# Calculation of Quark-antiquark Potential Coefficient and Charge Radius of Light Mesons

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**Abstract**

Mesons are composed of one quark and one anti quark which are considered to be governed by quantum chromo dynamic. In the present paper, it is considered that mesons are made up entirely, at least in principle, of light quarks (u, d, s). Since mesons consisting of light quarks are relativistic systems, we cannot use the Schrödinger equation, and the theory is rather limited. The light quark (u, d, s) mesons are intrinsically relativistic; because binding energies (typically a few hundred MeV) are not small compared to the constituent masses. Therefore they are based on nonrelativistic quantum mechanics. For the heavy quarks (c, b, t) though, the nonrelativistic theory should work reasonably well. For most part of this paper we restricted our attention to relativistic bound state of two particles and considered interaction between quark and antiquark, the essential point is that at short range we expect a columbic potential and at large distance we have to account for quark confinement, the simplest is a linear potential and other interaction oscillator potential. Then Dirac equation was solved with the central potential analytically and the wave function of system was determined. Finally, by using wavefunction we determined the charge radius of $p^0$ meson.

**Keywords:** Quark model, Central potential, Light meson, Dirac equation

## 1. Introduction

The Constituent Quark Model (CQM) has been extensively applied to the description of meson properties .In 1966 , Yoichiro Nambu first proposed the SU (3) gauge theory, and quantum chromo dynamics (QCD), as a candidate for the fundamental theory of the strong interaction, just after the introduction of the new quantum number color. QCD has been established as the fundamental theory of strong interaction (Giannini, et al., 2003) However, in spite of its simple form, QCD creates thousands of hadrons and leads to various interesting phenomena such as color confinement and oscillator potential. Mesons with heavy quark have been investigation by Schrödinger equation several times, but mesons consisting of light quark are relativistic systems (Giannini, et al., 2002). Since the light mesons are relativistic, we should utilize Dirac equation in order to investigation mesons with light quark. Then we also used central potentials (i.e. columbic, confiment and oscillator) to determine static properties of light mesons.( Bhaduri. et al., 1980) The purpose of the present paper is to provide a mechanism for investigation of mesons and interaction between quark and anti quark.( Tegen, et, al 1982) Since the meson consists of one quark and one antiquark, the wavefunction are dependent on two coordinates

$\vec{r}_1$ and $\vec{r}_2$, where $\vec{r}_1$, $\vec{r}_2$ are the quark and antiquark position vector (Lin, 1982).Then the internal quark and anti quark motion are described by the relative coordinates $x$ which is defined as follow: ( Shojaei, et al., 2008)

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \tag{1}$$

In this model we consider the central potentials between quark and antiq (Giannini , et al., 1999) uark . First, for small separations, the potential which has an attraction is columbic potential originating from the color charge (Rajabi, 2005).

$$V(r) = \frac{k\,\alpha_s}{r} = -\frac{c}{r}. \tag{2}$$

While at large separations a linear term gives rise to quark confinement (Rajabi, 2006).

$$V(r) = br. \tag{3}$$

In this article we have added the oscillator potential as follow ( Shojaei , et al 2009)

$$V = ar^2 \tag{4}$$

Here the total potential is assumed as below:

$$v(r) = ar^2 + br - \frac{c}{r}. \tag{5}$$

Where $a, b, c$ are potential coefficient .In section (2) we have calculated the relativistic wave function for quarks and anti quark .The results indicate that this potential is useful for quarks having masses in the range used in the phenomenological analysis of quark model and determined charge radius of $p^0$ meson.

## 2. Solution of Dirac equation for the meson

The Dirac equation may transform in various ways under a Lorentz transformation.The form in common use for scalar central potential $(u_0)$ and vector central $(v_0)$ is taken as (Shojaei ,et al 2007)

$$
\begin{aligned}
(\sigma . P)\chi + (m + u_0(r) + v_0(r))\varphi &= \varepsilon\varphi \\
(\sigma . P)\varphi - (m + u_0(r) - v_0(r))\chi &= \varepsilon\chi
\end{aligned}
\tag{6}
$$

Since the meson consists of one quark and one antiquark then we can use the Dirac equation for the quark as follows:

$$(\sigma . p_1)\chi_1 + (m_1 + u_0 + v_0)\varphi_1 = \varepsilon\varphi_1 \tag{7-a}$$

$$(\sigma . p_1)\varphi_1 - (m_1 + u_0 - v_0)\chi_1 = \varepsilon\chi_1 \tag{7-b}$$

We suppose the $u_0(r) = v_0(r)$ .Now by combining two equations (7-a) and (7-b), we obtain

$$p_1^{\,2}\varphi_1(r_1) + (m^2 - \varepsilon^2)\varphi_1(r_1) + 2v_0(m + \varepsilon)\varphi_1(r)_1 = 0 \tag{8}$$

For the antiquark as before it would by

$$p_2^{\,2}\varphi_2(r_2) + (m^2 - \varepsilon^2)\varphi_2(r_2) + 2v_0(m + \varepsilon)\varphi_2(r_2) = 0 \tag{9}$$

By combination equation (8) and (9) we have:

$$(P_1^2 + P_2^2)\varphi(r) + 2\left[(m^2 - \varepsilon^2) + 2(m + \varepsilon)v_0(r)\right]\varphi(r) = 0 \tag{10}$$

Where $j(r)$ is the upper component wavefunction of light meson. Since we suppose $2v_0(r) = 2u_0(r) = (ar^2 + br - \frac{c}{r})$ depends only on the relative distance $\vec{r}$ between quark and antiquark, it is more appropriate to use the center mass and relative coordinates $R$ and $\vec{r}$ instead of the coordinate $\vec{r_1}$ and $\vec{r_2}$. The transformation from $\vec{r_1}, \vec{r_2}$ to $\vec{r}, \vec{R}$ is given by (Lin, 1983).

$$\vec{r} = \vec{r_2} - \vec{r_1} \qquad\qquad \vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \qquad (11)$$

We can substitute the Laplacians based on the function of $\vec{r_1}$ and $\vec{r_2}$ with $\vec{r}, \vec{R}$ which are related. Therefore Eq. (10) in terms of relative coordinate becomes as follows.

$$-(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \frac{L^2(\Omega)}{r^2})\varphi(r) + \left[2(m^2 - \varepsilon^2) + 4(\varepsilon + m)v_0(r)\right]\varphi(r) = 0 \qquad (12)$$

With $v_0(r)$ given by (5), and $L^2(\Omega) = -l(l+1)$ is the grand orbital operator and $l$ is the grand angular quantum number. Now for the eigenfunction $\varphi(r)$, we make an ansatz, ( Shojaei , et al 2008)

$$\varphi(r) = f(r)\exp(h(r)) \qquad (13)$$

With $f(r)$ and $h(r)$ as:

$$f(r) = 1 \qquad\qquad h(r) = -\frac{1}{2}\alpha r^2 + \beta r + \delta \ln r \qquad (14)$$

This implies

$$g_\gamma''(r) + \frac{2}{x}g_\gamma'(r) = \left[h''(r) + h'^2(r) + \frac{2h'}{r}\right]g_\gamma(r) \qquad (15)$$

Equations (12) and (15) yield $\alpha, \beta, d$ and the constraints between the potential parameters $a, b$ and $c$. We have

$$\begin{cases} \alpha = 4a\sqrt{\varepsilon + m} & \text{(16-a)} \\ \beta = -\frac{2\sqrt{\varepsilon + m}b}{a} & \text{(16-b)} \\ c = \frac{\beta(\delta + 1)}{2(\varepsilon + m)} & \text{(16-c)} \\ \delta = l \end{cases}$$

Taking $\delta = l$ results a wavefunction which is well behaved at the origin .Therefore the upper component of Dirac spinor of the meson is as below:

$$\varphi(r) = r^l \exp(-\frac{1}{2}(4a(\varepsilon + m))^{\frac{1}{2}}r^2 - \frac{2(\varepsilon + m)^{\frac{1}{2}}b}{a}r) \qquad (17)$$

The lower component $\chi(r)$ of the Dirac hyper-central spinor can be found from (7-a), (7-b), for quark and antiquark we find

$$\chi(r) = \frac{(\vec{\sigma}.(\vec{p}_1 + \vec{p}_2))}{(m+\varepsilon)}\varphi(r) \tag{18}$$

We suppose $\vec{p} = \vec{p}_1 + \vec{p}_2$. We can $(\vec{\sigma}.\vec{p})\varphi(r)$ be written as:

$$(\vec{\sigma}.\vec{P})\varphi(r) = \frac{\vec{\sigma}.\vec{r}}{r^2}\left[(\vec{\sigma}.\vec{r})(\vec{\sigma}.\vec{P})\right]\varphi(r) \tag{19}$$

$\left(\vec{\sigma}.\vec{A}\right)\left(\vec{\sigma}.\vec{B}\right) = \vec{A}.\vec{B} + i\,\vec{\sigma}.(\vec{A}\times\vec{B})$     Using the equation and substituting it in equation(18)

$$\left(\vec{\sigma}.\vec{P}\right)\varphi(r) = \frac{\vec{\sigma}.\vec{r}}{r^2}\left[\vec{r}.\vec{P} + i\,\vec{\sigma}.(\vec{r}\times\vec{P})\right]\varphi(r) \tag{20}$$

Where $\vec{r}\times\vec{P} = \vec{L}$ and $P = -i\dfrac{d}{dr}$

$$\left(\vec{\sigma}.\vec{P}\right)\varphi(r) = -i\left[r\frac{d}{dr} + \vec{\sigma}.\vec{L}\right]\varphi(r) \tag{21}$$

By replace equation (21) in equation (18) and $\vec{r} = r\,\hat{r}$ the lower component obtained as so

$$\chi(r) = \frac{-i\,\vec{\sigma}.\hat{r}}{(m+\varepsilon)}\left[\frac{d}{dr} - \frac{\vec{\sigma}.\vec{L}}{r}\right]\varphi(r) \tag{22}$$

Therefore the wave functions for $l = 0$ is as the following form:

$$\psi(r) = \begin{bmatrix} \exp(-\frac{1}{2}(4a(\varepsilon+m))^{\frac{1}{2}}r^2 - \frac{2(\varepsilon+m)^{\frac{1}{2}}b}{a}r) \\[2mm] \frac{-i\,\vec{\sigma}.\hat{r}}{(m+\varepsilon)}\frac{d}{dr}(\exp(-\frac{1}{2}(4a(\varepsilon+m))^{\frac{1}{2}}r^2 - \frac{2(\varepsilon+m)^{\frac{1}{2}}b}{a}r)) \end{bmatrix} \tag{23}$$

By using this wavefunction we can determine the static properties of mesons. (Mass of meson, charge radius, magnetic moment…). In this present paper we calculated the charge radius of meson.

## 3. Charge Radius of $p^0$ meson by Using Quark Model

Let's take $p^0$ meson charge-radius. The charge-radius $<r_{em}^2>_p^{\frac{1}{2}}$ is defined as.

$$\langle r^2 \rangle_q = \int_0^\infty r^2\,\psi_\gamma(r)\psi_\gamma(r)d^3r \tag{24}$$

Here $\psi(\vec{r})$ is the quark wave function given by (23).Using from equation (16, 22) we can determined the potential coefficient and wavefunction. Then we calculated the charge radius of $p^0$ meson in Table 1. Which these results from the quark- anti quark substructure of the $p^0$ meson. Although the short lifetime of

the $p^0$ meson excludes the possibility of directly measuring the charge radius of the $p^0$ meson at present, one can define a diffractive radius from the $t$ dependence of diffractive $p^0$ meson electro production cross section.

## 4. Conclusion

An exact analytical solution for potential in the form of the confinent is presented. In this article we have shown the complete interaction including confinement, oscillator and coulomb in terms reproduces the position of the quark and antiquark. The central potential is a good starting point for construction of an unperturbed states and leads to realistic quark states, which shows the static properties of meson which are sensitive to the corrected wave functions. The central potential is a good stating point for investigation of meson structure A considerable improvement in the description of the static properties of nucleon is obtained with an isospin-dependent potential. By use this model we can investigation the other mesons. As we suggested the method for solving the wave function, essentially based on an ansatz for the eigenfunctions, works successfully for a large class of potentials for many particles interacting in D- dimension and at least for the study of the ground state of the system. Busied of these potentials we can consider the hyperfine potentials and calculate the shift energy. Finally one can use this model and determine the static properties of hadrons.

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Table 1. The charge radius of $p^0$ meson for different quark mass ($100 \# m_q \quad 300(MeV)$), b ,c are the of potential coefficient (a=1)

| $m_q (fm^{-1})$ | $\langle r^2 \rangle^{\frac{1}{2}} (fm)$ | $b (fm^{-2})$ | $c$ |
|---|---|---|---|
| .48 | .613 | 1.27 | 1.41 |
| .96 | .621 | .88 | .78 |
| 1.21 | .652 | .83 | .67 |
| 1.45 | .665 | .81 | .61 |