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On the Independence of KVL

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Abstract

Analyze the limitations of several methods to select cycle circuit, put forward a method to select independent cycle circuit freely, namely, to judge with symmetrical difference.

Keywords: Kirchhoff's law, Independent cycle circuit, Symmetrical difference

1. Introduction

For the complicated linear circuit, we can set out current equation and voltage equation to find the solution according to Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL). To a circuit with n knots and p branches, there are n-1 independent knot current equations, we can select any n-1 knots, and there are m(m=p-n+1) independent circuits, to which we can set out m independent cycle circuit voltage equations, therefore, get p independent equations and work out the current values of all branches. However, errors often occur when choosing independent cycle circuits from many cycle circuits to set out independent voltage equations. The phenomenon is researched in lots of literature. The method to judge the independence of circuit according to principle of symmetrical difference is put forward in this thesis.

2. Limitations of several rules to select cycle circuits

To a specific complicated circuit with n knots and p branches, the number of its independent equations is definite, namely, m=p-n+1. As for how to select m independent cycle circuits to set out these m independent equations, references (Li, Kemin, 2007; Cheng, Shouzhu, 2005; Liang, Chanbin, 1980) summed up some rules, but all these rules have limitations, some even have defects. The defects of rules in references (Cheng, Shouzhu, 2005; Liang, Chanbin, 1980) have been analyzed in reference (Li, Kemin, 2007), so it will not be discussed in this thesis. Two methods to select cycle circuit are introduced in references (Li, Kemin, 2007), one is method of single-chain cycle circuit, which needs the knowledge of topology; the other is the method of plane net hole, for the plane network, we can select all the net hole and set out a KVL equation for every net hole, which will constitute a group of independent equation. But it only fits plane network. For the plane network, we can also select in this way, namely, begin with a net hole, make the cycle circuit include a new net hole in the old net hole every time, then get m independent equations. The second method which to select cycle circuit conform to this rule: "In the newly select circuits, at least there is a branch which never appeared in the select circuit." But the rule is only the sufficient, not the necessary condition to make the select cycle circuit independent, if rigidly adhere to the rule, you can not select enough cycle circuit in many cases.

For example, for the circuit in the Figure 1(components are omitted), firstly, select cycle circuit *ADBECFA*, then select cycle circuit *DEFD*, the two cycle circuits include all the branches, henceforth, no matter how to select, new branch will not occur. But it still needs two equations to work out the current values of all branches.

The method of plane net hole does not fit three-dimensional network. The method of the single chain cycle circuit is too complicated in use. In fact, to a complicated circuit, there are lots of selections to set out KVL equations, as long as the listed *m* equations are independent to one another. The method of plane net hole and single-chain cycle circuit set way to select independent cycle circuit in prescribed order, lacking flexibility. The following is a method to judge the independence of selected cycle circuits, which will improve the flexibility to select cycle circuit.

3. Set out KVL equations by judging with symmetrical difference

Symmetrical difference is a kind of operation of set, often indicated with "⊿". The symmetrical difference of two sets (*A* and *B*) is defined as: A⊿B=A∪B-A∩B (Kuratowski, kazimierz & Mostowski, Andraej, 1976), and the operation of symmetrical difference conforms to commutative law and associative law. The simple operation of the symmetrical difference of a number of sets is put forward in reference (Deng, Shude, 2000), namely, if an element belongs to odd number of sets of *n* sets, then the element will belong to the symmetrical difference set of these *n* sets; whereas, if an element belongs to even number of sets of these *n* sets, then the element will not belong to the symmetrical difference set (particularly, if an element doesn't belong to any set of these *n* sets, then it doesn't belong to the symmetrical difference set too). From reference (Deng, Shude, 2000), it's easy to find the character of the operation of symmetrical difference of sets, if the symmetrical difference of *k* sets is set *C*, then these *k* sets plus set *C* is *k*+1 sets, its symmetrical difference is empty set *Φ*, whereas, if the symmetrical difference of *k* sets is*Φ*, then the symmetrical difference of any *k*-1 sets of these *k* sets is the left set.

Firstly, number the branches, for example in Figure 1, indicate branch *FAD* with 1, *DBE* with 2, and so on, just like Figure 2. Cycle circuit represents a set which is composed of branches, such as the cycle circuit *DBED*= {2, 5}, the cycle circuit *ADBECFA*= {1, 2, 3} in Figure 1. We could demonstrate that any *h* cycle circuits are dependent when the symmetrical difference of the *h* sets representing *h* cycle circuits is*Φ*; and any *j* ($2 \le j \le h$) cycle circuits which we select from *h* sets, if the symmetrical difference set of the *j* sets is not *Φ*, then the *h* cycle circuits are independent.

Therefore a method selecting independent cycle circuits is obtained.

For *m* independent cycle circuits, which is composed of *p* branches, each branch is given a number firstly, these number compose a set. Selecting any two cycle circuits, then the symmetrical difference of two cycle circuits (i.e. the symmetrical difference of two sets representing this two cycle circuits) is calculated. If the symmetrical difference set represents a cycle circuit, then the cycle circuit is dependent with the two cycle circuits, thus we can not select this cycle circuit yet.

Based on the previous principle, we select the third cycle circuit. It is calculated that the symmetrical difference of the third cycle circuit and every of previous two cycle circuits selected. And it is calculated that the symmetrical difference of the three cycle circuits too. Thus three symmetrical differences are obtained. If some sets or a set of the three symmetrical differences represent cycle circuits, then the cycle circuits are dependent with the three cycle circuits selected, thus we can not select this cycle circuits yet.

Then we select the fourth cycle circuit, and calculate the symmetrical differences of the fourth cycle circuit and every of previous three cycle circuits, the fourth cycle circuit and any two cycle circuits of previous three cycle circuits, and the four cycle circuits. Therefore the seven symmetrical differences are obtained, If some sets of the seven symmetrical differences represent cycle circuits, then the cycle circuits are dependent with the four cycle circuits selected, thus we can not select them yet.

Keep up to the *m*th cycle circuit. The *m* cycle circuits we select are independent each other, therefore *m* independent equations of KVL are obtained.

Such as *m*=4, *p*=6 in Figure 2, we select two cycle circuits *DBED*= {2, 5} and *FDEF*={4,5,6} firstly, then calculate the symmetrical difference of two cycle circuits, we have

$$\{2, 5\} ⊿ \{4.5,6\}=\{2,4,6\}.$$

{2, 4, 6} represents the cycle circuit *BDFEB*. We can not select the cycle circuit *BDFEB* later.

Second, we select the third cycle circuit, such as we select *CFEC*= {3, 6}. We have

$$\{2, 5\} ⊿ \{3,6\}=\{2,3,5,6\}, \qquad \{4,5,6\} ⊿ \{3,6\}=\{3,4,5\}$$

$$\{2, 5\} \triangle \{4,5,6\} \triangle \{3,6\}=\{2,3,4\}$$

In the three sets obtained, the set {3, 4, 5} represents the cycle circuit *CFDEC*, and the set {2, 3, 4} represents the cycle circuit *BDFCEB*. The fourth cycle circuit can not select *CFDEC* and *BDFCEB.*

Finally, we select the fourth cycle circuit. Except *CFDEC*, *BDFCEB* and *BDFEB*, any new cycle circuit can be selected. It is independent with previous three cycle circuits selected.

Obviously, Writing the KVL equations by symmetrical difference are not only flexibility, but also to judge the independence of cycle circuits

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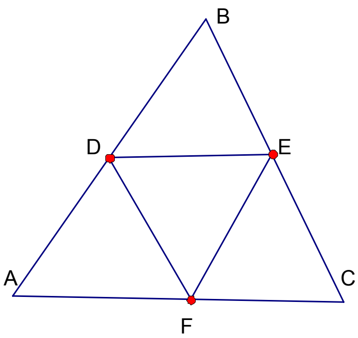

Figure 1.

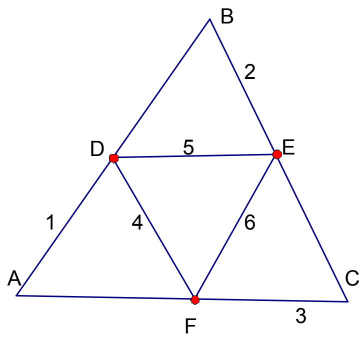

Figure 2.