear constraint

$g(x^0)+g(x^0)^T(x-x^0)\geq 0$,or $-0.0061x_1-0.0061x_2-0.0088x_3+000625\geq 0$

Now we solve the linear program:

$$\begin{aligned}
&\max f(x)=0.18x_1+0.19x_2+0.32x_3\\
&s.t\ -0.0061x_1-0.0061x_2-0.0088x_3+000625\geq 0\\
&\quad 0.1203x_1+0.1204x_2+0.2008x_3-0.123\geq 0\\
&\quad x_1+x_2+x_3=1\\
&\quad 0\leq x_j\leq 0.5, (j=1,2,3).
\end{aligned}$$

which leads to the optimal solution: $x^1=(0.4444444, 0.5, 00555556)$. Therefore, the return of the portfolio is 0.1927778, the risk(variance) of the portfolio is 0.00232747.

5. Conclusion

In this paper, we consider trapezoidal possibility distributions as possibility distribution of the rates of return on the securities, and propose a possibilistic mean variance safety-first portfolio selection model. The possibilistic mean variance safety-first portfolio selection model can be transformed into a nonlinear programming problem based on possibilistic theory. Using this approach, fuzzy variance and co-variance are derived directly from fuzzy numbers, which is different from the probability theory where variance and co-variance are derived from a great deal

of historical data such as Markowitz's mean variance framework and the safety-first portfolio model. This, on the one hand, reduces the computation complexity and on the other hand, overcomes the hurdle of semi-positive co-variance matrix as required in classical portfolio models based on the probability theory.

We have developed a cutting plane algorithm to solve the proposed fuzzy portfolio selection model. Two numerical examples are given to illustrate the proposed method. From the computation, we know that to obtain greater expected fuzzy returns on investments, one must be willing to take on greater risk. Interesting finding is regarding the change of the investor's strategy. As the investor's risk tolerance increases, the portfolio strategy changes by including more risky securities and exclude less risky securities. As a result, the investor's return increases.

Limitations of the proposed approach should be identified. First, the model did not consider dynamic situations, which means one further research can be an extension of the model to fuzzy dynamic portfolio selection and corresponding algorithms. Second, large computation based on real data should be explored by use of the proposed approach. This may allow us to test the computational advantage of our approach. Based on this study, techniques such as fuzzy data mining and fuzzy decision support system can be further developed for predicting the security market uncertainty to help improve investors' returns.

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