

Teaching of Symmetry at Mathematics Lessons in the First Forms of Azerbaijan Primary Schools

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Abstract

One of the effective ways of teaching of symmetry at the Mathematics lessons in the first forms of primary schools of the Republic of Azerbaijan on the basis of National Curriculum is discussed in the paper. The effective teaching of a symmetry conception enables also the pupils to study and understand Geometry, Physics, Chemistry, Music, Geography and other subjects deeply in the future and it lays down the foundations of preparation of future mathematicians, physicists, chemists, engineers, architects, musicians, artist etc. Taking into account the importance and universality of the concept of symmetry it is recommended to increase the number of teaching hours on “Symmetry” in the first forms of primary schools.

Keywords: Symmetry, First forms, Primary schools, Mathematics lessons, Symmetric objects, Symmetric geometric figures

1. Introduction

In 2007 “The General Educational Conception (National Curriculum) in the Republic of Azerbaijan” was confirmed by the Ministry of Education of the Republic of Azerbaijan (Ministry of Education, 2007). One of the structural parts of this document is called “The Mathematics Curriculum for the general educational schools of the Republic of Azerbaijan (the 1st – 4th forms)” (Ministry of Education, 2007). It should be noted that the 1st – 4th forms of a secondary school correspond to the primary school level. The main purposes aimed at the teaching of Mathematics are to form the mode of thinking on being of Mathematics as a method of describing and understanding reality, to create imaginations on being of Mathematics as a structural part of general human culture and the motivating forces of development of society and to create the real background for owning the necessary knowledge, abilities and skills with the purpose of continuing the education, learning other subjects and applying them in the practical activity. According to the “The Mathematics Curriculum for the general educational schools of the Republic of Azerbaijan (the 1st – 4th forms)” prepared on the basis of studying and analysing the existing world experience the following content lines of the teaching of Mathematics have been determined:

numbers and operations;

algebra and functions;

geometry;

measurements;

statistics and probability.

In this work we pay our main attention to geometry that is one of the indicated content lines. It is known (Ministry of Education, 2007) that the learning of properties of plane and spatial figures, the forming of spatial imaginations, the performing of analyses and solution of mathematical problems by using of properties of geometrical figures and geometrical methods are provided by means of the content line of geometry. In the 1st – 4th forms determination and description of characteristics of simple plane figures and spatial bodies are realized by means of the content line of geometry. A pupil achieves the following general results by means of the content line of geometry:

analyses the necessary and sufficient conditions of geometrical figures by using of observations and spatial imaginations;

knows geometrical relations and grounds them;

analyses situations connected with solutions of problems applying geometrical transformations and symmetry elements;

uses special reason or conjecture methods and geometrical modelling to analyse situations connected with solutions of problems.

According to the content standards of the subject of Mathematics at the end of the 1st academic year a pupil of the first form compares things on their necessary and sufficient conditions and spatial positions, knows simple geometrical figures (triangle, square, circle) and classifies them on their given necessary and sufficient conditions in the framework of the content line of geometry.

Symmetry is an important mathematical concept that plays an extremely important role not only in Mathematics but also in Physics, Chemistry, Music, Architecture, Geography, Engineering, Design etc. (Darvas, 2007; Leikin, Berman, & Zaslavsky, 1998; Leikin, Berman, & Zaslavsky, 2000; Petitjean, 2007; Weyl, 1982). Symmetric objects are distinguished in the world around us. Therefore mankind has always been interested in symmetries in Nature. Study of symmetry has fundamental importance for mankind. The interest to the investigations connected with symmetries in Nature is increased day by day. On 7 October 2008 the Royal Swedish Academy of Sciences awarded the 2008 Nobel Prize in Physics to two Japanese physicists M. Kobayashi and T. Maskawa for discovering the origin of the broken symmetry (Kobayashi, & Maskawa, 1973) and a Japanese-born American physicist Y. Nambu for his discovery of the mechanism of spontaneous broken symmetry (Nambu, & Jona-Lasinio, 1961; Nambu, & Jona-Lasinio, 1961).

A number of papers have been devoted to the teaching of the topic of “Symmetry” at a primary school (Dreyfus, & Eisenberg, 1990; Knuchel, 2004; Leikin, Berman, & Zaslavsky, 2000; Moore, 2002; Seidel, 1998). Here we have to note that the subject of Mathematics in the first forms of the primary schools of the Republic of Azerbaijan is taught on the basis of the textbook of “Mathematics (Textbook for the 1st form)” by Gahramanova, & Asgarova (Gahramanova, & Asgarova, 2008) and the teaching aid of “Mathematics (Teaching aid for a teacher)” by Gahramanova, & Asgarova (Gahramanova, & Asgarova, 2008). According to these books two hours are intended for the teaching of the topic of “Symmetry”.

2. Purpose of the Study

The main purpose of this work is determination of the effective way of the teaching of symmetry at Mathematics lessons in the 1st forms of primary schools of the Republic of Azerbaijan within the content line of geometry on the basis of National Curriculum. Achieving this purpose a pupil will be able to understand the concept of symmetry, to differ symmetrical objects and figures from asymmetrical ones, to demonstrate all these in the example of simple geometrical figures, to draw symmetry axes of simple geometrical figures, to restore the absent part of a symmetrically divided figure on its other given part.

3. Methods and Procedures

To teach the topic of “Symmetry” to the pupils effectively we recommend the organization of a lesson on the following plan.

Topic: Symmetry (2 hours).

Purpose:

to understand the concept of symmetry;

to be able to distinguish symmetric objects and figures from asymmetric ones and to demonstrate all these in the example of simple geometrical figures;

to be able to draw symmetry axes of simple geometrical figures;

to be able to restore the absent part of a symmetrically divided figure on its other given part.

Integration: integration between subjects and inner subject integration.

Lesson type: The lesson learning new mathematical concept.

Lesson form: combined lesson, lesson-discussion.

Methods: research, discussion, questions and answers (interrogation), working in group, brainstorming, independent working etc.

Resources: text-books and teaching aids, various symmetric and asymmetric objects, various pictures describing symmetric and asymmetric objects, a ruler, a pen, a sheet of paper, various geometrical figures, workbook.

Stages of the lesson:

- 1) Organization of the class
- 2) Motivation

It should be created the interest and inclination to the problem connected with “Symmetry” before starting the lesson. The created interest and inclination determine the motivation. The main purpose of the motivation is to organize the activity of the class so that the motivation should lead to the new knowledge and to be formed the desire of getting this knowledge in the class. During the motivation pupils approaches to the solution of the problem essentially. However pupils feel that they need something for solution of this problem. This “something” should be the subject of a new lesson (Davydov, 1991; Davydov, 1996). From this point of view the motivation is considered an essential stage of a lesson (Gahramanova, & Asgarova, 2008; Ministry of Education, 2007). The motivation connected with teaching of “symmetry” can be presented as follows.

The teacher demonstrates the pictures of a butterfly to the pupils and begins the lesson with the following dialogue:

Teacher: Whose picture is this?

Pupils (in chorus): Butterfly’s.

Teacher: How many wings has it got?

Tahir: Two.

Teacher: On what sign do the wings distinguish from each other?

Leyla: One of the wings is situated on the left, the other is situated on the right.

Teacher: What similarity is there between the left wing and the right one?

Rasim: They have the same shape.

Teacher: What is situated in the middle of the left and right wings?

Aysel: The line dividing them into the two parts.

Teacher: How do the left wing and the right wing are situated in relation to this line?

Arzu: They are situated face to face.

Hasan: They are situated at the same distance.

Jale: They are situated equally.

Then the teacher demonstrates other objects for the pupils: a leaf, an opened book, an opened copybook, sunglasses, a human picture, a small piece of wood, pieces of asymmetrically cut paper, and other asymmetric objects. The teacher gives the pupils a task to find the figures whose right and left sides are situated as butterfly’s wings. The pupils work in the form of a group consisting of 4 or 5 people and find this kind of objects: a leaf, an opened book, an opened copybook, sunglasses, human picture.

Teacher: What similarities are there between the picture of a butterfly and these pictures?

Vali: Each of them has two parts.

Teacher: That is right. What relations are there between these two parts?

Tural: These two parts look like each other.

Aynur: These are binary parts and they are situated equally.

The teacher shows one point on one of the butterfly’s wings and asks the pupils to find the similar or equal point on the other wing of the shown butterfly. The pupils perform this task successfully. Then the teacher continues the dialogue.

Teacher: What can you tell about the same points of these two equal parts?

Naila: The same points of these two equal parts are situated at the same distance in relation to the line dividing them into the two similar parts.

Teacher: What can you tell about the line dividing these pictures into the two equal or similar parts?

Tahira: The line dividing these pictures into the two equal or similar parts passes through the middle of each picture?

Teacher: What can you tell about the line passing through the middle of each picture presented here?

Sevda: The line passing through the middle of each picture presented here divides the picture into the two same parts.

The teacher shows the picture of a rectangle to the pupils and ask them to find the line dividing a rectangle into the two same parts. The pupils perform this task and they find out that a rectangle has a great (infinite) number of straight lines that pass through the middle of a rectangle.

Then the teacher demonstrates the picture of a rhombus and the pupils find the symmetry axes. When the teacher shows the pupils the picture of a parallelogram and give them a task to find its symmetry axes, the pupils make mistakes. They show each diagonal of a parallelogram as a symmetry axe. However, using this mistake the teacher explains that symmetry axes have the following necessary and sufficient conditions:

they divide the picture into the two same parts;

the two similar parts of a rectangle coincide when a rectangle is folded in two.

Then the teacher explains that the line that has the above indicated necessary and sufficient conditions is a symmetry line or a symmetry axes. Then the teacher explain that a parallelogram which is not a rhombus or a rectangle does not have a symmetry line or a symmetry axes.

Learning

The teacher explains that the right and left sides of a butterfly, a leaf, an opened book, an opened copybook etc. look like each other, these sides are binary and they are situated equally, these sides are divided by the line passing through the middle of each object. These features are the geometrical property that joins these objects. This geometrical property is called symmetry. Symmetry is the dividing of an object or a figure into two parts with regard to two same parts.

The teacher concludes this stage of the lesson as follows:

Teacher: What is symmetry? How do you understand the concept of symmetry?

Babek: Symmetry is a necessary and sufficient condition of objects belonging to a definite group.

Shamil: Symmetry is a geometrical property of objects.

Nazim: Symmetry is the dividing of an object or a figure into the two similar parts.

Aliya: If a symmetry is observed or met in an object, it means that it has a symmetry axes.

Rauf: The same points of the two equal parts of the objects having symmetry are situated at the same distance in relation to the symmetry axes.

Reyhan: A symmetry axes of an object divides it into the two equal or similar parts.

4) Application

Then the teacher explains what the symmetry line is. The teacher gives the pupils a task to show a symmetry line of a various objects. The pupils show the symmetry line of a butterfly, a leaf, an opened book, an opened copybook. The teacher gives the pupils a task to put a butterfly, a leaf, an opened book, an opened copybook onto one part (e.g., left part) of the table and other objects onto the other part (e.g., right part) of the table. The teacher explains again that a butterfly, a leaf, an opened book, an opened copybook are symmetric objects or figures. The teacher asks the pupils how to call the rest of the objects that are on the table. The pupils reply in chorus: asymmetric objects. Then the teacher divides the pupils into small groups, distributes the pictures of a rectangle, a square and a circle and asks whether they are symmetric or asymmetric. The teacher asks the pupils to show the symmetry lines of the indicated geometrical figures. The pupils determine that the indicated geometrical figures have great number (infinite number) of a symmetry line. The teacher shows the pupils the picture of an isosceles triangle. The pupils indicate that an isosceles triangle has one symmetry axis. Then the teacher gives the pupils a task to draw the symmetry axes of a rectangle, a square, a circle and an equilateral triangle using a ruler and a pen and to divide the indicated figures into two symmetric parts. The pupils perform this task.

Then the teacher tells the pupils to put their left hands on the table. All the pupils put their left hands on the table. The teacher asks the pupils to find an object that is symmetric to the left hand. The pupils show their right hands and put them next to the left hands.

Evaluation

Using “The Evaluation Conception in the General Educational System of the Republic of Azerbaijan” (Cabinet of Ministers, 2009) we recommend to perform the summative evaluation of this model of teaching of the topic of “Symmetry” at Mathematics lessons in the first forms according the following three questionnaires.

The first questionnaire (Table 1) enables us or primary class teachers to evaluate whether the pupil could achieve the aims determined in the given standard or group of standards.

<Insert Table 1 here>

The second questionnaire (Table 2) enables us or primary class teachers to evaluate whether the whole class could achieve the aims determined in the given standard or group of standards.

<Insert Table 2 here>

The summative evaluation table for the whole study (Table 3) is made up at the basis of the questionnaires collected from the teachers.

<Insert Table 3 here>

The achievements are classified on four levels. The first level is the lowest level and the fourth level is the highest level.

Homework

A teacher gives the pupils the following tasks as homework.

Task 1: Find a capital letters in the alphabet that have one or more symmetry axes and draw their symmetry axes.

Task 2: Find the numerals (from 0 to 9) that have symmetry axes in its writing and draw their symmetry axes.

Task 3: Write the names of symmetric objects that you usually use in winter when there is snow.

Task 4: Write the names of symmetric objects that you see in your flat.

Task 5: Restore the rest of the symmetrically divided picture of a boat on the basis of the given part of the picture (Exercise 3 on page 75 in the textbook (Gahramanova, & Asgarova, 2008).

At the second hour of a lesson devoted to the topic of “Symmetry” the pupils revise the gained knowledge on symmetry and the pupils apply their gained knowledge and skills on symmetry. The pupils perform Exercise 1, Exercise 2, Exercise 3 on page 76 in the textbook and Exercise 1, Exercise 2, Exercise 3 on page 75 in the workbook.

4. Results

To teach the topic of “Symmetry” to the pupils effectively we recommend the organization of a lesson on the following plan.

Topic: Symmetry (2 hours).

Purpose:

to understand the concept of symmetry;

to be able to distinguish symmetric objects and figures from asymmetric ones and to demonstrate all these in the example of simple geometrical figures;

to be able to draw symmetry axes of simple geometrical figures;

to be able to restore the absent part of a symmetrically divided figure on its other given part.

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Resources: text-books and teaching aids, various symmetric and asymmetric objects, various pictures describing symmetric and asymmetric objects, a ruler, a pen, a sheet of paper, various geometrical figures, workbook.

Stages of the lesson:

1) Organization of the class

- 2) Motivation
- 3) Learning
- 4) Application
- 5) Evaluation
- 6) Homework

At the end of the lesson the pupils could understand the concept of symmetry and distinguish symmetric objects and figures from asymmetric ones. They could demonstrate all these in the example of simple geometrical figures, draw symmetry axes of simple geometrical figures, restore the absent part of a symmetrically divided figure on its other given part.

The effective teaching of a symmetry conception at Mathematics lessons in the first forms enables the pupils to study and understand Geometry, Physics, Chemistry, Music, Geography and other subjects deeply in the future and to lay down the foundations of preparation of future mathematicians, physicists, chemists, engineers, architects, musicians, artist etc. Taking into account all these it is recommended to increase the number of teaching hours on the topic of "Symmetry" from two hours to four hours in the first forms of primary schools.

5. Conclusion

In this work we have investigated the effective way of teaching of symmetry at teaching of Mathematics in the first forms of primary schools of the Republic of Azerbaijan on the basis of National Curriculum. The realization of the offered effective way of the teaching of the topic of "Symmetry" in the first forms of primary schools enables the pupils to understand the concept of symmetry and distinguish symmetric objects and figures from asymmetric ones. The recommended effective way of the teaching of the topic of "Symmetry" in the first forms of primary schools also enables to demonstrate all these in the example of simple geometrical figures, draw symmetry axes of simple geometrical figures, restore the absent part of a symmetrically divided figure on its other given part.

As a result of the investigation we have come to the conclusion that the effective teaching of a symmetry concept enables the pupils to study and understand Geometry, Physics, Chemistry, Music, Geography and other subjects deeply in the future and it lays down the foundations of preparation of future mathematicians, physicists, chemists, engineers, architects, musicians, artist etc. Taking into account the importance and universality of the concept of symmetry we recommend to increase the number of teaching hours on "Symmetry" from two hours to four hours in the first forms of primary schools.

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Table 1. Summative evaluation table prepared for pupils (sample)

Topic: Symmetry

School name or No _____

Pupil's first name and surname: _____

No	Pupil's skills	Possible answers			
		Sometimes	Usually	Often	Always
1.	Can you learn the material?	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	Can you apply your knowledge?	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	Are you up to the mark to advance more?	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Table 2. Summative evaluation table prepared for teachers (sample)

Topic: Symmetry

School name or No _____

Teacher's first name and surname: _____

Levels: 1 - Sometimes, 2 – Usually, 3 – Often, 4 - Always

No	Pupil's skills	Levels			
		1	2	3	4
1.	Pupils can learn the material.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	Pupils can apply their knowledge.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	Pupils are up to the mark to advance more.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Table 3. Summative evaluation table for the whole study

Topic: Symmetry

Levels: 1 - Sometimes, 2 – Usually, 3 – Often, 4 - Always

No	Pupil's skills	Levels			
		1	2	3	4
1.	Pupils can learn the material.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	Pupils can apply their knowledge.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	Pupils are up to the mark to advance more.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>