



The Research of the Model of Communication System Depending on the Queuing Theory

Min Zhao, Jiyin Sun & Xiaochun Che
Xi'an 710025, China

Abstract

Queuing theory based on the actual situation in the communication system must be defined within the mathematical model of the building and analysis. To shorten the time for communication time as the target of communications equipment, the average rate services to scientific and reasonable request is analyzed in-depth in the paper.

Keywords: Queuing theory, Communication system, Mathematical model

1. Foreword

Under war circumstances, communications between equipments are quite different from usual circumstances, most of the equipments in the battlefields are high-tech electronic devices ,and has much high sensitivity. The research of the mathematical model of communications system is carried out to reasonably deploy communication resources, effectively and swiftly communicate, improve the capability, which enables us to make the best use of communication system at war.

Communication system is a big complex system, which consists of equipments, persons, information and some other factors. In an effort to make the best use of the system and fulfill the mission in the shortest time with limited resources, the mathematical model of the system is defined to adjust the factors that can affect average service rate such as equipments and information. Based on queuing theory, a research on the modeling and analysis of communication system is given in this paper.

2. Proposition of the problem

The communication procedure can be divided into three sub-processes. The process of sending out information is illustrated as an example in this paper, which is shown below.

This system can be concluded into a problem of queuing that consists of both sub-systems in series and sub-system in parallel serves the customers with information. According to this, the following objectives are supposed to achieve as follows

1) Allocating proper communication resources to adjust the whole communication time in order to fulfill certain communication efficiency, in other words, make the whole time within the time limit which is requested by war.

2) Allocating the communication resources properly to coordinate different sub-systems in order to make them work in certain intensity to improve system efficiency.

3. Mathematical model and Analysis

3.1 Assumptions

Based on Queuing theory, the mathematical model of the communication system is analyzed and the assumptions are made as follows:

- (1) All the input and output.
- (2) Equipments works on FCFS principle (First come, first serve).
- (3) All the equipments are in good shape.

3.2 Building of the model

In order to guarantee the consistence in symbols in the following passage, some regulations is made as follows,

λ_i =average information that system receives per time span;

μ_i = average serving rate of i ;

ρ_i = average serving intensity of equipment i ;

It is mainly concerned about two parameters of the system as follows:

- 1) W_{si} , average lingering time, which is the expected value of working time of equipments i .
- 2) W_{qi} , average waiting time, which is the expected value of waiting time of information at equipment i .

3.2.1 Building of the model of sub-systems

Seen from the Diagram 1, the system has both sub-system in series and sub-system in parallel, so it is complicated. But the system can be converted into an equivalent one which consists of only sub-systems in series by converging a_1, a_2 into sub-system1, and converging b_1, b_2, b_3 into sub-system2. As we can see below,

So the system can be regarded that consists of two M/M/1system in series, the I/O distribution and the time that information is served are listed below

$$P_n(t) = \frac{(\lambda_1 t)^n}{n!} e^{-\lambda_1 t}, \quad F_{V_i}(t) = \mu_1 e^{-\mu_1 t} \quad (t \geq 0)$$

In above formula, random variable V_i stands for the time that information is served($i=1,2,\dots$). Based on above conclusions, the serving intensity of exchange equipment is listed $\rho_1 = \frac{\lambda_1}{\mu_1}$.

Meanwhile, average lingering time and average waiting time are concluded

$$W_{s1} = \frac{1}{\mu_1 - \lambda_1}, \quad W_{q1} = \frac{\lambda_1}{\mu_1(\mu_1 - \lambda_1)}.$$

After analyzing and inducing, it is known that I/O of customers of transmitting equipments comply with Poisson distribution, and the time that information arrives complies with minus exponential distribution. So the average lingering time and average waiting time are correspondingly listed below,

$$W_{s2} = \frac{1}{\mu_2 - \lambda_2}, \quad W_{q2} = \frac{\lambda_2}{\mu_2(\mu_2 - \lambda_2)}$$

Based on Diagram2, the exchange equipment can be divided into three parts below,

As these two different equipments in exchange equipments are dependent to each other, information is allocated to two devices a_1, a_2 in proportion. For the equipments deal with the information randomly, the set of all the input incidence of the two equipments is $\Omega = \{A_1, A_2\}$, $\sum_{i=1}^2 A_i = \Omega$, the probability formula is listed $\sum_{i=1}^2 P(A_i) = 1$.

As information is allocated to exchange devices at random in line with probability distribution, it can be considered as two M/M/1-type serving system with different arrival rate of information. If the ratio of two equipments is supposed to $l_1 : l_2$, there are

$$\lambda_1 = \sum_{i=1}^2 \lambda_{1i} = \sum_{i=1}^2 P(A_i) \lambda_1, \quad \mu_1 = \sum_{i=1}^2 \mu_{1i} = \sum_{i=1}^2 l_i \mu_1 \quad (\lambda_{1i} = P(A_i) \lambda_1, \mu_{1i} = l_i \mu_1)$$

Provided that the system is constantly stable, the formula of average lingering time and average waiting time are concluded

$$W_{s1i} = \frac{1}{l_i \mu_1 - P(A_i) \lambda_1}, \quad W_{q1i} = \frac{P(A_i) \lambda_1}{l_i \mu_1 (l_i \mu_1 - P(A_i) \lambda_1)}, \quad (i = 1, 2).$$

The input incidence is $\Omega = \{B_1, B_2, B_3\}$, $\sum_{j=1}^3 B_j = \Omega$.

The probability formula is $\sum_{j=1}^3 P(B_j) = 1$.

If the serving rate ratio of the two equipments is defined as $k_1 : k_2 : k_3$, average lingering time and average waiting time of information are known as follows

$$W_{s2j} = \frac{1}{k_j \mu_2 - P(B_j) \lambda_2}, \quad W_{q2j} = \frac{P(B_j) \lambda_2}{k_j \mu_2 (k_j \mu_2 - P(B_j) \lambda_2)}, \quad (j = 1, 2, 3).$$

3.2.2 Modeling of the system and solution

Under specific circumstance, it is only need to know average lingering time and average waiting time of information and it's minimum. Based on the above conclusion, it is known that

$$W_{total(s)} = \max\{W_{s1i}\} + \max\{W_{s2j}\}, \quad W_{total(q)} = \max\{W_{q1i}\} + \max\{W_{q2j}\}$$

If $\mu_1 = \lambda_2$, it is known that

$$\left\{ \begin{array}{l} \min W_{total(s)} = \max\left\{\frac{1}{l_i \mu_1 - P(A_i) \lambda_1}\right\} + \max\left\{\frac{1}{k_j \mu_2 - P(B_j) \mu_1}\right\} \\ s.t. \sum_{i=1}^2 P(A_i) = 1, \sum_{j=1}^3 P(B_j) = 1, \sum_{i=1}^2 l_i = 1, \sum_{j=1}^3 k_j = 1 \\ l_i, k_j \geq 0, i = 1, 2; j = 1, 2, 3 \end{array} \right.$$

$$\left\{ \begin{array}{l} \min W_{total(q)} = \max\left\{\frac{P(A_i) \lambda_1}{l_i \mu_1 (l_i \mu_1 - P(A_i) \lambda_1)}\right\} + \max\left\{\frac{P(B_j) \mu_1}{k_j \mu_2 (k_j \mu_2 - P(B_j) \mu_1)}\right\} \\ s.t. \sum_{i=1}^2 P(A_i) = 1, \sum_{j=1}^3 P(B_j) = 1, \sum_{i=1}^2 l_i = 1, \sum_{j=1}^3 k_j = 1 \\ l_i, k_j \geq 0, i = 1, 2; j = 1, 2, 3 \end{array} \right.$$

$P(A_1) = P(A_2) = \frac{1}{2}, P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$, and when $l_1 = l_2 = \frac{1}{2}, k_1 = k_2 = k_3 = \frac{1}{3}$, the minimum of $W_{total(s)}$ and $W_{total(q)}$ are below

$$W_{total(s)} = \frac{2}{\mu_1 - \lambda_1} + \frac{3}{\mu_2 - \mu_1}, \quad W_{total(q)} = \frac{2\lambda_1}{\mu_1(\mu_1 - \lambda_1)} + \frac{3\mu_1}{\mu_2(\mu_2 - \mu_1)}.$$

3.3 Analysis of the model

The optimum can be gain by adjusting System 2 to make sub-systems' serving rate μ_1, μ_2, μ_3 equal.

According to the solution, the communication systems and the whole communication time can be further analyzed.

$$W_{total(s)} = \frac{2}{\mu_1 - \lambda_1} + \frac{3}{\mu_2 - \mu_1}, \quad W_{total(q)} = \frac{2\lambda_1}{\mu_1(\mu_1 - \lambda_1)} + \frac{3\mu_1}{\mu_2(\mu_2 - \mu_1)}$$

Based on the above conclusions, it is known that when λ_1 is fixed the serving rate of exchange devices and transmitting devices μ_1, μ_2 can be adjusted in order to make $W_{total(s)}$ and $W_{total(q)}$ minimized and reasonable, which also help us to minimize the whole communication time and get an optimum.

4. Conclusion

Through building and analyzing of the model, the efficiency of the system can be improved and the time can be reduced by adjusting the serving rate of the equipments. Besides, the model helps the commanders who organize the communication with the decision. What's more, it is still need some improvement on the model, as there are more things to do when a more complicated situation and the details are discussed.

References

Lu, Chuanfen, (1994). "Queuing Theory", Beijing: Beijing Youdian University Press.
 Qi, Shengli, Wu Chang, Yang Yi, "Based on Queuing Theory of Battlefield Equipment Repair System Modeling Analysis".
 Zhu, Detong, (2003). "Optimization Model and Experimental", Shanghai: Tongji University Press.

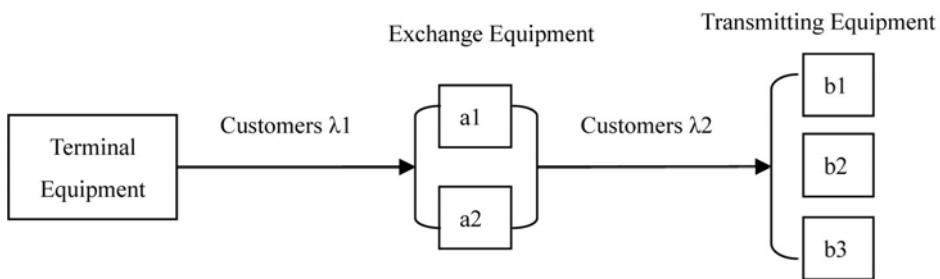


Figure 1. Communication procedure

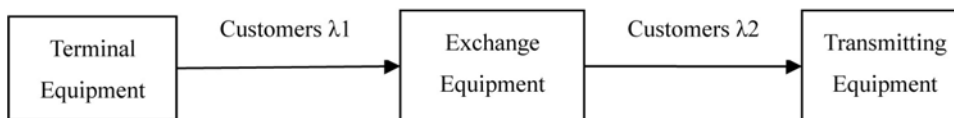


Figure 2. Serial communication system procedure

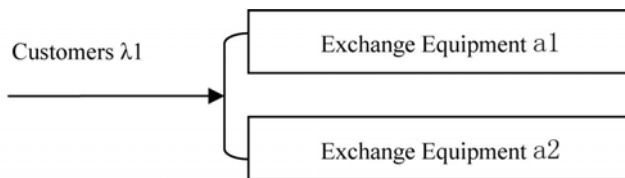


Figure 3. Parallel communication system procedure