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Enhancement of Electron Bunch Acceleration by the Wakefield Generated with Periodical Chirped Laser Pulse in under Dense Plasma

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Abstract

This paper presents a two dimensional analytical model for a linear laser wakefield generation by the chirped laser pulse and electron bunch acceleration has been studied numerically. It is found that the negative periodical chirp leads to increase the longitudinal and transverse components of the wakefield amplitude by one order of magnitude. In this status, we inject an electron bunch with the initial energy of about 2MeV in front of the periodical chirped laser pulse and we saw the final energy of the trapped electrons arrive to about 0.9Gev in 17mm of plasma length, with an energy spread about 1% and a the emmitance of the order of 0.12 mm mrad.

Keywords: laser wakefield; laser pulse, periodical chirp, electron acceleration

1. Introduction

Electron acceleration to deliver multi-GeV and making compact high-energy particle accelerators are the main goals of the plasma-based accelerators (Bingham, 2006; Blumenfeld et al., 2007; Martins, Fonseca, Lu, Mori & Silva, 2010; Mo et al., 2012). Among a number of laser-plasma accelerator concepts, laser wakefield accelerator (LWFA) has been revived because of a simple mechanism (Tajima& Dowson, 1979; Sprangle, Esarey, Ting & Joyce, 1988). In laser wakefield acceleration a short intense laser pulse with duration on the order of a plasma wavelength generates strong accelerating and focusing fields (Esarey, Schroeder, & Leemans, 2009; Kalmykov, Gorbunov, Mora & Shevets, 2006; Malka et al., 2008; Khachtryan, Irman, Vangoor & Boller, 2007). To date, intense short pulse lasers have been developed with light intensities as high as 10^{22}W/cm^2 with the development of chirp-pulse-amplification (CPA) technique (Danson et al., 2004). Recently, the laser chirped pulse effect in the laser acceleration in vacuum has been investigated (Sohbatzadeh, Mirzanejhad & Ghasemi, 2006; Sohbatzadeh, Mirzanejhad & Aku, 2009; Gupta & Suk, 2006; Singh & Sajal, 2009). Furthermore, the interaction of the chirped laser pulse with plasma has been investigated experimentally (Schroeder, Esarey, Shadwick, & Leemans, 2003; Schroeder et al., 2003; Tóth et al., 2003) and theoretically (Khachtryan, Vangoor, Verschuur, & Boller, 2005; Mirzanejhad, Sohbatzadeh, Asri, & Ghanbari, 2010). They had shown that the wakefield amplitude is increased due to the chirp effect. Moreover, an electron bunch injection in front of the laser pulse proposed (Khachtryan, 2001), the simple experimental setup and good quality of the final bunch are the excellences of this injection method.

In this work the effect of the linear and periodic profiles for frequency variation of the laser pulse are investigated for the 2D linear laser wakefield generation. We show that appropriate chirped laser pulse improves the wakefield amplitude in 2D case and so, increases accelerating and focusing forces. Moreover, we examined the external electron bunch injection in front of the laser pulse by a numerical simulation.

This paper is organized as follows. In sec 2, analytical calculations of the 2D linear wakefield are provided. In sec 3, the numerical results and discussions of the wakefield generation and electron bunch acceleration are considered, and in sec 4 some concluding remarks are noted.

2. 2D Wakefield in the Linear Regime

To describe the excitation of plasma waves by a laser pulse, we use the Maxwell's and hydrodynamic equations for cold plasma electrons: (Andreev, Gorbunov, Kirsanov, Nakajima, & Ogata, 1997)

$$\frac{\partial \vec{v}}{\partial t} = (\frac{e}{m_e})\vec{E} - c^2 \nabla \frac{|a|^2}{4} \quad , \tag{1}$$

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e v) = 0 \quad , \tag{2}$$

$$\frac{\partial \vec{E}}{\partial t} = -4\pi e n_e \vec{v} + c\nabla \times \vec{B} \quad , \tag{3}$$

$$\nabla \times \vec{v} = -\frac{e}{m_e c}\vec{B} \quad , \tag{4}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad , \tag{5}$$

Where e, m_e, n_e and v are the electron charge, rest mass, density and velocity, respectively. $\vec{E}$ and $\vec{B}$ are electric and magnetic fields and $\vec{a} = e\vec{A}/m_e c^2$ is the normalized amplitude of the vector of potential of the laser field.

For plasma with uniform density, combining these equations with assumption cylindrical symmetric, we can obtain following equations for the evolution of the 2D linear wakefield in cylindrical coordinate:

$$\frac{1}{c^2}\frac{\partial^2 E_r}{\partial t^2} + \frac{4\pi e^2 n_e}{m_e c^2} E_r = -\pi e n_e \frac{\partial a^2}{\partial r} \tag{6}$$

$$\frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} + \frac{4\pi e^2 n_e}{m_e c^2} E_z = -\pi e n_e \frac{\partial a^2}{\partial z} \tag{7}$$

In the quasi static approximation (Sprangle, Esarey, & Ting, 1990) by using coordinates moving with the laser pulse variables $\xi = z - v_g t$, $\zeta = z$, and for simplicity the spatial component are normalized to $1/k_p$, $k_p = \omega_p / v_g$ being the plasma wave number and v_g is the group velocity of the laser pulse in plasma.

The Equations (6) and (7) can be written in the form:

$$\frac{\partial^2 \tilde{E}_r}{\partial \xi^2} + N\tilde{E}_r = -\frac{N}{4\beta_g^2}\frac{\partial a^2}{\partial r} \tag{8}$$

$$\frac{\partial^2 \tilde{E}_z}{\partial \xi^2} + N\tilde{E}_z = -\frac{N}{4\beta_g^2}(\frac{\partial a^2}{\partial \xi} + \frac{\partial a^2}{\partial \zeta}) \tag{9}$$

Where $\tilde{E}_{r,z} = \frac{eE_{r,z}}{m_e v_g \omega_p}$ are the normalized axial and radial components of the wakefield ($\omega_p = \sqrt{4\pi e^2 n_p / m_e}$ is the plasma frequency), $N = n_e / n_p$ is the normalized plasma density.

If charged particles are trapped in the appropriate position in the wakefield the axial and radial forces accelerate and focus charge particles in the laser wakefield, respectively.

3. Numerical Results and Discussion

3.1 Periodical Chirp

The transverse electric field component of the chirped laser pulse can be described by

$$E_x(\xi-\xi_0)=E_{0x}Cos\left[\frac{\omega(\xi)}{v_p}(\xi-\xi_0)\right]Exp\left(-\frac{(\xi-\xi_0)^2}{\sigma_z^2}\right)Exp\left(-\frac{r^2}{\sigma_r^2}\right) \quad , \tag{10}$$

Where, $\omega(\xi)$ is an arbitrary chirp function. Usually, linear chirp, $\omega(\xi)=\omega_0+b(\xi-\xi_0)$ have been used in previous works, where b is the linear chirp parameter (Schroeder et al., 2003; Tóth et al., 2003; Khachtryan et al., 2005; Mirzanejhad et al., 2010). We use periodical function for the chirp relationship, $\omega(\xi)=\omega_0+d\omega\ Cos(\frac{\kappa(\xi-\xi_0)}{\sigma_z}+\varphi_0)$ where $d\omega$ is the periodical chirp parameter and $\omega_0+d\omega$ is the local frequency at the pulse center. κ and φ_0 are constant parameters, in this case the frequency changes periodically with the wavelength $2\pi\sigma_z/\kappa$.

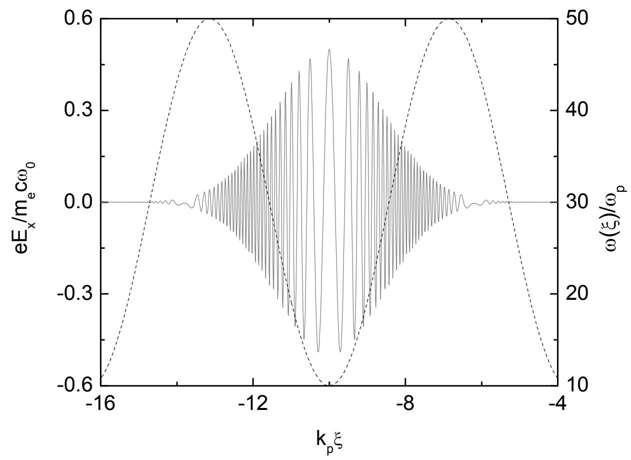

Figure 1. The normalized electric field amplitude and the normalized local frequency variation of the periodical chirp pulse is shown as a function of $k_p\xi$, for $d\omega/\omega_p=-20$, $eE_{0x}/m_ec\omega_0=0.5$, $k_p\xi_0=-10$

In Figure 1, the normalized electric field amplitude and the normalized local frequency variation of this pulse are shown as a function of $k_p\xi$, for $d\omega/\omega_p=-20$.

The vector potential of the chirped laser pulse is derived as

$$A(\xi)=\int_0^{\xi}E(\xi'-\xi_0)d\xi' \quad . \tag{11}$$

In next section wakefield amplitude is compared for linear and periodical chirp, numerically.

3.2 Wakefield Generation

In this section generation of the 2D linear wakefield is investigated, for the linear and periodical chirped laser pulse. In all figures, we assume a laser pulse with an intensity of $I_0\approx3.4\times10^{17}\ W/cm^2$ ($a_0=0.5$) which is centered at $k_p\xi_0=-10$ and density of plasma is $n_p=1.2\times10^{18}cm^{-3}$.

Equations 8 and 9 are solved numerically for linear and periodically chirp pulse. In Figure 2a and 2b the longitudinal wakefield amplitude on the axis ($r=0$) and transvers wakefield amplitude on the ($r=\sigma_r$) are shown for the periodical chirp ($d\omega/\omega_p=-20$), linear chirp ($b/\omega_pk_p=-3$) and unchirped laser pulses. It is seen that the periodically chirp can increase wakefield amplitude considerably and growth the wakefield amplitude for the linear chirp is very small, also shifted slightly compare with unchirped case.

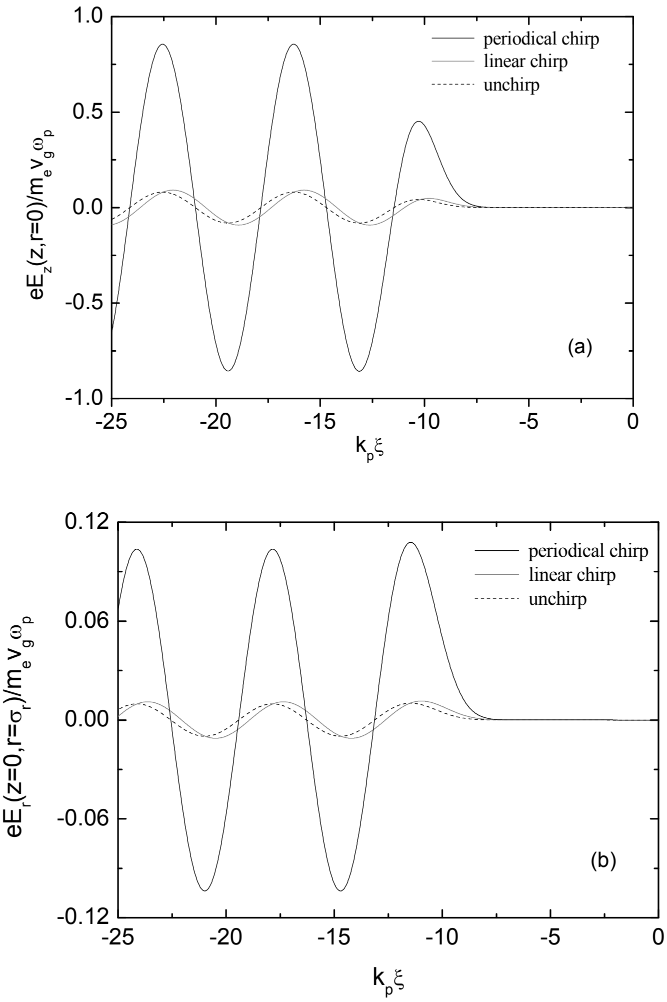

Figure 2. The normalized (a) longitudinal $(eE_z(z,r=0)/m_e v_g \omega_p)$ and (b) transverse $(eE_r(z=0,r=\sigma_r)/m_e v_g \omega_p)$ components of the wakefield versus the $k_p\xi$, for the periodical chirped laser pulse with $d\omega/\omega_p = -20$ (black solid line), linear chirped laser pulse with $b/\omega_p k_p = -3$ (gray solid line), and the constant frequency laser pulse (dash line)

Due to increasing amplitude of the longitudinal and transverse electric fields via periodic chirp rather than linear chirp, it seems the periodic chirp suitable candidate for high energy electron bunch generation. More, we will investigate efficient parameters of the periodic chirp pulse to generate the wakefield.

An effective parameter governing on the wakefield amplitude is the chirp parameter. The normalized longitudinal electric field of the wakefield in term of the different chirp parameters for periodic chirp pulse is shown in Figure 3. It is clear from this figure that the suitable value is $d\omega/\omega_p = -20$.

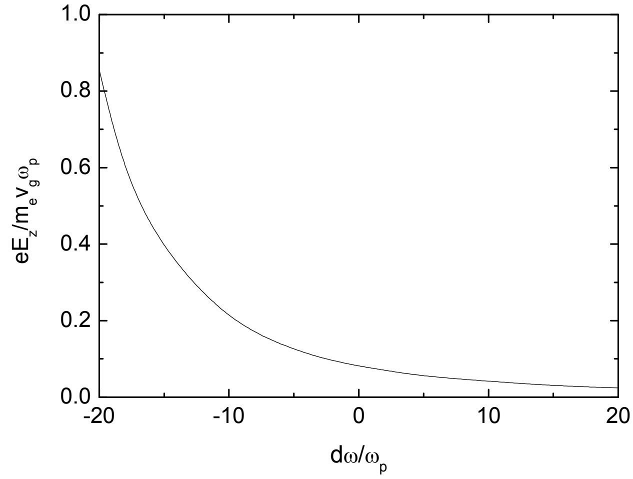

Figure. 3. Normalized longitudinal wakefield amplitude vs the normalized periodically chirp parameter

Another important parameter for increment the wakefield amplitude is the pulse length. Effect of this parameter on the acceleration field amplitude is shown in Figure (4). Clearly, the optimum value is $k_p\sigma_z \approx 3$, which is in agreement with theoretical results ($\sigma_z \approx \lambda_p / 2$) (Esarey et al., 2009). Also the optimum value of κ and φ_0 correspond to 1 and 0, respectively.

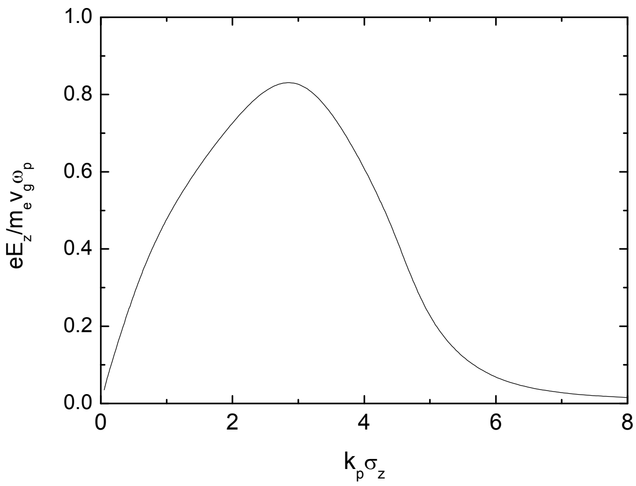

Figure 4. Normalized longitudinal wakefield amplitude vs the normalized pulse length

In Figure 5, the normalized longitudinal and transverse fields distribution E_z and E_r are plotted. We know that there are regions in the wakefield that overlap occurs between accelerating region and focusing region. Thus, a trapped electron in these regions can be accelerated to the ultra relativistic energies while the focusing force keeps the electron near the wakefield axis. Clearly, the spread of overlap region is about $\lambda_p / 4$, where shows by white rectangle in this figure.

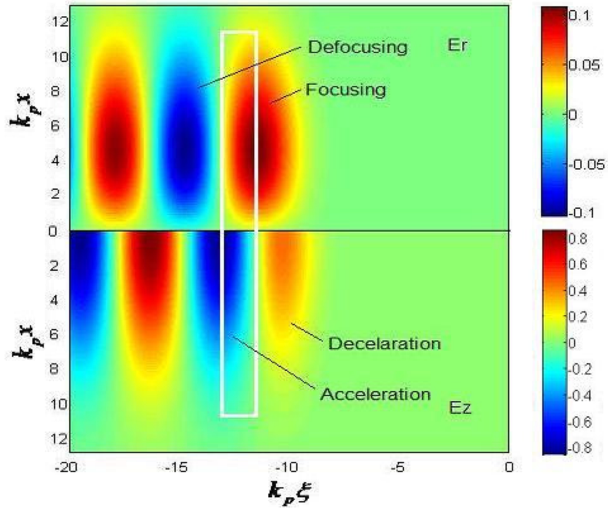

Figure 5. The normalized longitudinal and transverse fields distribution E_z and E_r versus the $k_p\xi$

In the next section we will inject an electron bunch in front of the laser pulse. In this scheme electrons accelerate to the ultra-relativistic velocities in the first cycle of the wakefield behind the laser pulse (Khachtryan, 2001).

3.3 Electron Bunch Acceleration

The normalized relativistic equation of motion for an electron is

$$\frac{d\tilde{\vec{P}}}{d\tau} = -\beta_g\left(\tilde{\vec{E}} + \vec{\beta}\times\tilde{\vec{B}}\right) - \frac{\nabla a^2}{4\beta_g\gamma}, \tag{12}$$

Where $\widetilde{P} = P/m_e c$ and $\vec{\beta} = \vec{v}/c$ are the normalized momentum and velocity of the electron, $\tilde{\vec{E}}$ and $\vec{B}$ are normalized electric and magnetic fields of the wake, respectively. $\beta_g = v_g/c$ and $\gamma = (1+\tilde{\vec{P}}^2 + a^2/2)^{1/2}$ is relativistic factor of the electron. The last term of right-hand-side represents the ponderomotive force (Quesnel & Mora, 1998). For convenience, we have introduced the dimensionless time $\tau = \omega_p t$, and we study the dynamic of electrons in laser wakefield in Cartesian coordinate.

We decompose Equation (12) into its three components:

$$\begin{aligned}
\frac{d\beta_x}{d\tau} &= -\frac{\beta_g}{\gamma}\left\{\left[(\frac{x}{r})(1-\beta_x^2) - (\frac{y}{r})\beta_x\beta_y\right]E_r - \beta_x\beta_z E_z\right\} + \frac{1}{4\beta_g^2\gamma^2}\left(\beta_x\frac{\partial a^2}{\partial\xi} - \frac{1}{\beta_g}(\frac{x}{r})\frac{\partial a^2}{\partial r}\right),\\
\frac{d\beta_y}{d\tau} &= -\frac{\beta_g}{\gamma}\left\{\left[(\frac{y}{r})(1-\beta_y^2) - (\frac{x}{r})\beta_x\beta_y\right]E_r - \beta_y\beta_z E_z\right\} + \frac{1}{4\beta_g^2\gamma^2}\left(\beta_y\frac{\partial a^2}{\partial\xi} - \frac{1}{\beta_g}(\frac{y}{r})\frac{\partial a^2}{\partial r}\right),\\
\frac{d\beta_z}{d\tau} &= -\frac{\beta_g}{\gamma}\left\{\left[(\frac{x}{r})\beta_x - (\frac{y}{r})\beta_y\right](-\beta_z E_r) + (1-\beta_z^2)E_z\right\} - \frac{1-\beta_g\beta_z}{4\beta_g^2\gamma^2}\frac{\partial a^2}{\partial\xi}.
\end{aligned} \tag{13}$$

The spatial component can be found from $dx/d\tau = \beta_x/\beta_g$, $dy/d\tau = \beta_y/\beta_g$ and $dz/d\tau = \beta_z/\beta_g$.

Equation (13) are coupled ordinary differential equations. We solve them numerically by the forth order Range-Kutta method and the results are discussed as follows.

In this simulation we choose a bunch with Gaussian distribution in both coordinates and velocities which consist of 10^5 electrons. The initial bunch energy spread is 5% and its emittance is $1\,mm\,mrad$. The plasma electron density is $n_p = 1.2\times10^{18}\,cm^{-3}$ which corresponds to a plasma wavelength of $\lambda_p = 30\mu m$. Moreover, we use a Gaussian laser pulse with $\lambda = 1\mu m$, $I \approx 3.4\times10^{17}\,W/cm^2$, ($a_0 = 0.5$), $\tau_p \approx 32\,fs$ ($\sigma_z \approx 9.5\ \mu m$), $\sigma_r \approx 47\mu m$ and periodic chirp parameter is $d\omega/\omega_p = -20$. Initial position of the laser pulse is $k_p\xi_0 = -10$. We compare only periodical chirp with unchirp laser pulse, because the final results of the linear chirp are not suitable.

More, we examine injection of electron bunch with mean energy about 2MeV ($\gamma = 5$) in front of the laser pulse. The initial position of the bunch is $k_p\xi = -3$. Average energy of electrons for the periodical chirp (soiled line) and unchirp laser pulse (dashed line) are shown in Fig. (6) in terms of $k_p z$. Clearly, periodical chirp increases mean energy of the electrons to 0.9GeV after 17 mm, i.e., ~45 GeV/m acceleration gradient, whereas the maximum energy of electron for unchirped case arrived about 100MeV.

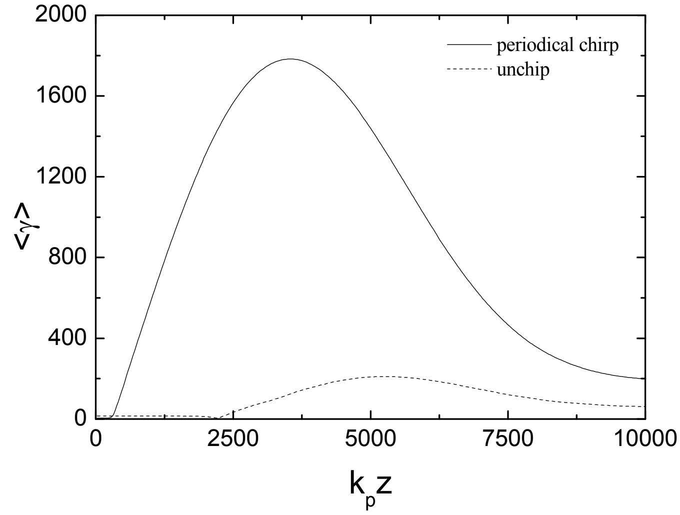

Figure 6. Average energy of electrons for injection in front of the periodical chirp (soiled line) and unchirp laser pulse (dashed line) as a function of $k_p z$

Initial and final structures of electron bunch before and after acceleration are plotted in Figures 7a and 7b, respectively. In this case 46% of the electrons are trapped and accelerated. The emittance is important quantity of an electron bunch. This is a measure of the divergence of the bunch. The final emmitance of electron bunch is 0.12 mm mrad and its final energy spread is 1%.

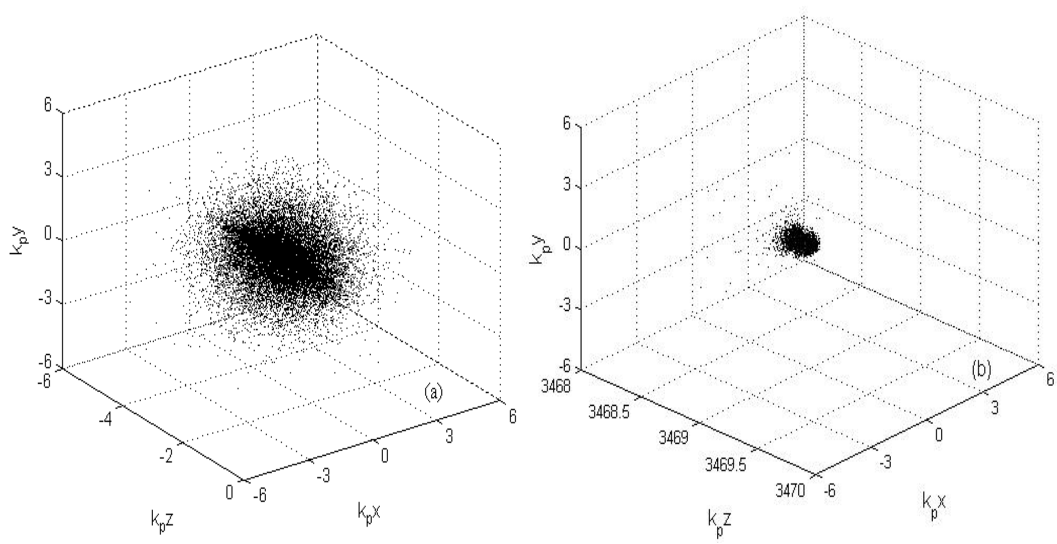

Figure 7. Electron bunch spatial distribution (a) before acceleration in $k_p z = -3$, (b) after acceleration in $k_p z \approx 3469$

4. Conclusions

We have studied the effect of the frequency variation of the laser pulse on the 2D linear laser wakefield acceleration. It was indicated that the effect of the periodically frequency variation profile is better than conventional linear chirp and unchirped case. We trust that employing the appropriate negative periodical chirp can excite a strong wakefield (about one order of magnitude) toward linear chirp and unchirped case.

Finally we tested electron bunch acceleration by the periodical chirp laser wake field with numerical simulation. Results of this simulation indicate accelerating gradient of the electron beam is about 45GeV/m with an energy spread about 1% and the emmitance 0.12mm mrad, with the help of periodical frequency variation laser pulse.

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References

Andreev, N. E., Gorbunov, L. M., Kirsanov, V. I., Nakajima, K., & Ogata, A. (1997). Structure of the wake field in plasma channels. *Phys. Plasmas, 4*, 1145. http://dx.doi.org/10.1063/1.872186

Bingham, R. (2006). Basic concepts in plasma accelerators. *Phil. Trans. R. Soc. A., 364*, 559. http://dx.doi.org/10.1098/rsta.2005.1722

Blumenfeld, I., Clayton, C. E., Decker, F. J., Hogan, M. J., Huang, C., Ischebeck, R., ... Zhou, M. (2007). Energy doubling of 42GeV electrons in a metre-scale plasma wakefield accelerator. *Nature. 445*, 741. http://dx.doi.org/10.1038/nature05538

Danson, C. N., Brummitt, P. A., Clarke, R. J., Collier, J. L., Fell, B., Frackiewicz, A. J., ... Wyborn, B. E. (2004). Vulcan Petawatt an ultra high intensity interaction facility. *Nuclear Fusion, 44*, 239. http://dx.doi.org/10.1088/0029-5515/44/12/S15

Esarey, E., Schroeder, C. B., & Leemans, W. P. (2009). Physics of laser-driven plasma-based electron accelerators. *Rev. Mod Phys., 81*, 0034. http://link.aps.org/doi/10.1103/RevModPhys.81.1229

Gupta, D. N. & Suk, H. (2006). Combined role of frequency variation and magnetic field on laser electron acceleration. *Phys. Plasmas, 13*, 013105. http://dx.doi.org/10.1063/1.2164809

Kalmykov, S. Yu., Gorbunov, L. M., Mora, P., & Shvets, G. (2006). Injection, trapping, and acceleration of electrons in a three-dimensional nonlinear laser wakefield. *Phys. Plasmas, 13*, 113102. http://dx.doi.org/10.1063/1.2363172

Khachatryan, A. G., Van Goor, F. A., Verschuur, J. W. J., & Boller, K. J. (2005). Effect of frequency variation on electromagnetic pulse interaction with charges and plasma. *Phys. Plasmas, 12*, 062116. http://dx.doi.org/10.1063/1.1938167

Khachatryan, A. G., Irman, A., Vangoor, F. A., & Boller, K. J. (2007). Femtosecond electron-bunch dynamics in laser wakefields and vacuum. *Phys. Rev. STAccel. Beams, 10*, 121301. http://link.aps.org/doi/10.1103/PhysRevSTAB.10.121301

Khachatryan, A. G. (2001). Trapping, compression, and acceleration of an electron beam by the laser wake wave. *JETP. Lett., 74*, 371. http://dx.doi.org/10.1134/1.1427124

Khachatryan, A. G. (2002). Trapping, compression, and acceleration of an electron bunch in the nonlinear laser wakefield. *Phys. Rev. E., 65*, 046504. ttp://link.aps.org/doi/10.1103/PhysRevE.65.046504

Malka, V., Faure, J., Gauduel, Y. A., Lefebvre, E., Rousse, A., & Phuoc, K. T. (2008). Principles and applications of compact laser-plasma accelerators. *Nature Physics, 4*, 447. http://dx.doi.org/10.1038/nphys966;

Martins, S. F., Fonseca, R. A., Lu, W., Mori, W. B., & Silva, L. O. (2010). Exploring laser-wakefield accelerator regimes for near-term lasers using particle-in-cell simulation in Lorentz boosted frames. *Nature Phys., 6*, 311. http://dx.doi.org/10.1038/nphys1538

Mirzanejhad, S., Sohbatzadeh, F., Asri, M., & Ghanbari, K. (2010). Quasimonoenergetic GeV electron bunch generation by the wake-field of the chirped laser pulse. *Phys. Plasmas, 17*, 033103. http://dx.doi.org/10.1063/1.3339908

Mo, M. Z. , Ali, A., Fourmaux, S., Lassonde, P., Kieffer, J. C., & Fedosejevs, R. (2012). Quasimonoenergetic electron beams from laser wakefield acceleration in pure nitrogen. *Appl. Phys. Lett., 100*, 074101. http://dx.doi.org/10.1063/1.3685464

Quesnel, B., & Mora, P. (1998). Theory and simulation of the interaction of ultraintense laser pulses with electrons in vacuum. *Phys. Rev. E., 58*, 3719. http://link.aps.org/doi/10.1103/PhysRevE.58.3719

Schroeder, C. B., Esarey, E., Shadwick, B. A., & Leemans, W. P. (2003). Raman forward scattering of chirped laser pulses. *Phys. Plasmas, 10*, 285. http://dx.doi.org/10.1063/1.1528901

Schroeder, C. B., Esarey, E., Geddes, C. G. R., Toth, Cs., Shadwick, B. A., Tilborg J. V., ... Leemans, W. P. (2003). Frequency chirp and pulse shape effects in self-modulated laser wakefield accelerators. *Phys. Plasmas, 10*, 2039. http://dx.doi.org/10.1063/1.1560614

Singh, K. P., & Sajal, V. (2009). Quasimonoenergic collimated electrons from the ionization of nitrogen by a chirped intense laser pulse. *Phys. Plasmas, 16*, 043113. http://dx.doi.org/10.1063/1.3116646

Sohbatzadeh, F., Mirzanejhad, S., & Ghasemi, M. (2006). Electron acceleration by a chirped Gaussian laser pulse in vacuum. *Phys. Plasmas, 13*, 123108. http://dx.doi.org/10.1063/1.2405345

Sohbatzadeh, F., Mirzanejhad, S., & Aku, H. (2009). Synchronization scheme in electron vacuum acceleration by a chirped Gaussian laser pulse. *Phys. Plasmas, 16*, 023106. http://dx.doi.org/10.1063/1.3077666

Sprangle, P., Esarey, E., Ting, A., & Joyce, G. (1988). Laser wakefield acceleration and relativistic optical guiding. *Appl. Phys. Lett., 53*, 2146. http://dx.doi.org/10.1063/1.100300

Sprangle, P., Esarey, E., & Ting, A. (1990). Nonlinear theory of intense laser-plasma interactions. *Phys. Rev. Lett., 64*, 2011. http://link.aps.org/doi/10.1103/PhysRevLett.64.2011

Tajima, T., & Dawson, J. M. (1979). Laser Electron Accelerator. *Phys. Rev. Lett., 43*, 267 http://dx.doi.org/10.1103/PhysRevLett.43.267

Tóth, Cs., Faure, J., Tilborg, J. V., Geddes, C. G. R., Schroeder, C. B., Esarey, E., & Leemans, W. P. (2003). Tuning of laser pulse shapes in grating-based compressors for optimal electron acceleration in plasmas. *Opt. Lett., 28*, 1823. http://dx.doi.org/10.1364/OL.28.001823