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The Cantor Set Constructed from an Infinite Number of Quarks Constituting the Nucleon and the Dark Matter

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Abstract

In a previous paper, the present author proposed the infinite sub-layer quark model, in which the proton and the neutron are composed of an infinite number of point-like (structure-less) quarks μ_∞ and anti-quarks μ_∞^{CP} at an infinite sub-layer level. The limit particle μ_∞ has all one-half quantum numbers of spin $S=1/2$, isospin $I=1/2$, third component of isospin $I_3=1/2$, and fractional electric charge $Q=(1/2)|e|$, where e is the electron charge. This fermion will behave as if it was lepton, since the baryon number vanishes at an infinite sub-layer level. Thus, this ultimate particle is considered as constituting the non-baryonic cold dark matter. A pair of an infinite number of μ_∞ and μ_∞^{CP} quarks would be produced in the first moments after the Big Bang and form the nucleons and remain as the dark matter for all time, stable against decay. It was then shown that CP is violated in the doublet of μ_∞ and μ_∞^{CP} quarks to account for the asymmetry of the number of particles and antiparticles in the present universe. Furthermore, it was shown that the Higgs bosons are composed of μ_∞ and μ_∞^{CP} dark matter particles. In this paper, we will show that the Cantor set is constructed from an infinite number of point-like quarks μ_∞ and anti-quarks μ_∞^{CP} which constitutes the nucleon and the dark matter. It is also shown that the color charges in quantum chromodynamics (QCD) of μ_∞ and μ_∞^{CP} quarks vanish and these limiting particles of dark matter have no color force with the strong interaction and subject only to the weak interaction and gravity.

Keywords: cantor set, nucleon, dark matter

1. Introduction

Since the introduction of the parton model (Feynman, 1969; Bjorken & Paschos, 1969), one of the most important problems in high energy physics has been the relationship between an infinite number of partons and the three quarks inside the nucleon. At low energies, we see the three quarks, while at higher energies, we detect an infinite number of partons.

In modern particle physics, the partons are considered as the quarks, quark and anti-quark sea and gluons (Feynman, 1972). Although this quark-parton model has been established and has a very strong appeal, we cannot exclude the existence of a further sub-layer quark (Kogut & Susskind, 1974a, 1974b). By analyzing deep inelastic electron-proton scattering at high q^2 (the 4-momentum transfer squared), each of the valence quarks will be itself resolved into infinitely many constituents called "partons".

In a previous paper (Sekine, 1985), we proposed the infinite sub-layer quark model. This model implied that the proton (p) and the neutron (n) are made of u_1 and d_1 quarks, so that $p=u_1u_1d_1$ and $n=u_1d_1d_1$. Furthermore, u_1 and d_1 quarks are made of u_2 and d_2, etc. In summary, u_N and d_N quarks at level N are made of u_{N+1} and d_{N+1} quarks at level $N+1$, such as $u_N=(u_{N+1},\ u_{N+1},\ d_{N+1})$ and $d_N=(u_{N+1},\ d_{N+1},\ d_{N+1})$ where

$$N = 1, 2, 3, \cdots, \infty$$

Here, the u_N and d_N quarks have quantum numbers of spin $S=1/2$, isospin $I=1/2$, baryon number $B=1/3^N$, third component of isospin $I_3=+1/2$ and fractional electric charge $Q=\frac{1+3^N}{2\times 3^N}|e|$ for the u_N quark, and $I_3=-1/2$ and $Q=\frac{1-3^N}{2\times 3^N}|e|$ for the d_N quark. The anti-particle of u_∞ is the d_∞ quark, since the baryon number vanishes at $N=\infty$. The number of quarks at level N is 3^N. Thus, at $N=\infty$, an infinite number of point-like quarks

($u_\infty = \mu_\infty$) and anti-quarks ($d_\infty = \overline{u_\infty} = \mu_\infty^{CP}$) is considered as constituting the nucleon. The superscript *CP* means charge conjugation and parity transformation. The ultimate particle μ_∞ has quantum numbers of $S = 1/2$, $I = 1/2$, $I_3 = 1/2$ and $Q = (1/2)|e|$. Thus, all quantum numbers of the μ_∞ quark are just one-half and this quark is non-baryonic, since the baryon number *B* is zero at an infinite sub-layer level. In a previous paper (Sugita, Okamoto, & Sekine, 2008), we proposed the non-baryonic and exotic quark μ_∞ as an excellent candidate for non-baryonic cold dark matter to comprise the universe, since they are absolutely stable and the non-baryonic particles with the baryon number 0. In this model, a pair of an infinite number of μ_∞ and μ_∞^{CP} quarks can be produced thermally in the hot early universe of the Big Bang and form the nucleons, and leave approximately the right relic abundance to account for the observed dark matter. Also, in this paper, we first showed that *CP* is violated in only one doublet of the ultimate quarks μ_∞ and anti-quarks μ_∞^{CP} to account for the asymmetry of the number of particles and antiparticles in the present universe. To this end, we considered the SU(2) noncommutative geometry from our published paper (Sugita, Okamoto & Sekine, 2006) and our published book (Sugita, Okamoto, & Sekine, 2007).

Next, we showed that the Higgs bosons are composed of μ_∞ and μ_∞^{CP} dark matter particles and give the masses to gauge bosons, quarks and leptons in the framework of the standard $SU(2)_L \times U(1)$ electroweak model. Moreover, in our previous paper (Sugita, Okamoto, & Sekine, 2011), we replaced the Higgs potential by the gravitational potential and it was then shown that the masses are produced and a cosmological constant is derived. It was then emphasized that if we can write down the *n*th order T product Green's function in the path-integral representation, then it is expected to construct a quantization theory including the cosmological constant. This prescription of the quantization by path-integral representation without the gravitational field is suggested from our published paper (Okamoto, Sugita, & Sekine, 1999) and our published book (Sugita, Okamoto, & Sekine, 1998).

In the following, we will show that the Cantor set (Cantor, 1883) is constructed from an infinite number of point-like μ_∞ and μ_∞^{CP} quarks.

2. The Cantor Set Applied to an Infinite Number of Quarks Constituting the Nucleon and the Dark Matter

The Cantor set is an example of a fractal and self-similarity (Peitgen, Jürgen, & Saupe, 2004). Recently, there has been great interest in the Cantor set, fractal and chaos in a field of quantum mechanics, elementary particle physics and quantum gravity. As one of important applications in this field, E infinity theory was introduced into elementary particle physics from Cantorian fractal space-time (EI Naschie, 2004). Here we apply the Cantor set to an infinite number of quarks constituting the proton and the neutron and the dark matter.

The Cantor middle-thirds or ternary set is described by repeatedly removing the open middle thirds of a set of line segments. Begin with the interval [0, 1] and divide it into three equal open intervals, that is, [0, 1/3], [1/3, 2/3], [2/3,1]. Remove the open middle third (1/3, 2/3), leaving two line segments: [0, 1/3]∪[2/3,1]. Next, subdivide each of these two remaining intervals into three equal open subintervals and again delete the open middle third of each of these remaining intervals, and so on, ad infinitum. In this infinite process, finally, the Cantor set contains an infinite number of points. We apply this process to the infinite sub-layer quark model. We introduce the color charges of red(R), green (G) and blue (B) in quantum chromodynamics (QCD). The proton is made of $u^R u^G d^B$ or $u^R u^B d^G$ or $u^B u^G d^R$. The neutron is made of $u^R d^G d^B$ or $u^R d^B d^G$ or $u^B d^G d^R$. The color-neutral system, that is, "white" color requires R+G+B = 1 with 1/3 color charge quantum number for each R, G, B color charges. We begin with $p = (u_1^R u_1^G d_1^B)$ quarks which constitute the proton. We introduce $1/3^N$ color charge quantum numbers for each R, G, B color charges with N=1, 2, 3,⋯, ∞. Thus, at N=∞, color charges vanish. Now we remove the u_1^G quark of the middle quark, leaving $(u_1^R, , d_1^B)$ behind. Next, the u_1^R and d_1^B quarks are subdivided into $u_1^R = (u_2^R, u_2^R, d_2^R)$ and $d_1^B = (u_2^B, d_2^B, d_2^B)$ and again we remove the $u_2^R$and d_2^B quarks of the middle quark, leaving us with four quarks$((u_2^R, \ldots, d_2^R), \ldots, (u_2^B, \ldots, d_2^B))$. This process is to be continued infinitely. The number of leaving quarks at level N is 2^N, where N=1, 2, 3,⋯, ∞. Thus, at N=∞, there exists an infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) and color charges vanish. Finally, it is concluded that an infinite number of point-like μ_∞ and μ_∞^{CP} quarks that remain after all these middle quarks have been removed is called the Cantor set. In a middle-thirds Cantor set, the sum of the lengths of the removed intervals is restored and equal to the length of the original interval. Similarly, we will show that the sum of the removed middle quarks of the proton $p = (u_1^R u_1^G d_1^B)$ is restored and equal to the original proton.The removed middle quarks are arranged as follows:

$$u_1^G, (u_2^R, d_2^B), \quad 2(u_3^R, d_3^B), \quad 2^2(u_4^R, d_4^B), \ldots, 2^{N-2}(u_N^R, d_N^B), \ldots \tag{1}$$

where N=1, 2, 3,⋯, ∞. Thus, at N=∞, there exists an infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) with all half- quantum numbers. We will show that the proton is constructed from these removed quarks of (1).

First, consider the baryon number. If we add up the baryon number from (1), we obtain

$$\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots=1$$

This total is the geometric progression. The electric charge Q is calculated from (1) as

$$\mathrm{Q}=\frac{2}{3}|e|+\left(\frac{1}{9}|e|+\frac{2}{27}|e|+\frac{4}{81}|e|+\ldots\right)=\frac{2}{3}|e|+\frac{1}{3}|e|=|e|.$$

For color charges, green u_1^{G} quark has 1/3. Furthermore, the net red color charge of u_2^{R}, $2\,u_3^{\mathrm{R}}$, $2^2\,u_4^{\mathrm{R}}$, ... ,$2^{N-2}\,u_N^{\mathrm{R}}$, … quarks has 1/3, since $\frac{1}{9}+\frac{2}{27}+\frac{4}{81}+\ldots=\frac{1}{3}$. Similarly, the net blue color charge of d_2^{B}, $2\,d_3^{\mathrm{B}}$, $2^2\,d_4^{\mathrm{B}}$,... ,$2^{N-2}d_N^{\mathrm{B}}$, … quarks also has 1/3. Therefore, the net color charge of (1) is R+G+B = 1. This is a color neutral white of the proton. Thus, the constituents from the removed middle quarks of (1) are the proton. It is fun to speculate a model of (1) which looks like the atom: in place of the positively charged nucleus, we have u_1^{G} quark, and instead of negatively charged electrons, we have $(u_2^{\mathrm{R}}, d_2^{\mathrm{B}})$, $2(u_3^{\mathrm{R}}, d_3^{\mathrm{B}})$, $2^2(u_4^{\mathrm{R}}, d_4^{\mathrm{B}})$, ... ,$2^{N-2}(u_N^{\mathrm{R}}, d_N^{\mathrm{B}})$, … quarks surrounded by the orbit, or shell. At N =∞, that is, the most outer orbit, there exists an infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) having all half- quantum numbers.

Now consider n= $(u_1^{\mathrm{R}}d_1^{\mathrm{G}}d_1^{\mathrm{B}})$ quarks which constitute the neutron. In the same way to the p= $(u_1^{\mathrm{R}}u_1^{\mathrm{G}}d_1^{\mathrm{B}})$ quarks, we repeat any step in this infinite process. The removed quarks are as follows:

$$d_1^{\mathrm{G}},\ (u_2^{\mathrm{R}}, d_2^{\mathrm{B}}),\quad 2(u_3^{\mathrm{R}}, d_3^{\mathrm{B}}),\quad 2^2(u_4^{\mathrm{R}}, d_4^{\mathrm{B}}),\ \ldots,2^{N-2}(u_N^{\mathrm{R}}, d_N^{\mathrm{B}}),\ \ldots \qquad (2)$$

where N=1, 2, 3,⋯, ∞. At N=∞, there exists also an infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) having all half- quantum numbers.

From (2), the baryon number is calculated as $\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots=1$ and the electric charge Q is

$$\mathrm{Q}=-\frac{1}{3}|e|+\left(\frac{1}{9}|e|+\frac{2}{27}|e|+\frac{4}{81}|e|+\ldots\right)=-\frac{1}{3}|e|+\frac{1}{3}|e|=0.$$

In a similarly way to (1), we have also the net color charge R+G+B = 1.

Thus, the constituents from (2) are the neutron.

An infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) is an excellent candidate for non-baryonic cold dark matter to comprise the universe, since the baryon number is 0. Also the color charges vanish. Therefore, a theory of the strong interaction (color force) does not exist between quarks (μ_∞) and anti-quarks (μ_∞^{CP}) and gluons.

3. Discussions

At low energies, we see the three quarks inside the nucleon, while at higher energies, we detect an infinite number of quarks. Although the quarks, quark and anti-quark sea and gluons are regarded as partons in modern particle physics, some attempts were made to solve this problem. For example, a Lorentz-squeezed hadron becomes a collection of partons (Kim, 1989). There exist valence-quark clusters in the nucleon (Hwa, 1980; Hussar, 1981).

We assumed that when probed at high q^2, each of the three quarks will itself be resolved into infinitely many constituents called partons. To this end, we investigated the infinite sub-layer quark model and the Cantor set model. In both models, there exists an infinite number of point-like quarks (μ_∞) and anti-quarks (μ_∞^{CP}) having all half- quantum numbers including a half electron charge. It is very interesting to note that a point Dirac particle with charge ±1/2 e was already predicted in the total backward electron-proton scattering at fixed large four-momentum transfer q^2 (Bjorken, 1967). A pair of these ultimate quarks would be produced in the first moments after the Big Bang and form the nucleons and then remain as the non-baryonic cold dark matter for all time, stable against decay and subject only to the weak interaction and gravity, since both the baryon number and

the color charges vanish. *CP* is also violated in the doublet of (μ_∞) quarks and (μ_∞^{CP}) anti-quarks to account for the asymmetry of the number of particles and antiparticles in the present universe.

We have shown that infinite sets are constructed from an infinite number of point-like μ_∞ and μ_∞^{CP} quarks. In infinite sets theory, the continuum hypothesis (Cantor, 1878) is an important problem, because of the possible sizes of infinite sets. The continuum hypothesis can neither be proved nor be disproved using the ZFC axioms (Gödel, 1940; Cohen, 1963). Here ZFC means the Zermelo-Fraenkel set theory (Zermelo, 1908; Fraenkel, 1922) and axiom of choice. In addition to these ZFC axioms, a new axiom is formulated and it is realized that these axioms resolve the continuum hypothesis (Woodin, 2001a, 2001b).

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